Lecture 26

習題

Chapter 11 Sequences; Indeterminate forms

[Part I]

1. Find the limit.

① $\lim\limits_{\theta\to\frac{\pi}{2}} \dfrac{1-\sin\theta}{\csc\theta}$ ② $\lim\limits_{x\to 0} \dfrac{x}{\tan^{-1}(4x)}$ ③ $\lim\limits_{x\to\infty}(x^3 e^{-x^2})$ ④ $\lim\limits_{x\to 0^+}(\sin x \ln x)$

⑤ $\lim\limits_{x\to\infty}(x\tan\frac{1}{x})$. ⑥ $\lim\limits_{x\to\infty}(xe^{\frac{1}{x}}-x)$.

2. If f' is cont., $f(2)=0$ and $f'(2)=7$, evaluate $\lim\limits_{x\to 0}\dfrac{f(2+3x)+f(2+5x)}{x}$

3. If f' is cont., use l'Hospital Rule to show that $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x-h)}{2h}=f'(x)$.

[Part II]

1. Determine whether the sequence converges or diverges. If it converges find the limit.

① $a_n=\dfrac{3+5n^2}{n+n^2}$ ② $a_n=\dfrac{2^n}{3^{n+1}}$ ③ $a_n=\dfrac{(n+2)!}{n!}$ ④ $a_n=\dfrac{(-1)^n n^3}{n^3+2n^2+1}$

⑤ $a_n=\cos(\frac{2}{n})$ ⑥ $a_n=\dfrac{\cos n}{2^n}$ ⑦ $\{n\cos(n\pi)\}$ ⑧ $\{0,1,0,0,1,0,0,0,1,\cdots\}$

⑨ $a_n=\dfrac{(\ln n)^2}{n}$.

2. ⓐ If $\{a_n\}$ is convergent, show that $\lim\limits_{n\to\infty} a_{n+1}=\lim\limits_{n\to\infty} a_n$.

ⓑ A sequence $\{a_n\}$ is defined by $a_1=1$ and $a_{n+1}=\dfrac{1}{1+a_n}$, $\forall n\geq 1$. Assuming that $\{a_n\}$ is convergent, find its limit.

3. A sequence $\{a_n\}$ is given by $a_1=\sqrt{2}$, $a_{n+1}=\sqrt{2+a_n}$.

ⓐ By induction, show that $\{a_n\}$ is increasing and bounded above by 3.

ⓑ Show that $\lim\limits_{n\to\infty} a_n$ exists and find its limit.

4. ⓐ Show that the sequence defined by $a_1=2$, $a_{n+1}=\dfrac{1}{3-a_n}$ satisfies $0\leq a_n\leq 2$ and is decreasing. ⓑ Show that it converges and find its limit.