

微分積分の練習問題:

§17.3

I Evaluate the double integral.

- ① $\iint_D x \cos y \, dx \, dy$, D is bounded by $y=0$, $y=x^2$, $x=1$.
 ② $\iint_D y^3 \, dx \, dy$, D is the triangular region with vertices $(0,2)$, $(1,1)$, $(3,2)$.
 ③ $\iint_D (2x-y) \, dx \, dy$, D is bounded by the circle with center the origin and radius 2.

II Find the volume of the given solid.

- ① Under the surface $z=xy$ and above the triangle with vertices $(1,1)$, $(4,1)$ and $(1,2)$.
 ② Enclosed by the surfaces $z=x^2$, $y=x^2$ and the planes $z=0$ and $y=4$.
 ③ Bounded by the cylinder $x^2+y^2=1$ and the planes $y=z$, $x=0$, $z=0$ in the first octant.

III Sketch the region of integration and change the order of integration.

- ① $\int_0^3 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} f(x,y) \, dx \, dy$ ② $\int_1^2 \int_0^{\ln x} f(x,y) \, dy \, dx$

IV Evaluate the integral by reversing the order of integration.

- ① $\int_0^1 \int_{3y}^3 e^{x^2} \, dx \, dy$ ② $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1} \, dx \, dy$ ③ $\int_0^1 \int_{\sin^{-1} y}^{\frac{\pi}{2}} (\cos x) \sqrt{1+\cos^2 x} \, dx \, dy$.

V In evaluate a double integral over a region D , a sum of iterated integrals was obtained as follows:

$$\iint_D f(x,y) \, dx \, dy = \int_0^1 \int_0^{2-y} f(x,y) \, dx \, dy + \int_0^3 \int_0^{3-y} f(x,y) \, dx \, dy.$$

Sketch the region D and express the double integral as an iterated integral with reversed order of integration.