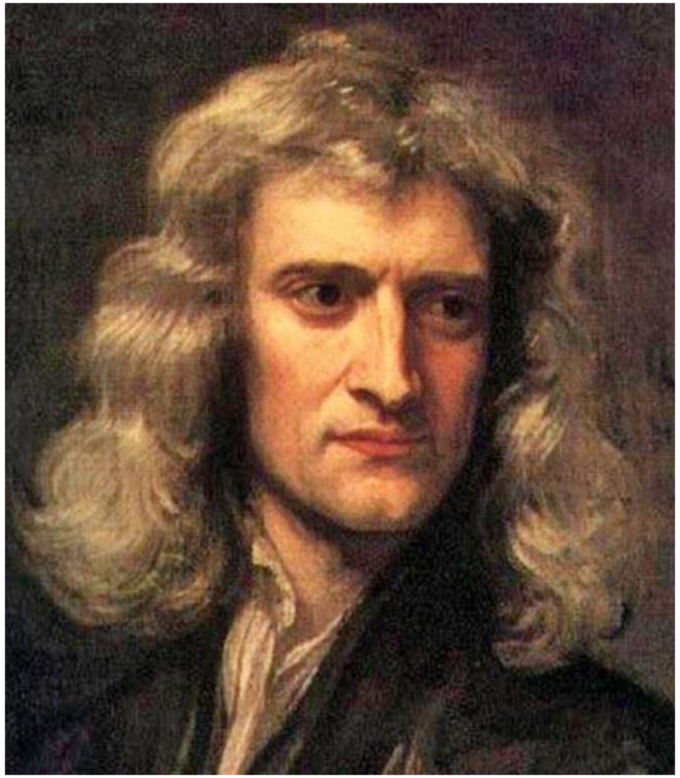
Chap.2 Supplement

For the EM Course Lectured by Prof. Tsun-Hsu Chang

Teaching Assistants: Hung-Chun Hsu, Yi-Wen Lin, and Tien-Fu Yang

2022 Fall at National Tsing Hua University

Brief History of Field theory



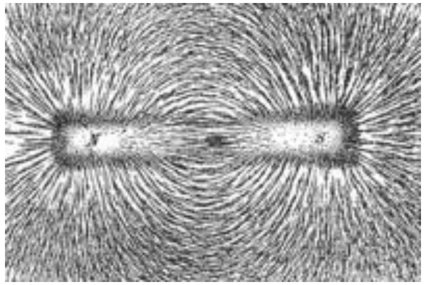
Issac Newton
(1643-1727)

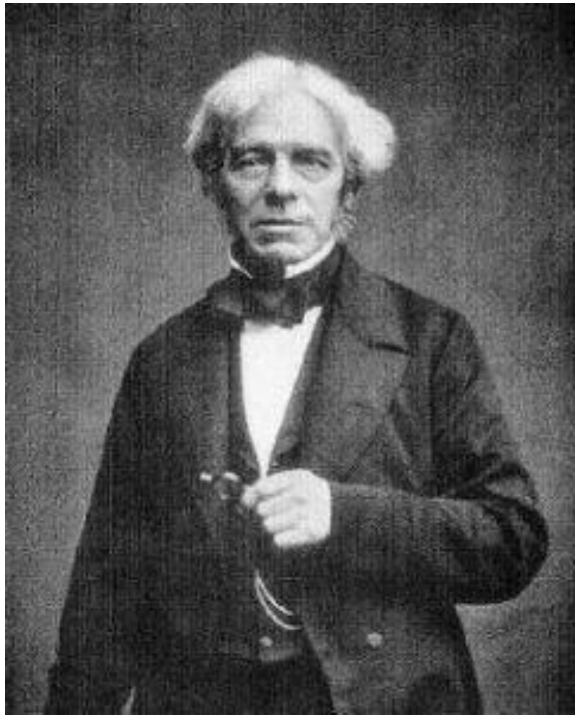
- Classical field theory can be traced back to Newton’s law of universal gravitation.
- “Action at a distance”:

  Instantaneous effect on faraway objects

- Newton couldn’t find empirical evidence to support a causal explanation of gravity, and any description remained purely hypothetical.
- Concept of fields was proposed by mathematical physicists in the 18th century as a mathematical trick without physical interpretation.

Brief History of Field theory





Michael Faraday
(1791-1867)

- Michael Faraday coined the term “field” in 1845.
- He stated
  - Interactions between objects occur via space-filling “lines of force” that can move, expand, and contract.
  - Atoms are convergences of the lines of force.
  - Non-instantaneous EM interactions are fatal to AD.
  - Magnetic lines of force are physical conditions of mere space (space without substance).

Brief History of Field theory

James Maxwell
(1831-1879)

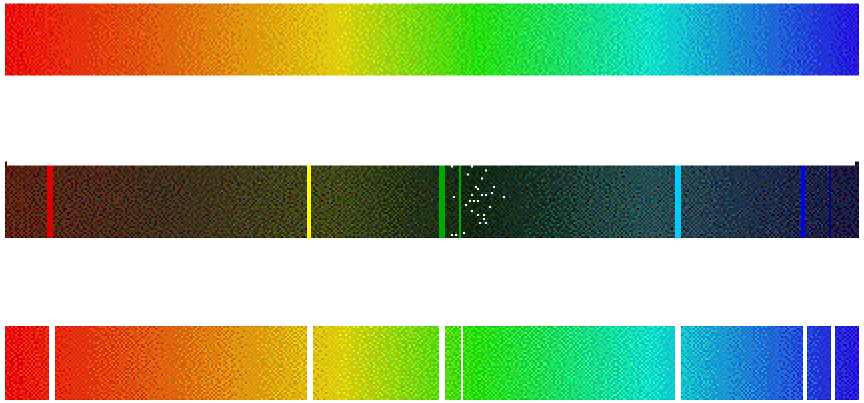
- Maxwell was more "Newtonian" and more mathematical than Faraday.
- Faraday's lines of force is the state of the ether as a velocity field is a state of a material fluid.
- His analysis led to the predictions of
  - a finite transmission time for EM actions
  - light as an EM field phenomenon
- A time delay demands the presence of energy in the intervening space to conserve energy.
- Action-at-a-distance was thus conclusively refuted.

Brief History of Field theory

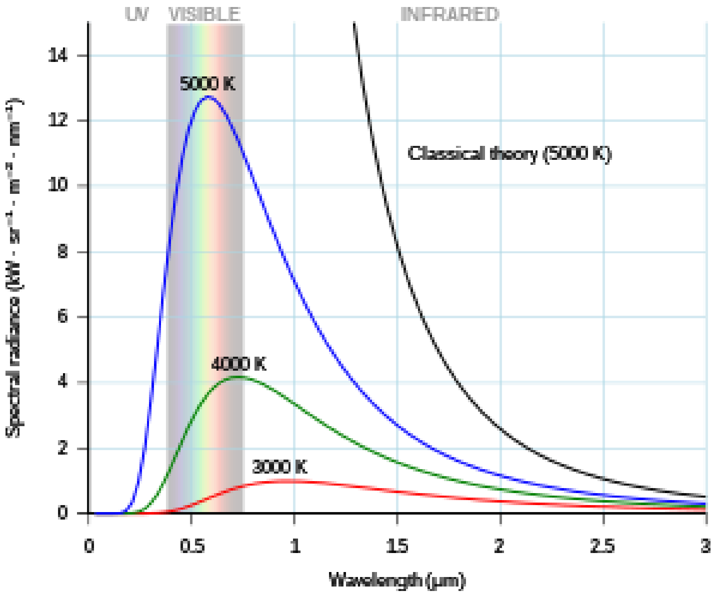

"Before Maxwell, Physical Reality was thought of as consisting in material particles … Since Maxwell's time, Physical Reality has been thought of as represented by continuous fields, … and not capable of any mechanical interpretation. This change in the conception of Reality is the most profound and fruitful that physics has experienced since the time of Newton."

— Albert Einstein —

Brief History of Field theory



Discrete lines in atomic spectra



Distribution of blackbody radiation
in different wavelengths

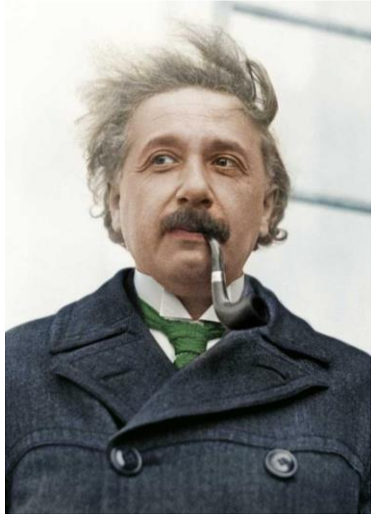
→ The beginning of quantum mechanics!

Brief History of Field theory

Max Planck (1858-1947)
Atoms discretely absorb and emit EM radiation as tiny oscillators.
→ Quantization

Louis de Broglie(1892-1987)
Microscopic particles exhibit both wave- and particle-like properties under different circumstances.
→ Wave-Particle Duality

Werner Heisenberg (1901-1976)
Matrix formulation and Law of multiplication (Commutator)
→ Creation of Quantum Mechanics (with help of Born and Jordan)



Albert Einstein (1879-1955)

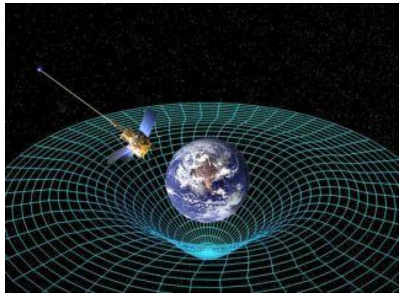
Light is composed of individual packets of energy called photons.
→ Photoelectric effect

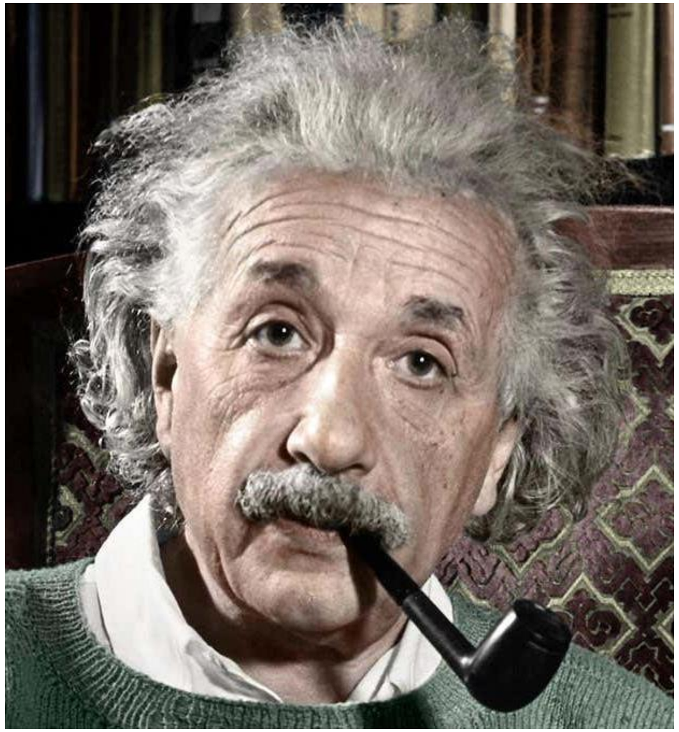
The distinction between time and space was blurred.
→Special Relativity

Two Difficulties Remained

- Schrödinger equation explains stimulated emission but not spontaneous emission.
- Schrödinger equation treats time as an ordinary number while promoting spatial coordinates to linear operators.

Brief History of Field theory





Albert Einstein
(1879-1955)

- Einstein supported a "fields are all there is" view of classical physics.

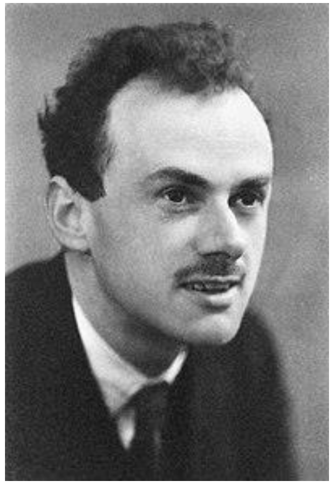
→ Fields are states of space.

- The general theory of relativity (GR) resolves Newton's dilemma about the "absurdity of gravitational Action at a distance".

$$\underbrace{R_{\mu\nu}(x)-\frac{1}{2}g_{\mu\nu}(x)R(x)+g_{\mu\nu}(x)\Lambda}_{\text{Geometry of space-time}}=\underbrace{kT_{\mu\nu}(x)}_{\text{Energy and momentum of matter}}$$

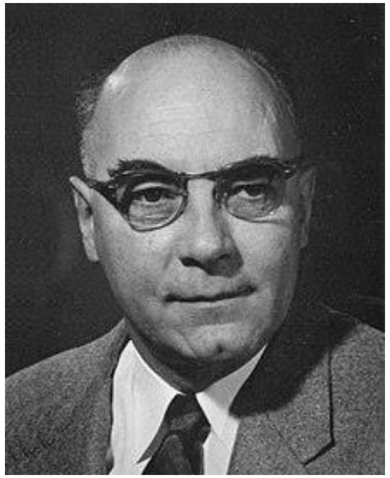
Matter tells space-time how to curve; curved space-time tells matter how to move.

Brief History of Field theory







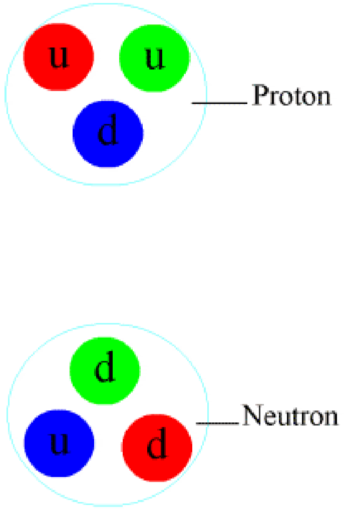




Brief History of Field theory

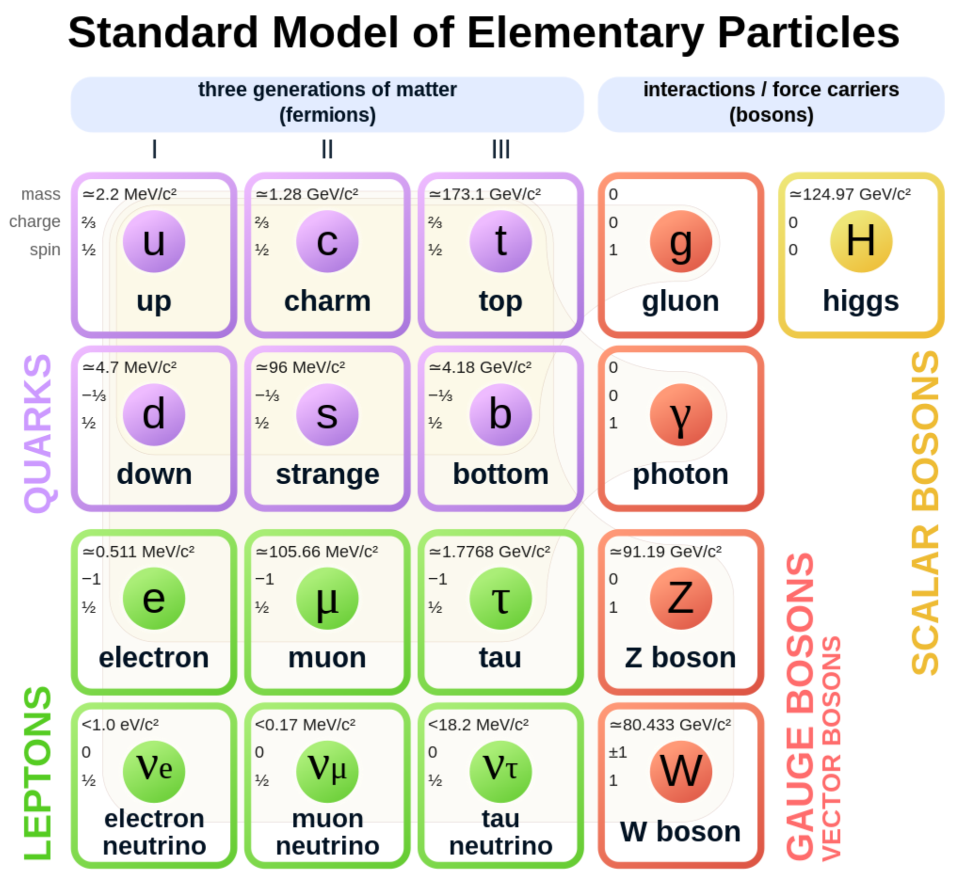
Further development of fields leads to the conclusion:

> Charges are emitting particles constantly, and forces/ interactions result from exchanging particles between charges.

Yukawa (1949 Nobel)



Brief History of Field theory



The view of gauge bosons is now the basis for “Standard Model” of elementary particles.

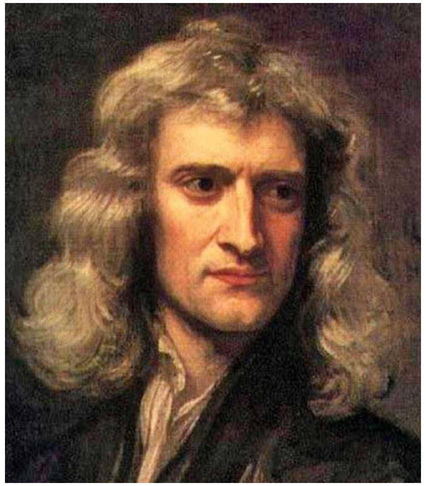
- Strong: Gluons
- EM： Photons
- Weak：W, Z bosons
- Gravitational： Graviton? (not observed yet)

Higgs bosons: the mechanism “to get mass”

延伸閱讀：上帝的粒子—希格斯粒子的發明與發現

Brief History of Field theory

From the above brief introduction, it is clear that electro-magnetism is the first example of the modern theory of interaction. That's why we need tensor/vector analysis to describe fields.

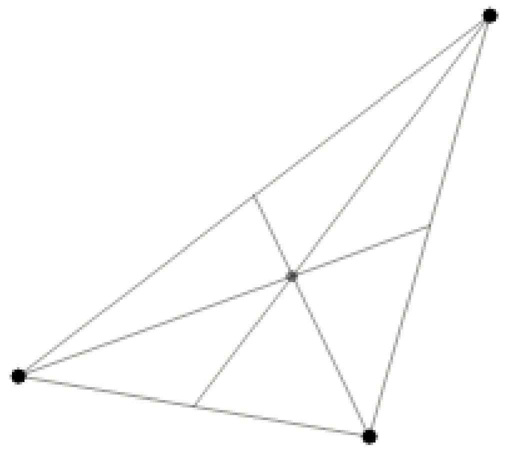


Three-body Problem

$$\ddot{\mathbf{r}}_1 = -Gm_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - Gm_3 \frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3}$$

$$\ddot{\mathbf{r}}_2 = -Gm_3 \frac{\mathbf{r}_2 - \mathbf{r}_3}{|\mathbf{r}_2 - \mathbf{r}_3|^3} - Gm_1 \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3}$$

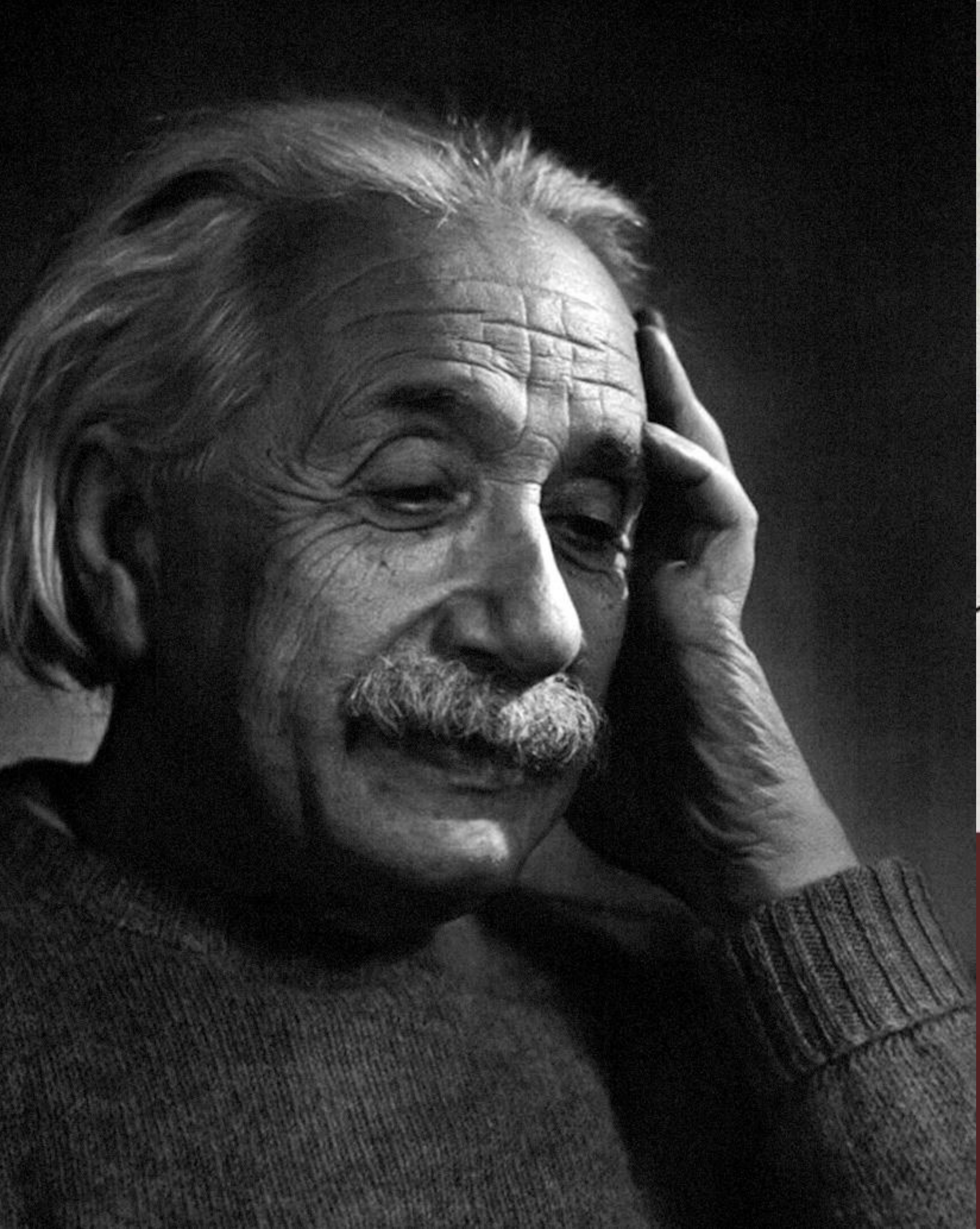
$$\ddot{\mathbf{r}}_3 = -Gm_1 \frac{\mathbf{r}_3 - \mathbf{r}_1}{|\mathbf{r}_3 - \mathbf{r}_1|^3} - Gm_2 \frac{\mathbf{r}_3 - \mathbf{r}_2}{|\mathbf{r}_3 - \mathbf{r}_2|^3}$$

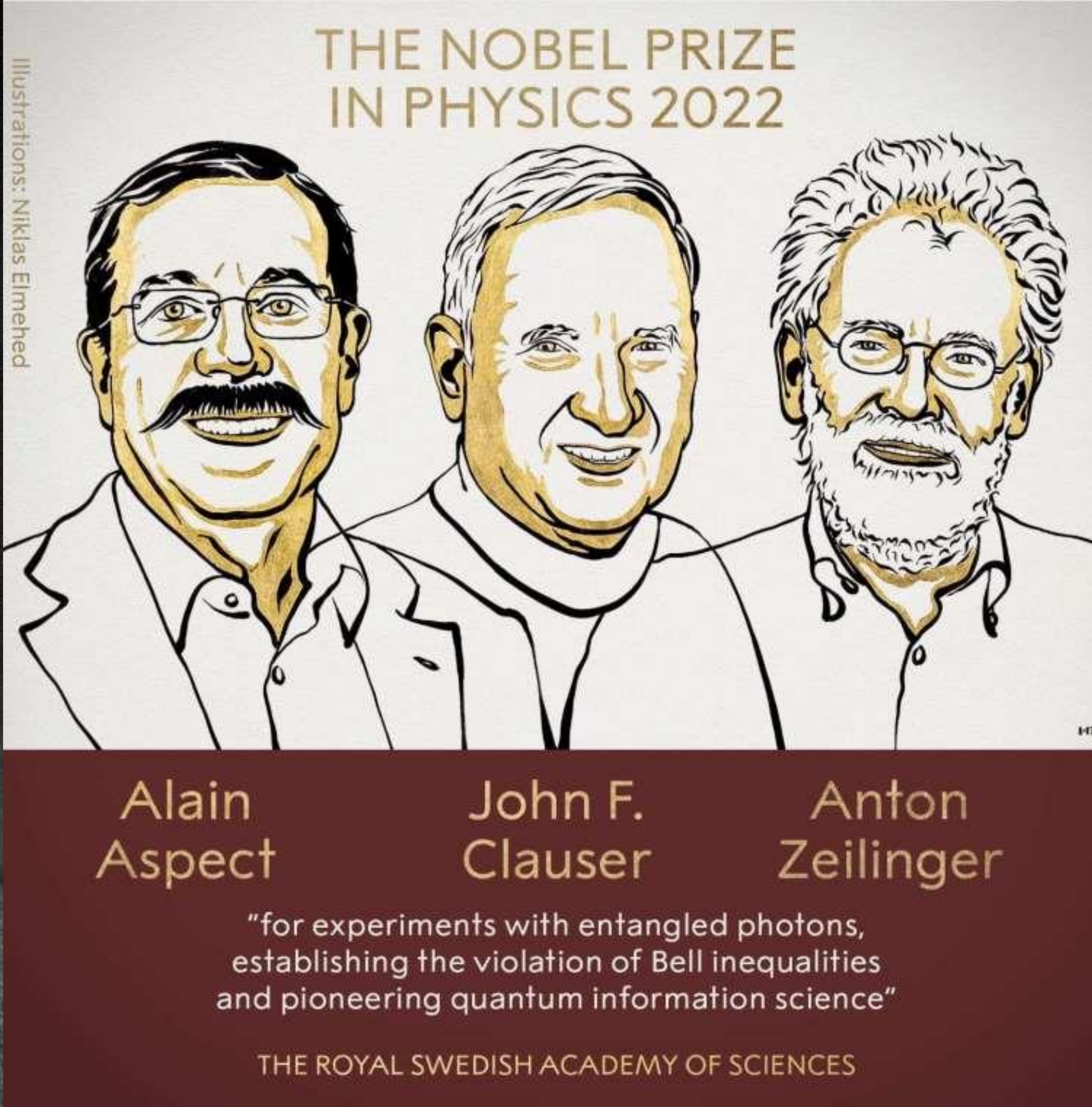


$$\mathcal{H} = -\frac{Gm_1m_2}{|\mathbf{r}_1 - \mathbf{r}_2|} - \frac{Gm_2m_3}{|\mathbf{r}_3 - \mathbf{r}_2|} - \frac{Gm_3m_1}{|\mathbf{r}_3 - \mathbf{r}_1|} + \frac{\mathbf{p}_1{}^2}{2m_1} + \frac{\mathbf{p}_2{}^2}{2m_2} + \frac{\mathbf{p}_3{}^2}{2m_3}$$

Chaos!

So… Can we find an analytic solution by “Field”?




THE NOBEL PRIZE
IN PHYSICS 2022
Illustrations: Niklas Elmehed
Alain
Aspect
John F.
Clauser
Anton
Zeilinger
"for experiments with entangled photons,
establishing the violation of Bell inequalities
and pioneering quantum information science"
THE ROYAL SWEDISH ACADEMY OF SCIENCES

Homework Exercises

Griffiths:
2.5, 2.6, *2.9, 2.10, 2.14, 2.16, 2.18, *2.20, 2.21, 2.22, 2.25, 2.27, 2.29, 2.30, 2.33, 2.34, *2.36, 2.38, *2.39, 2.42, *2.43, *2.53, 2.60

Open question: Is electric/magnetic force generally a conservative force? Why or why not?

Griffiths Prob 2.53

Problem 2.53 In a vacuum diode, electrons are "boiled" off a hot **cathode**, at potential zero, and accelerated across a gap to the **anode**, which is held at positive potential V_0. The cloud of moving electrons within the gap (called **space charge**) quickly builds up to the point where it reduces the field at the surface of the cathode to zero. From then on, a steady current I flows between the plates.

Suppose the plates are large relative to the separation ($A \gg d^2$ in Fig. 2.55), so that edge effects can be neglected. Then V, ρ, and v (the speed of the electrons) are all functions of x alone.

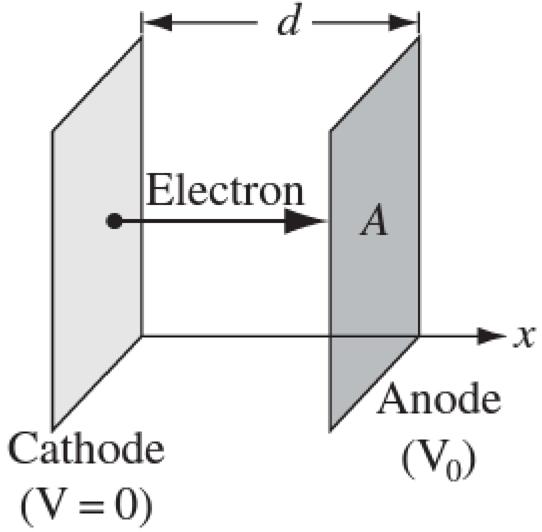
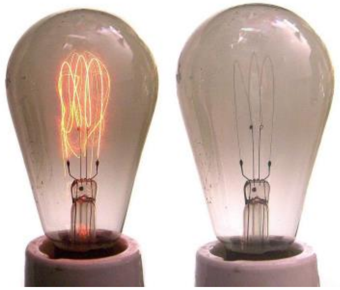
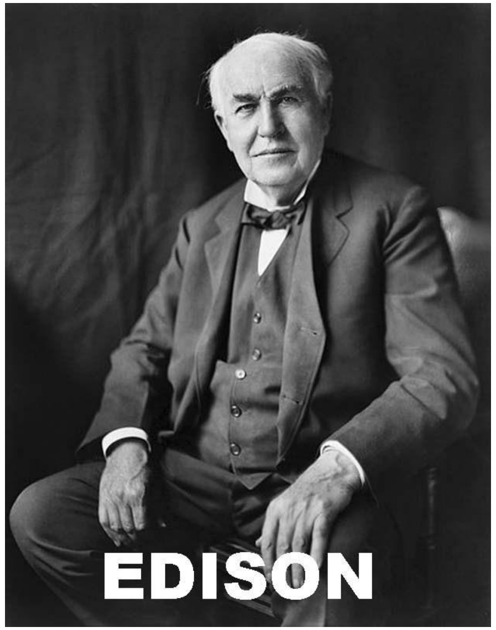
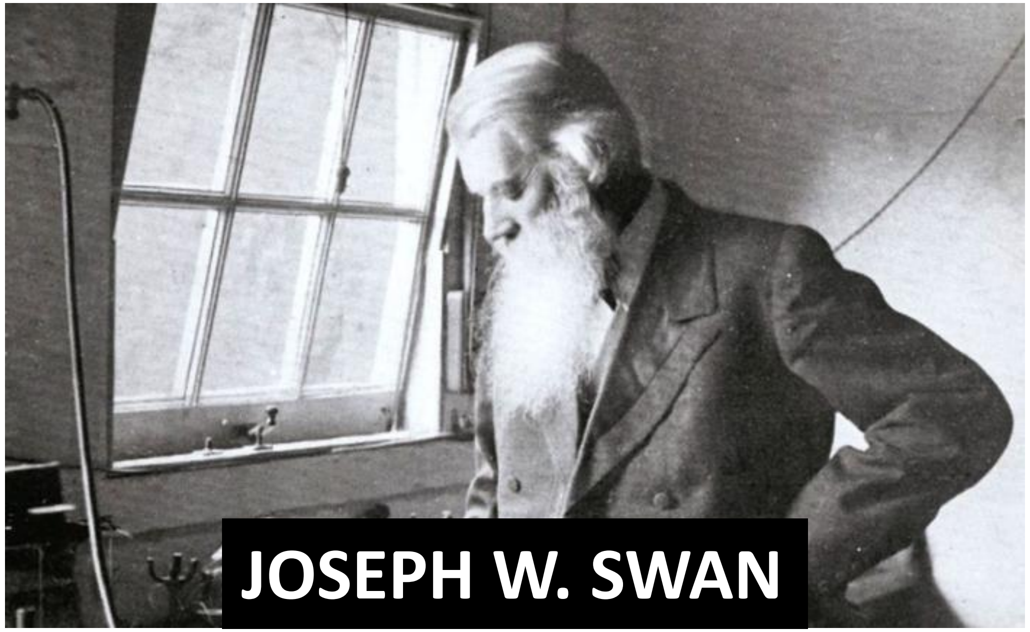


FIGURE 2.55

Inventor of Light Bulb





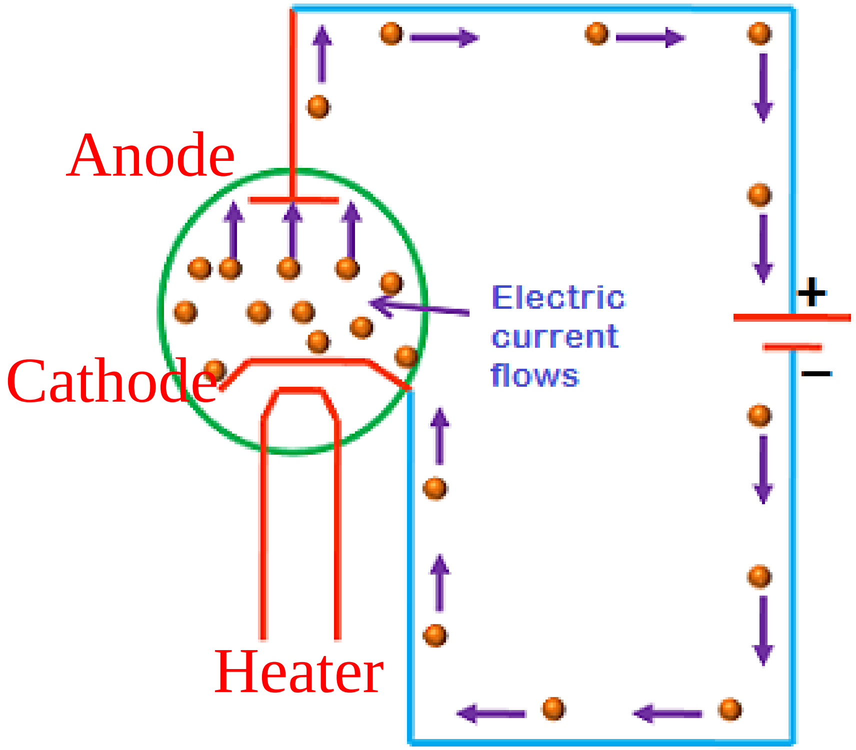


Further reading: 怪奇科學研究所：42個腦洞大開的趣味科學故事

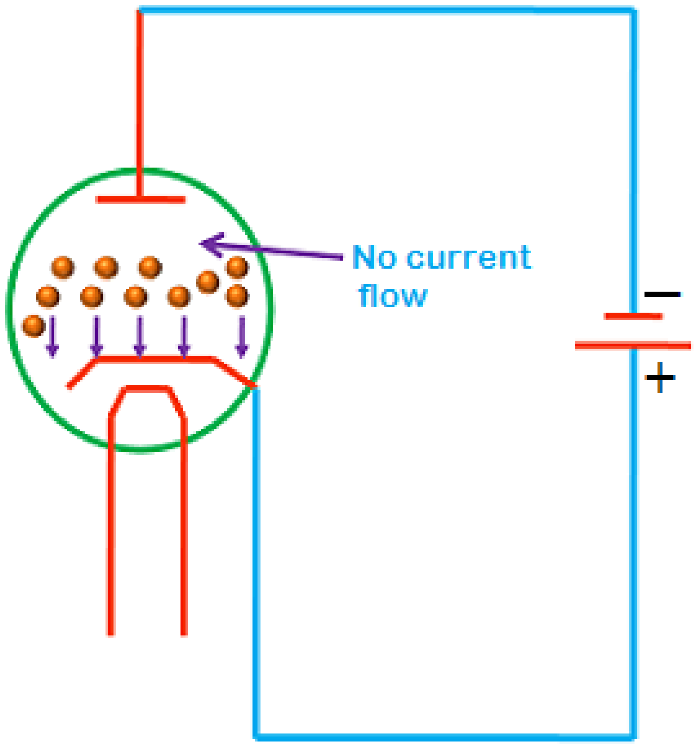
Pictor credit:
https://reurl.cc/yMq4xM

"Edison" effect

Vacuum diode with forward voltage



Vacuum diode with reverse voltage



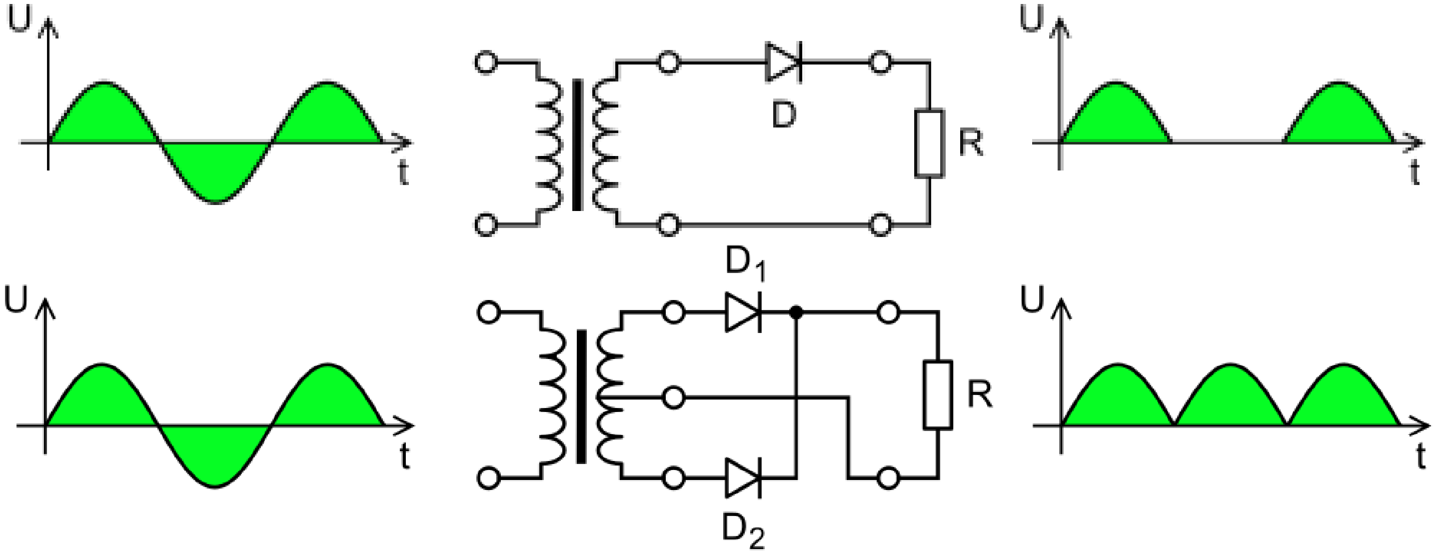
Richardson-Dushman Equation

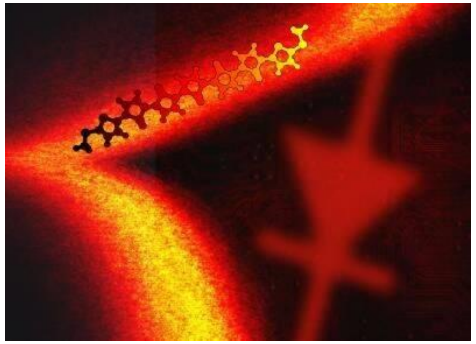
$$J = aT^2 \exp\left(-W/RT\right)$$

Edison saw no special value in this effect, but he patented it anyway.
Today we call the effect by the more descriptive term, "thermionic emission."

Further Applications

John Ambrose Fleming
(1849 ~ 1945)

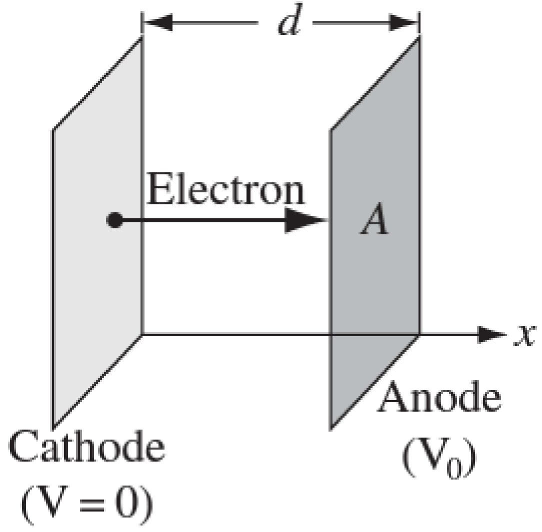




Single-molecule optoelectronic devices promise a potential solution for miniaturization and functionalization of silicon-based microelectronic circuits in the future.

Ref.: P. Li et al., *Rep. Prog. Phys.* **85**, 086401 (2022).

Griffiths Prob 2.53



Before the calculation: What is the physics? → Dimensional Analysis

Current Density $\mathbf{J} = \rho \mathbf{v} \;\leftarrow$ Two scales needed

Length: d (m), A (m^2)

Mass: m (kg)

Charge: e $(A \cdot s)$

Voltage: $V_0\ (kg \cdot m^2 \cdot s^{-3} \cdot A^{-1})$

Permittivity: $\varepsilon_0\ (kg^{-1} \cdot m^{-3} \cdot s^4 \cdot A^2)$

Velocity scale $v_s = \sqrt{eV_0/m}$

Charge Density scale $\rho_s = \varepsilon_0 V_0 / d^2$

$$\therefore J_s \sim v_s \rho_s \sim \varepsilon_0 \sqrt{\frac{e}{m}} \frac{V_0^{3/2}}{d^2}$$

Known as Child-Langmuir Law

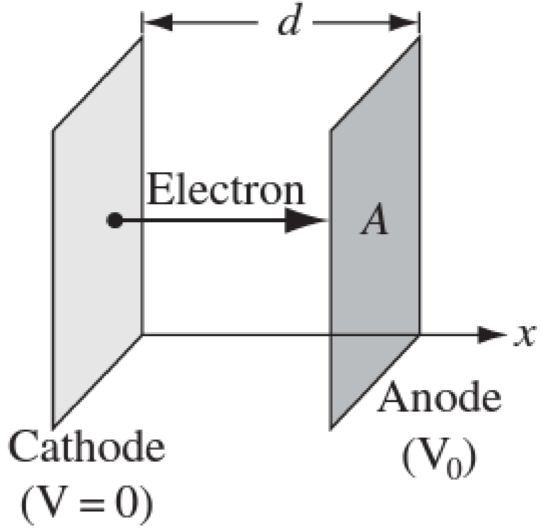
For moving electrons in solids: introduce mobility μ (response to E field)

Velocity scale $v_s = \mu \cdot V_0 / d \quad \therefore J_s \sim v_s \rho_s \sim \varepsilon_0 \mu \dfrac{V_0^2}{d^3}$ Known as Mott–Gurney law

Griffiths Prob 2.53

Velocity scale $v_s = \sqrt{eV_0/m}$

Charge Density scale $\rho_s = \varepsilon_0 V_0 / d^2$



$$J_s \sim v_s \rho_s \sim \varepsilon_0 \sqrt{\frac{e}{m}} \frac{V_0^{3/2}}{d^2} \propto \frac{[\phi(z)]^{3/2}}{d^2} \leftarrow \text{Constant for all z}$$

$$\Rightarrow \phi(z) = V_0 \left(\frac{z}{d}\right)^{4/3} \Rightarrow \rho(z) = -\varepsilon_0 \cdot \phi'' = \frac{4}{9}\varepsilon_0 \frac{V_0}{d^2}\left(\frac{z}{d}\right)^{-2/3}$$

$$\therefore J = -\rho(z) v(z) = \frac{4}{9}\varepsilon_0 \frac{V_0}{d^2}\left(\frac{z}{d}\right)^{-2/3} \sqrt{\frac{2e\phi(z)}{m}} = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2e}{m}} \frac{V_0^{3/2}}{d^2}$$

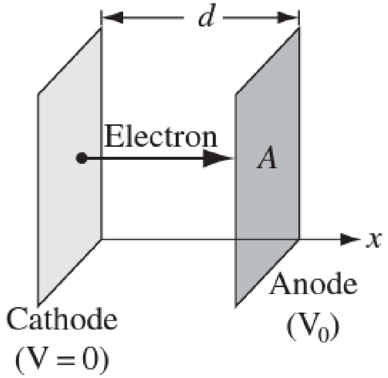
Conservation of energy

$$\frac{1}{2} m v^2(z) - e\phi(z) = \text{Const.} = 0$$

$$\Rightarrow I = JA = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2e}{m}} V_0^{3/2}$$

Ref.: R. J. Umstattd et al., *Am. J. Phys.* **73**, 160 (2005).

Griffiths Prob 2.53



Or you can solve this by brutal force……

$$\nabla^2\phi(z) = -\frac{\rho(z)}{\varepsilon_0} \qquad \frac{1}{2}mv^2(z) - e\phi(z) = 0 \Rightarrow v = \sqrt{\frac{2e\phi}{m}}$$

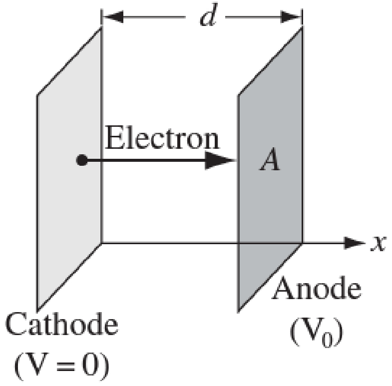
$$J(z) = \rho v \Rightarrow \rho = J\sqrt{\frac{m}{2e\phi}}$$

B.C.: $\phi(0) = \phi'(0) = 0,\ \phi(d) = V_0$

$$\phi''\phi' = -\frac{J}{\varepsilon_0}\sqrt{\frac{m}{2e}}\frac{1}{\sqrt{\phi}}\phi' \Rightarrow \int \phi' d\phi' = -\frac{J}{\varepsilon_0}\sqrt{\frac{m}{2e}}\int \phi'^{-1/2} d\phi'$$

$$\frac{1}{2}\phi'^2 = -2\frac{J}{\varepsilon_0}\sqrt{\frac{m}{2e}}\phi'^{1/2} \Rightarrow \frac{d\phi}{dz} = \ldots\ldots \Rightarrow \phi = V_0\left(\frac{z}{d}\right)^{4/3} \Rightarrow \rho(z) = -\varepsilon_0 \cdot \phi'' = \frac{4}{9}\varepsilon_0\frac{V_0}{d^2}\left(\frac{z}{d}\right)^{-2/3}$$

Griffiths Prob 2.53



$$J = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2e}{m}} \frac{V_0^{3/2}}{d^2}$$

Assumptions:

1. Steady-state in the vacuum gap
2. Single species charged particles with zero initial velocity
3. Nonrelativistic and non-quantum under electrostatic approximation.

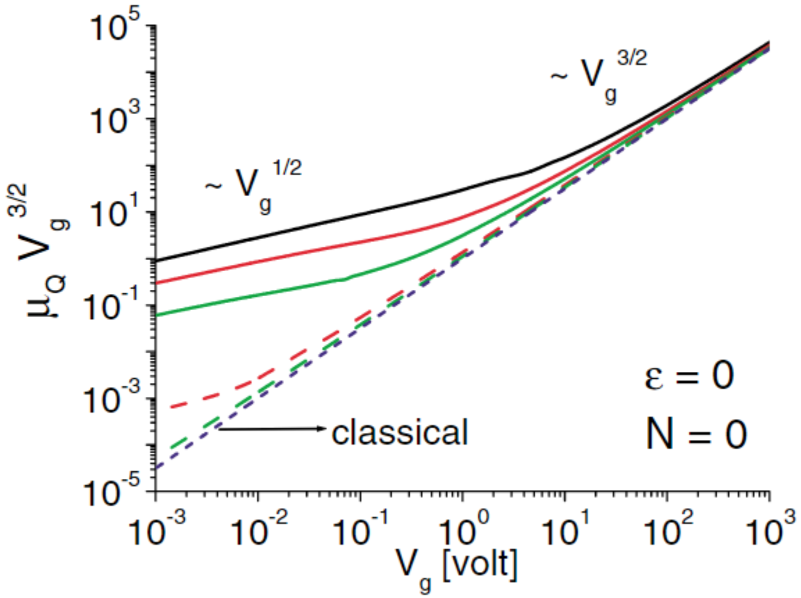


FIG. 2 (color online). The values of $\mu_Q V_g^{3/2}$ as a function of gap voltage V_g (volt) for various gap spacing $D = 1$, 10, and 100 nm (solid lines: top to bottom). The dashed lines are without an exchange-correlation term ($\phi_{xc} = 0$) for $D = 10$ and 100 nm (top to bottom), and the short-dashed line is the classical limit.

Ref.: L. K. Ang, T. J. T. Kwan, and Y. Y. Lau, *Phys. Rev. Lett.* .**91**, (2003).

Further reading

APPLIED PHYSICS REVIEWS **4**, 011304 (2017)

CrossMark

APPLIED PHYSICS REVIEWS

100 years of the physics of diodes

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The Child–Langmuir Law (CL), discovered a century ago, gives the maximum current that can be transported across a planar diode in the steady state. As a quintessential example of the impact of space charge shielding near a charged surface, it is central to the studies of high current diodes, such as high power microwave sources, vacuum microelectronics, electron and ion sources, and high current drivers used in high energy density physics experiments. CL remains a touchstone of fundamental sheath physics, including contemporary studies of nanoscale quantum diodes and nano gap based plasmonic devices. Its solid state analog is the Mott–Gurney law, governing the maximum charge injection in solids, such as organic materials and other dielectrics, which is important to energy devices, such as solar cells and light emitting diodes. This paper reviews the important advances in the physics of diodes since the discovery of CL, including virtual cathode formation and extension of CL to multiple dimensions, to the quantum regime, and to ultrafast processes. We review the influence of magnetic fields, multiple species in bipolar flow, electromagnetic and time dependent effects in both short pulse and high frequency THz limits, and single electron regimes. Transitions from various emission mechanisms (thermionic-, field-, and photoemission) to the space charge limited state (CL) will be addressed, especially highlighting the important simulation and experimental developments in selected contemporary areas of study. We stress the fundamental physical links between the physics of beams to limiting currents in other areas, such as low temperature plasmas, laser plasmas, and space propulsion.
[http://dx.doi.org/10.1063/1.4978231]

P. Zhang, Á. Valfells, L. K. Ang, J. W. Luginsland, and Y. Y. Lau, *Appl. Phys. Rev.* **4**, 011304 (2017).