微積分習題

17.2

① If $R=[-1,3]\times[0,2]$, Use a Riemann sum with $m=4$ and $n=2$ to estimate the value of $\iint_R (y^2-2x^2)\,dx\,dy$. Take the sample points to be the upper left corners of the sub-rectangle R_{ij}.

② Calculate the iterated integral.

ⓐ $\int_1^4\int_0^2 (x+\sqrt{y})\,dx\,dy$ ⓑ $\int_0^1\int_1^2 \frac{xe^x}{y}\,dy\,dx$

ⓒ $\int_1^4\int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right)dy\,dx$ ⓓ $\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx$.

③ Calculate the double integral.

ⓐ $\iint_R \frac{xy^2}{x^2+1}\,dx\,dy$, $R=\{(x,y)\mid 0\le x\le 1,\ -3\le y\le 3\}$

ⓑ $\iint_R xye^{x^2y}\,dx\,dy$, $R=[0,1]\times[0,2]$.

④ The average value of a function $f(x,y)$ over a rectangle R is defined to by $f_{ave}=\frac{1}{\text{area}(R)}\iint_R f(x,y)\,dx\,dy$.

Find the average value of f over the given rectangle.

ⓐ $f(x,y)=x^2y$, R has vertices $(-1,0),(-1,5),(1,5),(1,0)$.

ⓑ $f(x,y)=e^y\sqrt{x+e^y}$, $R=[0,4]\times[0,1]$.

⑤ Let $R=[a,b]\times[c,d]$. Show that if f is continuous on $[a,b]$ and g is continuous on $[c,d]$, then

$$\iint_R f(x)g(y)\,dx\,dy=\left[\int_a^b f(x)\,dx\right]\cdot\left[\int_c^d g(y)\,dy\right].$$

微積分習題

17.2

① If $R=[-1,3]\times[0,2]$, Use a Riemann sum with $m=4$ and $n=2$ to estimate the value of $\iint\limits_R (y^2-2x^2)\,dx\,dy$. Take the sample points to be the upper left corners of the sub-rectangle R_{ij}.

② Calculate the iterated integral.

ⓐ $\int_1^4\int_0^2 (x+\sqrt{y})\,dx\,dy$ ⓑ $\int_0^1\int_1^2 \frac{xe^x}{y}\,dy\,dx$

ⓒ $\int_1^4\int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right)dy\,dx$ ⓓ $\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx$.

③ Calculate the double integral.

ⓐ $\iint\limits_R \frac{xy^2}{x^2+1}\,dx\,dy$, $R=\{(x,y)\mid 0\le x\le 1,\ -3\le y\le 3\}$

ⓑ $\iint\limits_R xye^{x^2y}\,dx\,dy$, $R=[0,1]\times[0,2]$.

④. The average value of a function $f(x,y)$ over a rectangle R is defined to by $f_{ave}=\frac{1}{\text{area}(R)}\iint\limits_R f(x,y)\,dx\,dy$.

Find the average value of f over the given rectangle.

ⓐ $f(x,y)=x^2y$, R has vertices $(-1,0)$, $(-1,5)$, $(1,5)$, $(1,0)$.

ⓑ $f(x,y)=e^y\sqrt{x+e^y}$, $R=[0,4]\times[0,1]$.

⑤ Let $R=[a,b]\times[c,d]$. Show that if f is continuous on $[a,b]$ and g is continuous on $[c,d]$, then

$$\iint\limits_R f(x)g(y)\,dx\,dy=\left[\int_a^b f(x)\,dx\right]\cdot\left[\int_c^d g(y)\,dy\right].$$