



# 微積分一 題庫

## 【章節 14.1】

p.700 (45、46、51、54、55、56)

45. Set  $\mathbf{f}(t) = t\mathbf{i} + f(t)\mathbf{j}$  and calculate  $f'(t_0)$ ,  $\int_a^b f(t)dt$ ,  $\int_a^b f'(t)dt$  given that  $f'(t_0) =$

$m$ ,  $f(a) = c$ ,  $f(b) = d$ ,  $\int_c^b f(t)dt = A$ .

46. Find  $f(t)$  from the information given.  $f'(t) = t\mathbf{i} + t(1+t^2)^{-1/2}\mathbf{j} + te^t\mathbf{k}$  and  $f(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ .

51. (a) Show that, if  $f'(t) = \mathbf{0}$  for all  $t$  in an interval  $I$ , then  $f$  is a constant vector on  $I$ .

(b) Show that, if  $f'(t) = g'(t)$  for all  $t$  in an interval  $I$ , then  $f$  and  $g$  differ by a constant vector on  $I$ .

54. A vector-valued function  $G$  is called an antiderivative for  $f$  on  $[a, b]$  provided that

(i)  $G$  is continuous on  $[a, b]$  and (ii)  $G'(t) = f(t)$  for all  $t \in (a, b)$ . Show that:

(a) If  $f$  is continuous on  $[a, b]$  and  $G$  is an antiderivative for  $f$  on  $[a, b]$ , then

$$\int_a^b f(t)dt = G(b) - G(a).$$

(b) If  $f$  is continuous on an interval  $I$  and  $F$  and  $G$  are antiderivatives for  $f$ , then  $F = G + C$  for some constant vector  $C$ .

55. Is it necessarily true that  $\int_a^b [f(t) \cdot g(t)]dt = \left[ \int_a^b f(t)dt \right] \cdot \left[ \int_a^b g(t)dt \right]$ ?

56. Prove that, if  $f$  is continuous on  $[a, b]$ , then for each constant vector  $c$ ,

$$\int_a^b [c \cdot f(t)]dt = c \cdot \int_a^b f(t)dt \quad \int_a^b [c \times f(t)]dt = c \times \int_a^b f(t)dt.$$

## 【章節 14.2】

p.701 (57、58)

57. Let  $f$  be a differentiable vector-valued function. Show that if  $\|f(t)\| \neq 0$ , then

$$\frac{d}{dt} (\|f(t)\|) = \frac{f(t) \cdot f'(t)}{\|f(t)\|}.$$

58. Let  $f$  be a differentiable vector-valued function. Show that where  $\|f(t)\| \neq 0$ ,

$$\frac{d}{dt} \left( \frac{f(t)}{\|f(t)\|} \right) = \frac{f'(t)}{\|f(t)\|} - \frac{f(t) \cdot f'(t)}{\|f(t)\|^2} f(t).$$

p.704 (6、11、12、29、30、31、33)

6. Calculate  $f'(t)$  and  $f''(t)$ .  $f(t) = [(3ti - t^2j + k) \cdot (i + t^3j - 2tk)]k$ .

11. Calculate  $f'(t)$  and  $f''(t)$ .  $f(t) = t g(\sqrt{t})$ .

12. Calculate  $f'(t)$  and  $f''(t)$ .  $f(t) = (e^{2t}i + e^{-2t}j + k) \times (e^{2t}i - e^{-2t}j + k)$ .

29. Assume the rule for differentiating a cross product and show the following.

$$\frac{d}{dt} [f(t) \times f'(t)] = f(t) \times f''(t).$$

30. Assume the rule for differentiating a cross product and show the following.

$$\frac{d}{dt} [u_1(t)r_1(t) \times u_2(t)r_2(t)] = u_1(t)u_2(t) \frac{d}{dt} [r_1(t) \times r_2(t)] + [r_1(t) \times$$

$$r_2(t)] \frac{d}{dt} [u_1(t)u_2(t)].$$

31. Set  $E(t) = f(t) \cdot [g(t) \times h(t)]$  and show that  $E'(t) = f'(t) \cdot [g(t) \times h(t)] + f(t) \cdot [g'(t) \times h(t)] + f(t) \cdot [g(t) \times h'(t)]$ .

33. Show that  $\|r(t)\|$  is constant iff  $r(t) \cdot r'(t) = 0$  identically.

※補充題：True or False. If  $f: [a, b] \rightarrow \mathbb{R}^n$  is differentiable, then there is a  $c \in (a, b)$

such that  $f'(c) = \frac{1}{b-a} [f(b) - f(a)]$ .

### 【章節 14.3】

p.713 (5、8、11、13、14、43)

5. Find the tangent vector  $r'(t)$  at the indicated point and parametrize the tangent line at that point.  $r(t)=2t^2i + (1 - t)j + (3 + 2t^2)k$  at  $P(2, 0, 5)$ .
8. Find the tangent vector  $r'(t)$  at the indicated point and parametrize the tangent line at that point.  $r(t)=t\sin t i + t \cos t j + 2tk$ ;  $t=\pi/2$ .
11. Find (a) the points on the curve  $r(t)=ti + (1 + t^2)j$  at which  $r(t)$  and  $r'(t)$  are perpendicular; (b) the points at which they have the same direction; (c) the points at which they have opposite directions.
13. Suppose that  $r'(t)$  and  $r(t)$  are parallel for all  $t$ . Show that, if  $r'(t)$  is never 0, then the tangent line at each point passes through the origin.
14. The curves intersect at the point give. Find the angle of intersection. Express your answer in radians.  $r_1(t) = ti + t^2j + t^3k$ ,  $r_2(u) = \sin 2u i + u \cos u j + uk$ ;  $P(0,0,0)$ .
43. Let  $r=r(t)$ ,  $t \in [a, b]$  be a differentiable curve with nonzero tangent vector  $r'(t)$ . We know that the vector function  $R(u)=r(a+b-u)$ ,  $u \in [a, b]$  traces out the same curve but in the opposite direction. Verify that this change of parameter changes the sign of the unit tangent but does not alter the principal normal.

### 【章節 14.4】

p.720 (5、6、18、19)

5. Find the length of the curve.  $r(t)=ti + \ln(\sec t)j + 3k$  from  $t=0$  to  $t=\frac{1}{4}\pi$ .
6. Find the length of the curve.  $r(t)=\tan^{-1} t i + \frac{1}{2}\ln(1 + t^2)j$  from  $t=0$  to  $t=1$ .
18. Use vector methods to show that, if  $y=f(x)$  has a continuous first derivative, then the length of the graph from  $x=a$  to  $x=b$  is given by the integral  $\int_a^b \sqrt{1 + [f'(x)]^2} dx$ .
19. (*Important*) Let  $y=f(x)$ ,  $x \in [a, b]$ , be a continuously differentiable function. Show that, if  $s$  is the length of the graph from  $(a, f(a))$  to  $(x, f(x))$ , then

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

(14.4.7)

## 【章節 15.2】

p.757 (13、21、22、26、30、31、32、34、36)

13. Sketch the cylinder.  $25y^2 + 4z^2 - 100 = 0$

21. Sketch the cylinder.  $y^2 - 4x^2 = 4$ .

22. Sketch the cylinder.  $z = x^2$ .

26. Identify the surface and find the traces. Then sketch the surface.  $9x^2 + 4y^2 + 36z^2 - 36 = 0$ .

30. Identify the surface and find the traces. Then sketch the surface.  $9x^2 - 4y^2 - 36z = 0$ .

31. Identify the surface and find the traces. Then sketch the surface.  $9x^2 - 4y^2 - 36z^2 = 36$ .

32. Identify the surface and find the traces. Then sketch the surface.  $4x^2 + 4z^2 - 36y^2 - 36 = 0$ .

34. Identify the surface and find the traces. Then sketch the surface.  $9x^2 + 4z^2 - 36y^2 = 0$ .

36. Identify the surface and find the traces. Then sketch the surface.  $9y^2 + 4z^2 - 36x = 0$ .

【章節 15.4】 見附件 A

【章節 15.6】 見附件 A

## 【章節 16.1】

p.794 (29、31、32、41<sup>Ⓐ</sup>、43)

29. Obtain the gradient directly from Definition 16.1.2.  $f(x, y) = 3x^2 - xy + y$ .

31. Obtain the gradient directly from Definition 16.1.2.  $f(x, y, z) = x^2y + y^2z + z^2x$ .

32. Obtain the gradient directly from Definition 16.1.2.  $f(x, y, z) = 2x^2y - \frac{1}{z}$ .

41. (a) Show that, if  $c \cdot h$  is  $o(h)$ , then  $c=0$ . HINT: First set  $h=hi$ , then set  $h=hj$ , then  $h=hk$ .

43. Set  $f(x, y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

(a) Show that  $f$  is not differentiable at  $(0, 0)$ .

(b) In Section 15.6 you saw that the first partials  $\partial f/\partial x$  and  $\partial f/\partial y$  exist at  $(0, 0)$ . Since these partials obviously exist at every other point of the plane, we can conclude from Theorem 16.1.3 that at least one of these partials is not continuous in a neighborhood of  $(0, 0)$ . Show that  $\partial f/\partial x$  is discontinuous at  $(0, 0)$ .

## 【章節 16.2】

p.803 (4、13、15、19、21、28、30、31、33、45)

4. Find the directional derivative at the point  $P$  in the direction indicated.  $f(x, y) = \frac{2x}{x-y}$  at  $P(1, 0)$  in the direction of  $i - \sqrt{3}j$ .

13. Find the directional derivative at the point  $P$  in the direction indicated.  $f(x, y, z) = x \tan^{-1}(y + z)$  at  $P(1, 0, 1)$  in the direction of  $i + j - k$ .

15. Find the directional derivative of  $f(x, y) = \ln \sqrt{x^2 + y^2}$  at  $(x, y) \neq (0, 0)$  toward the origin.

19. Find the directional derivative of  $f(x, y, z) = xe^{y^2-z^2}$  at  $(1, 2, -2)$  in the direction of increasing  $t$  along the path  $r(t) = ti + 2 \cos(t - 1)j - 2e^{t-1}k$ .

21. Find the directional derivative of  $f(x, y, z) = x^2 + 2xyz - yz^2$  at  $(1, 1, 2)$  in the direction parallel to the line  $\frac{x-1}{2} = y - 1 = \frac{z-2}{-3}$ .

28. Suppose that  $f$  is differentiable at  $(x_0, y_0)$  and  $\nabla f(x_0, y_0) \neq 0$ . Calculate the rate of change of  $f$  in the direction of  $\frac{\partial f}{\partial y}(x_0, y_0)i - \frac{\partial f}{\partial x}(x_0, y_0)j$ . Give a geometric

interpretation to your answer.

30. Verify that, if  $g$  is continuous at  $x$ , then

(a)  $g(x+h) - g(x) = o(h)$  and

(b)  $[g(x+h) - g(x)]\nabla f(x \cdot h) = o(h)$ .

31. Given the density function  $\lambda(x, y) = 48 - \frac{4}{3}x^2 - 3y^2$ , find the rate of density

change (a) at  $(1, -1)$  in the direction of the most rapid density decrease; (b) at  $(1, 2)$  in the  $i$  direction; (c) at  $(2, 2)$  in the direction away from the origin.

33. Determine the path of steepest descent along the surface  $z = x^2 + 3y^2$  from each of the following points: (a)  $(1, 1, 4)$ ; (b)  $(1, -2, 13)$ .

45. Assume that  $\nabla f(x)$  exists. Prove that, for each integer  $n$ ,  $\nabla f^n(x) = n f^{n-1}(x) \nabla f(x)$ . Does this result hold if  $n$  is replaced by an arbitrary real number?

### 【章節 16.3】

p.815 (9、12、18、23、32、34、36、41<sup>ⓑ</sup>、42<sup>ⓑ</sup>、43、44、45<sup>ⓐ</sup>、50、56、57)

9. Find the rate of change of  $f$  with respect to  $t$  along the curve.  $f(x, y) = \tan^{-1}(y^2 - x^2)$ ,  $r(t) = \sin t i + \cos t j$ .

12. Find the rate of change of  $f$  with respect to  $t$  along the curve.  $f(x, y, z) = \ln(x^2 + y^2 + z^2)$ ,  $r(t) = \sin t i + \cos t j + e^{2t} k$ .

18. Find  $du/dt$  by applying (16.3.5) or (16.3.6).  $u = x + 4\sqrt{xy} - 3y$ ;  $x = t^3, y = t^{-1}$  ( $t > 0$ ).

23. Find  $du/dt$  by applying (16.3.5) or (16.3.6).  $u = xy + yz + zx$ ;  $x = t^2, y = t(1 - t), z = (1 - t)^2$ .

32. Find  $\partial u/\partial s$  and  $\partial u/\partial t$ .  $u = z^2 \sec xy$ ;  $x = 2st, y = s - t^2, z = s^2 t$ .

34. Find  $\partial u/\partial s$  and  $\partial u/\partial t$ .  $u = x e^{yz^2}$ ;  $x = \ln st, y = t^3, z = s^2 + t^2$ .

36. (Important) Set  $r = \|r\|$  where  $r = xi + yj + zk$ . If  $f$  is a continuously differentiable function of  $r$ , then

$$\nabla[f(r)] = f'(r) \frac{r}{r} \quad \text{where } r \neq 0.$$

(16.3.11)

41. (b) Set  $u=u(x, y, z)$  where  $x=x(w, t)$ ,  $y=y(w, t)$ ,  $z=z(w, t)$ ,  $w=w(r, s)$ ,  $t=t(r, s)$ . Calculate  $\partial u / \partial r$  and  $\partial u / \partial s$ .

42. (b) Set  $u=u(x, y, z, w)$  where  $x=x(r, s, t)$ ,  $y=y(s, t, v)$ ,  $z=z(r, t)$ ,  $w=w(r, s, t, v)$ . Calculate  $\partial u / \partial r$  and  $\partial u / \partial v$ .

43. Let  $u=u(x, y)$ , where  $x=x(t)$  and  $y=y(t)$ , and assume that these functions have continuous second derivatives. Show that  $\frac{d^2u}{dt^2} = \frac{\partial^2u}{\partial x^2} \left(\frac{dx}{dt}\right)^2 + 2 \frac{\partial^2u}{\partial x \partial y} \frac{dx}{dt} \frac{dy}{dt} +$

$$\frac{\partial^2u}{\partial y^2} \left(\frac{dy}{dt}\right)^2 + \frac{\partial u}{\partial x} \frac{d^2x}{dt^2} + \frac{\partial u}{\partial y} \frac{d^2y}{dt^2}.$$

44. Let  $u=u(x, y)$ , where  $x=x(s, t)$  and  $y=y(s, t)$ , and assume that all these functions have continuous second partials. Show that  $\frac{d^2u}{ds^2} = \frac{\partial^2u}{\partial x^2} \left(\frac{\partial x}{\partial s}\right)^2 + 2 \frac{\partial^2u}{\partial x \partial y} \frac{\partial x}{\partial s} \frac{\partial y}{\partial s} +$

$$\frac{\partial^2u}{\partial y^2} \left(\frac{\partial y}{\partial s}\right)^2 + \frac{\partial u}{\partial x} \frac{\partial^2x}{\partial s^2} + \frac{\partial u}{\partial y} \frac{\partial^2y}{\partial s^2}.$$

45. (a) Assume that  $u=u(x, y)$  is differentiable. Show that the change of variables to polar coordinates  $x=r \cos \theta$  and  $y = r \sin \theta$  gives  $\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \cos \theta + \frac{\partial u}{\partial y} \sin \theta$ ,  $\frac{\partial u}{\partial \theta} =$

$$-\frac{\partial u}{\partial x} r \sin \theta + \frac{\partial u}{\partial y} r \cos \theta.$$

50. Let  $u=u(x, y)$ , where  $x=r \cos \theta$  and  $y = r \sin \theta$ , and assume that  $u$  has continuous second partials. Derive a formula for  $\partial^2u / \partial r \partial \theta$ .

56. Assume that  $z$  is a differentiable function of  $(x, y)$  which satisfies the given equation. Obtain  $\partial z / \partial x$  and  $\partial z / \partial y$  by implicit differentiation.  $z^4 + x^2z^3 + y^2 + xy = 2$ .

57. Assume that  $z$  is a differentiable function of  $(x, y)$  which satisfies the given equation. Obtain  $\partial z / \partial x$  and  $\partial z / \partial y$  by implicit differentiation.  $\cos xyz = \ln(x^2 + y^2 + z^2)$ .

## 【章節 16.4】

p.823 (3、9、14、18、26、27、31、34)

3. Find a normal vector and a tangent vector at the point P. Write an equation for the tangent line and an equation for the normal line.  $(x^2 + y^2)^2 = 9(x^2 - y^2)$ ;  $P(\sqrt{2}, 1)$ .
9. Find the slope of the curve  $x^2y = a^2(a - y)$  at the point  $(0, a)$ .
14. Find an equation for the tangent plane at the point P and scalar parametric equations for the normal line.  $\sqrt{x} + \sqrt{y} + \sqrt{z} = 4$ ;  $P(1, 4, 1)$ .
18. Find an equation for the tangent plane at the point P and scalar parametric equations for the normal line.  $z = \sin(x \cos y)$ ;  $P(1, \frac{1}{2}\pi, 0)$ .
26. Show that in the case of a surface of the form  $z = xf(x/y)$  with  $f$  continuously differentiable, all the tangent planes have a point in common.
27. Given that the surfaces  $F(x, y, z) = 0$  and  $G(x, y, z) = 0$  intersect at right angles in a curve  $\gamma$ , what condition must be satisfied by the partial derivatives of  $F$  and  $G$  on  $\gamma$ ?
31. The curve  $r(t) = 2t\mathbf{i} + 3t^{-1}\mathbf{j} - 2t^2\mathbf{k}$  and the ellipsoid  $x^2 + y^2 + 3z^2 = 25$  intersect at  $(2, 3, -2)$ . What is the angle of intersection?
34. Show that the sphere  $x^2 + y^2 + z^2 - 8x - 8y - 6z + 24 = 0$  is tangent to the ellipsoid  $x^2 + 3y^2 + 2z^2 = 9$  at the point  $(2, 1, 1)$ .

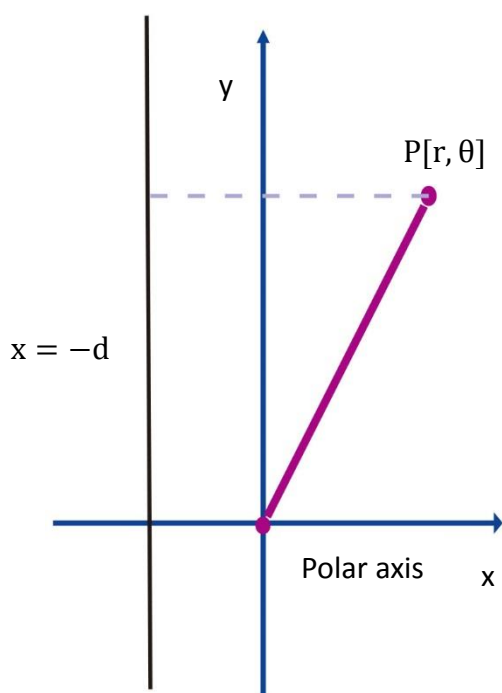
## 【章節 10.2】

p.483 (3、4、8、10、13、21、23、24)

3. Plot the point with these polar coordinates.  $[-1, \frac{1}{3}\pi]$ .
4. Plot the point with these polar coordinates.  $[-1, -\frac{1}{3}\pi]$ .
8. Plot the point with these polar coordinates.  $[\frac{1}{3}, \frac{2}{3}\pi]$ .
10. Give the rectangular coordinates of the point.  $[4, \frac{1}{6}\pi]$ .
13. Give the rectangular coordinates of the point.  $[-3, -\frac{1}{3}\pi]$ .
21. Below some points are specified in rectangular coordinates. Give all possible polar coordinates for each point.  $(2, -2)$ .



23. Below some points are specified in rectangular coordinates. Give all possible polar coordinates for each point.  $(4\sqrt{3}, 4)$ .
24. Below some points are specified in rectangular coordinates. Give all possible polar coordinates for each point.  $(\sqrt{3}, -1)$ .
- p.484 (39、45、53、57、63、64)
39. Write the equation in polar coordinates.  $2xy=1$ .
45. Write the equation in polar coordinates.  $x^2 + y^2 + x = \sqrt{x^2 + y^2}$ .
53. Identify the curve and write the equation in rectangular coordinates.  $r = 2(1 - \cos \theta)^{-1}$ .
57. Identify the curve and write the equation in rectangular coordinates.  $\tan \theta = 2$ .
63. Show that if a and b are not both zero, then the curve  $r = 2a \sin \theta + 2b \cos \theta$  is a circle. Find the center and the radius.
64. Find a polar equation for the set of points  $P[r, \theta]$  such that the distance from P to the pole equals the distance from P to the line  $x=-d$ . Take  $d \geq 0$ . See the figure.



※補充題： sketch the region in the plane consisting of points whose polar coordinates satisfying the following conditions.

- Ⓐ  $1 \leq r \leq 2$    Ⓑ  $r \geq 0, \frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3}$    Ⓒ  $2 < r < 3, \frac{5\pi}{3} \leq \theta \leq \frac{7\pi}{3}$    Ⓓ  $-1 < r < 1, \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

### 【章節 10.3】

Sketch the curve with the given polar equation.

Ⓐ  $r = \ln \theta, \theta \geq 1$    Ⓑ  $r = \sin(2\theta)$    Ⓒ  $r^2 = 4 \cos(3\theta)$    Ⓓ  $r = 1 + 2 \cos\left(\frac{\theta}{2}\right)$

### 【章節 17.2】 見附件 B

### 【章節 17.3】 見附件 C

### 【章節 17.4】

p.894 (4、8、11、16、19、22、23、25、26、28)

4. Calculate  $\int_{-\pi/3}^{2\pi/3} \int_0^{2 \cos \theta} r \sin \theta \, dr d\theta$ .

8. Integrate  $f(x, y) = \sqrt{x^2 + y^2}$  over the triangular region with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(1, \sqrt{3})$ .

11. Calculate using polar coordinates  $\int_{1/2}^1 \int_0^{\sqrt{1-x^2}} dy dx$ .

16. Calculate using polar coordinates  $\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} (x^2 + y^2) dy dx$ .

19. Find the area of the region by double integration. The region inside the circle  $r = 4 \cos \theta$  but outside the circle  $r=2$ .

22. Find the area of the region by double integration. The region inside the circle  $r = 3 \cos \theta$  but outside the cardioid  $r=1+\cos \theta$ .

23. Find the volume of the solid bounded above by the plane  $z=y+b$ , below by the  $xy$ -plane, and on the sides by the circular cylinder  $x^2 + y^2 = b^2$ .

25. Find the volume of the ellipsoid  $x^2/4 + y^2/4 + z^2/3 = 1$ .

26. Find the volume of the solid bounded below by the  $xy$ -plane and above by the surface  $x^2 + y^2 + z^6 = 5$ .

28. Find the volume of the solid bounded above by the surface  $z = 1 - (x^2 + y^2)$ , below by the  $xy$ -plane, and on the sides by the cylinder  $x^2 + y^2 - x = 0$ .

## 【章節 17.6】

p.907 (1)

1. Let  $f(x, y)$  be a function continuous and nonnegative on a basic region  $\Omega$  and set

$T = \{(x, y, z): (x, y) \in \Omega, 0 \leq z \leq f(x, y)\}$ . Compare  $\iiint_T dx dy dz$  to

$$\iint_{\Omega} f(x, y) dx dy.$$

## 【章節 17.7】

p.914 (11、23、24、28、30、31、32、44、48、50)

11. (Separated variables over a box) Set  $\Pi: a_1 \leq x \leq a_2, b_1 \leq y \leq b_2, c_1 \leq z \leq c_2$ .

Show that, if  $f$  is continuous on  $[a_1, a_2]$ ,  $g$  is continuous on  $[b_1, b_2]$ , and  $h$  is continuous on  $[c_1, c_2]$ , then

$$\iiint_{\Pi} f(x)g(y)h(z) dx dy dz = \left( \int_{a_1}^{a_2} f(x) dx \right) \left( \int_{b_1}^{b_2} g(y) dy \right) \left( \int_{c_1}^{c_2} h(z) dz \right)$$

(17.7.2)

23. The volume of the solid bounded above by the parabolic cylinder  $z = 1 - y^2$ , below by the plane  $2x + 3y + z + 10 = 0$ , and on the sides by the circular cylinder  $x^2 + y^2 - x = 0$ .

24. The volume of the solid bounded above by the parabolic cylinder  $z = 4 - x^2 - y^2$  and bounded below by the parabolic cylinder  $z = 2 + y^2$ .

28. Evaluate the triple integral.  $\iiint_T 2ye^x dx dy dz$ , where  $T$  is the solid given by  $0 \leq y \leq 1, 0 \leq x \leq y, 0 \leq z \leq x + y$ .

30. Evaluate the triple integral.  $\iiint_T xy dx dy dz$  where  $T$  is the first-octant solid bounded by the coordinate planes and the upper half of the sphere  $x^2 + y^2 + z^2 = 4$ .

31. Evaluate the triple integral.  $\iiint_T y^2 dx dy dz$  where  $T$  is the tetrahedron in the first octant bounded by the coordinate planes and the plane  $2x + 3y + z = 6$ .

32. Evaluate the triple integral.  $\iiint_T y^2 dx dy dz$  where  $T$  is the solid in the first

octant bounded by the cylinders  $x^2 + y = 1, z^2 + y = 1$ .

44. Integrate  $f(x, y, z) = x^2 y^2$  over the solid bounded above by the cylinder  $y^2 + z = 4$ , below by the plane  $y + z = 2$ , and on the sides by the planes  $x = 0$  and  $x = 2$ .

48. Find the volume of the solid bounded by the paraboloids  $z = 2 - x^2 - y^2$  and  $z = x^2 + y^2$ .

50. Let  $T$  be a solid with volume

$$V = \iiint_T dx dy dz = \int_0^2 \int_0^{9-x^2} \int_0^{2-x} dz dy dx.$$

Sketch  $T$  and fill in the blanks.

(a)  $V = \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} dy dx dz.$

(b)  $V = \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} dy dz dx.$

(c)  $V = \int_0^5 \int_{\square}^{\square} \int_{\square}^{\square} dz dx dy + \int_5^9 \int_{\square}^{\square} \int_{\square}^{\square} dz dx dy.$

## 【章節 17.8】

p.921 (4、5、9、13、16、21、24、26、28、30)

4. Express in cylindrical coordinates and sketch the surface.  $x = 4z$ .

5. Express in cylindrical coordinates and sketch the surface.  $4x^2 + 4y^2 - z^2 = 0$ .

9. The volume of a solid  $T$  is given in cylindrical coordinates. Sketch  $T$  and evaluate

the repeated integral.  $\int_0^{2\pi} \int_0^2 \int_0^{r^2} r dz dr d\theta$ .

13. Evaluate using cylindrical coordinates.  $\iiint_T \frac{1}{\sqrt{x^2 + y^2}} dx dy dz$ ;  $T: 0 \leq x \leq$

$\sqrt{9 - y^2}, 0 \leq y \leq 3, 0 \leq z \leq \sqrt{9 - (x^2 - y^2)}$ .

16. Evaluate using cylindrical coordinates.  $\iiint_T \sqrt{x^2 + y^2} dx dy dz$ ;  $T: -1 \leq x \leq 1,$

$-\sqrt{1 - x^2} \leq y \leq \sqrt{1 - x^2}, x^2 + y^2 \leq z \leq 2 - (x^2 + y^2)$ .

21. Find the volume of the solid bounded above by the cone  $z^2 = x^2 + y^2$ , below by the  $xy$ -plane, and on the sides by the cylinder  $x^2 + y^2 = 2ax$ . Take  $a > 0$ .

24. Find the volume of the solid bounded above by the plane  $2z = 4 + x$ , below by the  $xy$ -plane, and on the sides by the cylinder  $x^2 + y^2 = 2x$ .

26. Find the volume of the solid bounded above by  $x^2 + y^2 + z^2 = 25$  and below by  $z = \sqrt{x^2 + y^2} + 1$ .

28. Find the volume of the solid bounded by the hyperboloid  $z^2 = a^2 + x^2 + y^2$  and by the upper nappe of the cone  $z^2 = 2(x^2 + y^2)$ .

30. Find the volume of the solid that lies between the cylinders  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$ , and is bounded above by the ellipsoid  $x^2 + y^2 + 4z^2 = 36$  and below by the  $xy$ -plane.

### 【章節 17.9】

p.929 (9、18、19、33、34、35、37)

9. Equations are given in spherical coordinates. Interpret each one geometrically.  
 $\rho \sin \phi = 1$ .

18. Each expression represents the volume of a solid as calculated in spherical coordinates. Sketch the solid and carry out the integration.

$$\int_0^{\pi/4} \int_0^{2\pi} \int_0^{\sec \phi} \rho^2 \sin \phi \, d\rho d\theta d\phi.$$

19. Evaluate using spherical coordinates.  $\iiint_T dx dy dz$ ;  $T: 0 \leq x \leq 1, 0 \leq y \leq$

$$\sqrt{1-x^2}, \sqrt{x^2+y^2} \leq z \leq \sqrt{2-(x^2+y^2)}.$$

33. Find the volume of the solid common to the sphere  $\rho = a$  and the cone  $\phi = a$ .

Take  $a \in (0, \frac{1}{2}\pi)$ .

34. Let  $T$  be the solid bounded below by the half-cone  $z = \sqrt{x^2 + y^2}$  and above by the spherical surface  $x^2 + y^2 + z^2 = 1$ . Use spherical coordinates to evaluate

$$\iiint_T e^{(x^2+y^2+z^2)^{3/2}} dx dy dz.$$

35. (a) Find an equation in spherical coordinates for the sphere  $x^2 + y^2 + (z - R)^2 = R^2$ .

(b) Express the upper half of the ball  $x^2 + y^2 + (z - R)^2 \leq R^2$  by inequalities in spherical coordinates.

37. Find the volume of the solid common to the spheres  $\rho = 2\sqrt{2} \cos \phi$  and  $\rho = 2$ .

### 【章節 11】 習題請見附件 D

## 【章節 12.2】

p.584 (3、7、12、30、32、33、34、42)

3. Find the sum of the series.  $\sum_{k=1}^{\infty} \frac{1}{k(k+3)}$ .

7. Find the sum of the series.  $\sum_{k=0}^{\infty} \frac{1-2^k}{3^k}$ .

12. (a) Let  $j$  be a positive integer. Show that  $\sum_{k=0}^{\infty} a_k$  converges iff  $\sum_{k=j}^{\infty} a_k$  converges.

(b) Show that if  $\sum_{k=0}^{\infty} a_k = L$ , then  $\sum_{k=j}^{\infty} a_k = L - \sum_{k=0}^{j-1} a_k$ .

(c) Show that if  $\sum_{k=j}^{\infty} a_k = M$ , then  $\sum_{k=0}^{\infty} a_k = M + \sum_{k=0}^{j-1} a_k$ .

30. (a) Show that if the series  $\sum a_k$  converges and the series  $\sum b_k$  diverges, then the series  $\sum(a_k + b_k)$  diverges.

(b) Give examples to show that if  $\sum a_k$  and  $\sum b_k$  both diverge, then each of the series  $\sum(a_k + b_k)$  and  $\sum(a_k - b_k)$  may converge or may diverge.

32. (a) Prove that if  $\sum_{k=0}^{\infty} a_k$  is a convergent series with all terms nonzero, then  $\sum_{k=0}^{\infty} (1/a_k)$  diverges.

(b) Suppose that  $a_k > 0$  for all  $k$  and  $\sum_{k=0}^{\infty} a_k$  diverges. Show by example that  $\sum_{k=0}^{\infty} (1/a_k)$  may converge and it may diverge.

33. Show that  $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$  diverges although  $\ln\left(\frac{k+1}{k}\right) \rightarrow 0$ .

34. Show that  $\sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^k$  diverges.

42. Prove that the series  $\sum_{k=1}^{\infty} (a_{k+1} - a_k)$  converges iff the  $a_n$  tend to a finite limit.

### 【章節 12.3】

p.592 (3、12、13、26、32、50)

3. Determine whether the series converges or diverges.  $\sum \frac{1}{(2k+1)^2}$ .

12. Determine whether the series converges or diverges.  $\sum \frac{1}{k(k+1)(k+2)}$ .

13. Determine whether the series converges or diverges.  $\sum \left(\frac{4}{3}\right)^k$ .

26. Determine whether the series converges or diverges.  $\sum \frac{2k+1}{\sqrt{k^3+1}}$ .

32. Determine whether the series converges or diverges.  $\sum \frac{2+\cos k}{\sqrt{k+1}}$ .

50. Let  $\sum a_k$  be a series with nonnegative terms. Show that if  $\sum a_k^2$  converges, then  $\sum (a_k/k)$  converges.

### 【章節 12.4】

1. Determine the convergence or divergence of the following series.

①  $\sum \frac{10^n}{(n+1)4^{2n+1}}$    ②  $\sum \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{n!}$    ③  $\sum \frac{\sin^2(4n)}{4^n}$    ④  $\sum \frac{\cos^2 n}{n!}$    ⑤  $\sum \frac{2^n n!}{5 \cdot 8 \cdot 11 \cdots (3n+2)}$

2. (a) show that  $\sum_{n=1}^{\infty} \frac{x^n}{n!}$  Converges for all (fixed)  $x \geq 0$ .

(b) Deduce that  $\lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0$  for all  $x \geq 0$ .

### 【章節 12.5】

p.601 (6、10、11、12、19、21、22、24、26、28、44、48)

6. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum (-1)^k \frac{k}{\ln k}$$

10. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum (-1)^k \frac{(k!)^2}{(2k)!}$$

11. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum \frac{k!}{(-2)^k}.$$

12. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum \sin\left(\frac{k\pi}{4}\right).$$

19. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum \frac{(-1)^k}{k-2\sqrt{k}}.$$

21. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum (-1)^k \frac{4^{k-2}}{e^k}.$$

22. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum (-1)^k \frac{k^2}{2^k}.$$

24. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum (-1)^{k+1} \frac{k^k}{k!}.$$

26. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum \frac{\cos \pi k}{k}.$$

28. Test these series for (a) absolute convergence, (b) conditional convergence.

$$\sum \frac{\sin(\pi k/2)}{k\sqrt{k}}.$$

44. Show that if  $\sum a_k$  is absolutely convergent and  $|b_k| \leq |a_k|$  for all  $k$ , then  $\sum b_k$  is absolutely convergent.

48. Form the series  $a - \frac{1}{2}b + \frac{1}{3}a - \frac{1}{4}b + \frac{1}{5}a - \frac{1}{6}b + \dots$ .

(a) Express this series in  $\sum$  notation.

(b) For what positive values of  $a$  and  $b$  is this series absolutely convergent? conditionally convergent?



## 【章節 12.6】

p.611 (3、8、9、11、16、21、36、39)

3. Find the Taylor polynomial  $P_4$  for the function  $f$ .  $f(x) = \ln \cos x$ .
8. Find the Taylor polynomial  $P_5$  for the given function  $f$ .  $f(x) = x \cos x^2$ .
9. Determine  $P_0(x), P_1(x), P_2(x), P_3(x)$  for  $f(x) = 1 - x + 3x^2 + 5x^3$ .
11. Determine the  $n$ th Taylor polynomial  $P_n$  for the function  $f$ .  $f(x) = e^{-x}$ .
16. Determine the  $n$ th Taylor polynomial  $P_n$  for the function  $f$ .  $f(x) = \cos bx$ ,  $b$  a real number.
21. Assume that  $f$  is a function with  $|f^{(n)}(x)| \leq 3$  for all  $n$  and all real  $x$ . Find the least integer  $n$  for which you can be sure that  $P_n(1/2)$  approximates  $f(1/2)$  with four decimal place accuracy.
36. Find the Lagrange form of the remainder  $R_n(x)$ .  $f(x) = \sqrt{x+1}$ ;  $n=3$ .
39. Find the Lagrange form of the remainder  $R_n(x)$ .  $f(x) = \tan^{-1} x$ ;  $n=2$ .

## 【章節 12.7】

p.616 (1、7、10、12、14、16、18、22、23、24、26、27、33)

1. Find the Taylor polynomial of the function  $f$  for the given values of  $a$  and  $n$  and give the Lagrange form of the remainder.  $f(x) = \sqrt{x}$ ;  $a=4$ ,  $n=3$ .
7. Expand  $g(x)$  as indicated and specify the values of  $x$  for which the expansion is valid.  $g(x) = 3x^3 - 2x^2 + 4x + 1$  in powers of  $x-1$ .
10. Expand  $g(x)$  as indicated and specify the values of  $x$  for which the expansion is valid.  $g(x) = x^{-1}$  in powers of  $x-1$ .
12. Expand  $g(x)$  as indicated and specify the values of  $x$  for which the expansion is valid.  $g(x) = (b+x)^{-1}$  in powers of  $x-a$ ,  $a \neq -b$ .
14. Expand  $g(x)$  as indicated and specify the values of  $x$  for which the expansion is valid.  $g(x) = e^{-4x}$  in powers of  $x+1$ .
16. Expand  $g(x)$  as indicated and specify the values of  $x$  for which the expansion is valid.  $g(x) = \sin x$  in powers of  $x - \frac{1}{2}\pi$ .
18. Expand  $g(x)$  as indicated and specify the values of  $x$  for which the expansion is valid.  $g(x) = \cos x$  in powers of  $x - \frac{1}{2}\pi$ .
22. Expand  $g(x)$  as indicated and specify the values of  $x$  for which the expansion is

valid.  $g(x) = \ln(2 + 3x)$  in powers of  $x-4$ .

23. Expand  $g(x)$  as indicated.  $g(x) = x \ln x$  in powers of  $x-2$ .

24. Expand  $g(x)$  as indicated.  $g(x) = x^2 + e^{3x}$  in powers of  $x-2$ .

26. Expand  $g(x)$  as indicated.  $g(x) = \ln(x^2)$  in powers of  $x-1$ .

27. Expand  $g(x)$  as indicated.  $g(x) = (1 - 2x)^{-3}$  in powers of  $x+2$ .

33. (a) Expand  $e^x$  in powers of  $x-a$ .

(b) Use the expansion to show that  $e^{x_1+x_2} = e^{x_1}e^{x_2}$ .

(c) Expand  $e^{-x}$  in powers of  $x-a$ .

## 【章節 12.8】

p.622 (8、9、22、25、28、36、39、43、46、48)

8. Find the interval of convergence.  $\sum \frac{(-1)^k}{\sqrt{k}} x^k$ .

9. Find the interval of convergence.  $\sum \frac{1}{k2^k} x^k$ .

22. Find the interval of convergence.  $\sum k! x^k$ .

25. Find the interval of convergence.  $\sum (-1)^k \frac{k!}{k^3} (x-1)^k$ .

28. Find the interval of convergence.  $\sum \frac{\ln k}{k} (x+1)^k$ .

36. Find the interval of convergence.  $\sum \frac{2^{1/k} \pi^k}{k(k+1)(k+2)} (x-2)^k$ .

39. Find the interval of convergence.  $\frac{3x^2}{4} + \frac{9x^4}{9} + \frac{27x^6}{16} + \frac{81x^8}{25} + \dots$

43. Let  $\sum a_k x^k$  be a series with radius of convergence  $r > 0$ .

(a) Show that if the series is absolutely convergent at one endpoint of its interval of convergence, then it is absolutely convergent at the other endpoint.

(b) Show that if the interval of convergence is  $(-r, r]$  then the series is only conditionally convergent at  $r$ .

46. Find the interval of convergence of the series  $\sum s_k x^k$  where  $s_k$  is the  $k$ th

partial sum of the series  $\sum_{n=1}^{\infty} \frac{1}{n}$ .

48. Let  $\sum a_k x^k$  be a power series with finite radius of convergence  $r$ . Show that the power series  $\sum a_k x^{2k}$  has radius of convergence  $\sqrt{r}$ .

## 【章節 12.9】

p.632 (2、5、6、7、8、10、12、13、15、17、20、22、28、29、43、44、47、49、51)

2. Expand  $f(x)$  in powers of  $x$ , basing your calculations on the geometric series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots.$$

$$f(x) = \frac{1}{(1-x)^3}.$$

5. Expand  $f(x)$  in powers of  $x$ , basing your calculations on the geometric series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots.$$

$$f(x) = \ln(1 - x^2).$$

6. Expand  $f(x)$  in powers of  $x$ , basing your calculations on the geometric series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots.$$

$$f(x) = \ln(2 - 3x).$$

7. Expand  $f(x)$  in powers of  $x$ , basing your calculations on the tangent series

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots.$$

$$f(x) = \sec^2 x.$$

8. Expand  $f(x)$  in powers of  $x$ , basing your calculations on the tangent series

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots.$$

$$f(x) = \ln \cos x.$$

10. Find  $f^{(4)}(0)$ .  $f(x) = x \cos x^2$ .

12. Expand  $f(x)$  in powers of  $x$ .  $f(x) = x^2 \tan^{-1} x$ .

13. Expand  $f(x)$  in powers of  $x$ .  $f(x) = e^{3x^3}$ .

15. Expand  $f(x)$  in powers of  $x$ .  $f(x) = \frac{2x}{1-x^2}$ .

17. Expand  $f(x)$  in powers of  $x$ .  $f(x) = \frac{1}{1-x} + e^x$ .

20. Expand  $f(x)$  in powers of  $x$ .  $f(x) = (x^2 + x) \ln(1 + x)$ .

22. Expand  $f(x)$  in powers of  $x$ .  $f(x) = x^5(\sin x + \cos 2x)$ .

28. Find a power series representation for the improper integral.  $\int_0^x \frac{1-\cos t}{t^2} dt$ .

29. Find a power series representation for the improper integral.  $\int_0^x \frac{\tan^{-1} t}{t} dt$ .

43. Sum the series.  $\sum_{k=0}^{\infty} \frac{3^k}{k!} x^{3k-1}$ .

44. Set  $f(x) = \frac{e^x - 1}{x}$ .

(a) Expand  $f(x)$  in a power series.

(b) Differentiate the series and show that  $\sum_{n=1}^{\infty} \frac{1}{n!(n+2)} = \frac{1}{2}$ .

47. Show that, if  $\sum a_k x^k$  and  $\sum b_k x^k$  both converge to the same sum on some interval, then  $a_k = b_k$  for each  $k$ .

49. Suppose that the function  $f$  has the power series representation  $f(x) = \sum_{k=0}^{\infty} a_k x^k$ .

(a) Show that if  $f$  is an even function, then  $a_{2k+1} = 0$  for all  $k$ .

(b) Show that if  $f$  is an odd function, then  $a_{2k} = 0$  for all  $k$ .

51. Suppose that the function  $f$  is infinitely differentiable on an open interval that contains 0, and suppose that  $f''(x) = -2f(x)$  for all  $x$  and  $f(0) = 0$ ,  $f'(0) = 1$ . Express  $f(x)$  as a power series in  $x$ . What is the sum of this series?