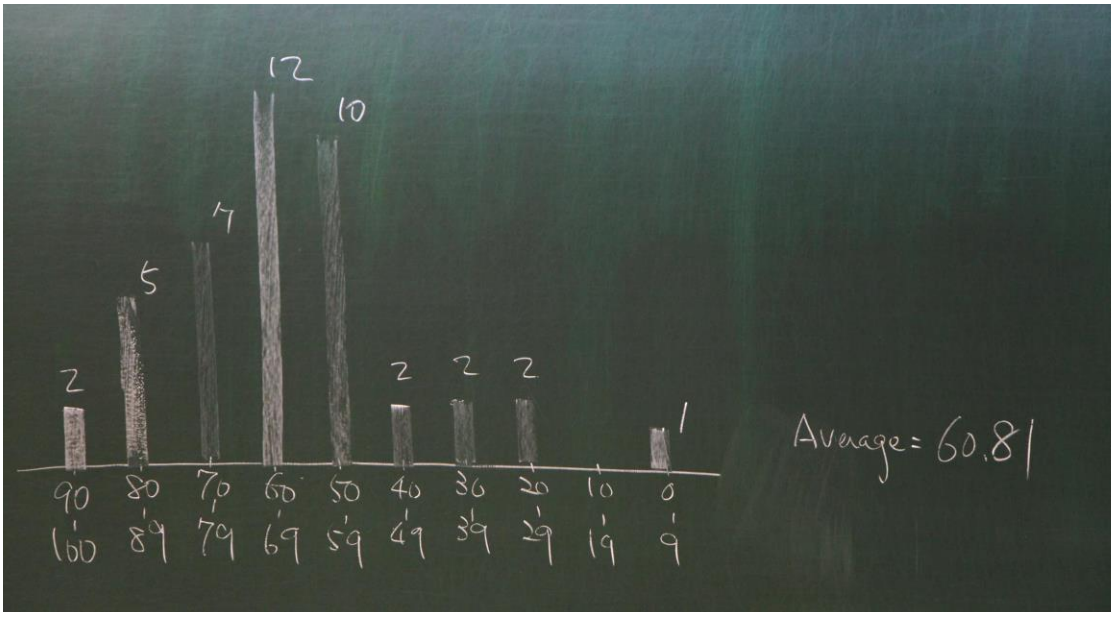
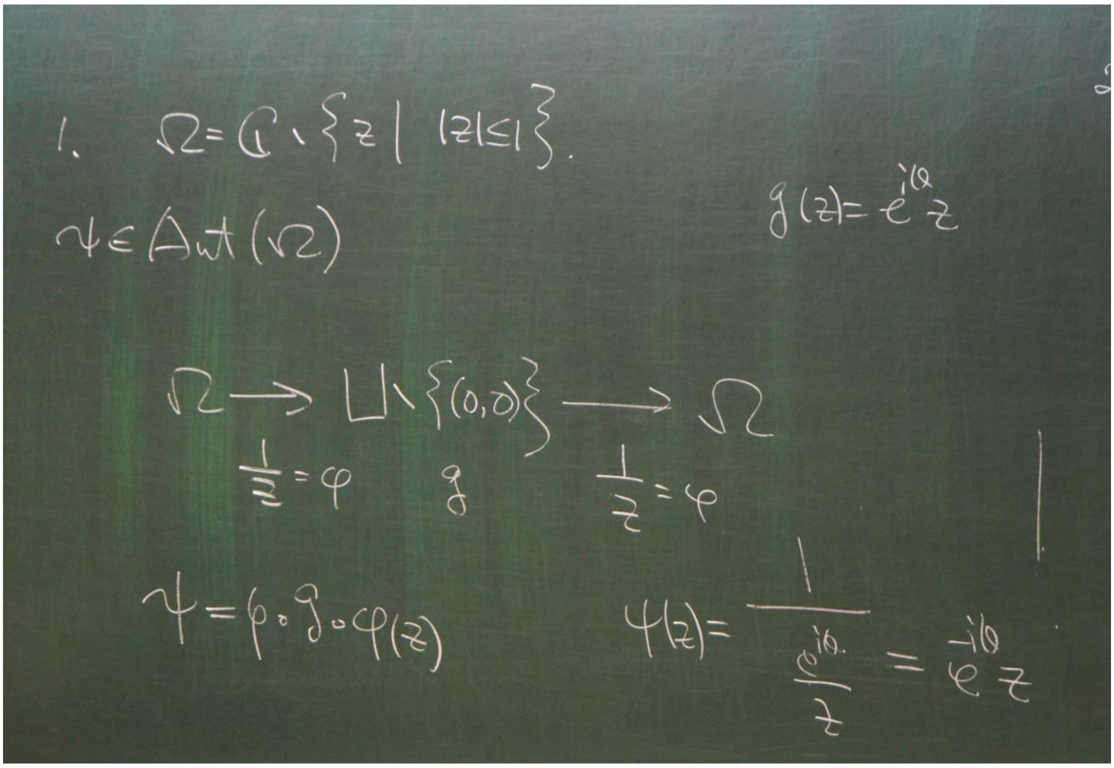
【10920 程守慶教授複變數函數論 / 第 29 堂版書】





2.

$\mathbb{H} \longrightarrow \mathbb{C}$

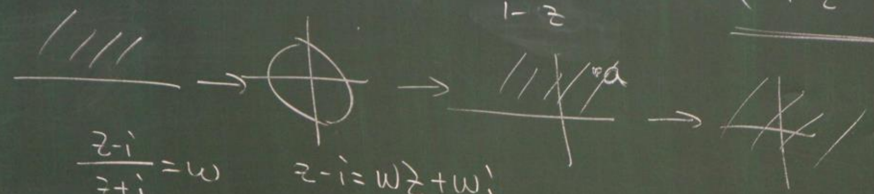
holo. ft. onto?

Choose a with $\operatorname{Im} a > 0$

$(w-a)^2$

$w = \frac{i(1+z)}{1-z}$

$\left(i\frac{1+z}{1-z} - i\right)^2 = f(z)$



$\frac{z-i}{z+i} = w$ $\quad z-i = wz+wi$

$-i-wi = (w-1)z$

$\Big)^2 = f(z)$

3. $\Omega \subsetneq \mathbb{C}$ domain

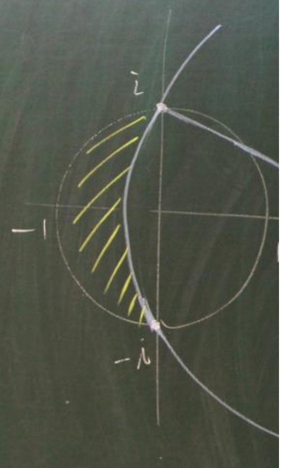
simply-connected

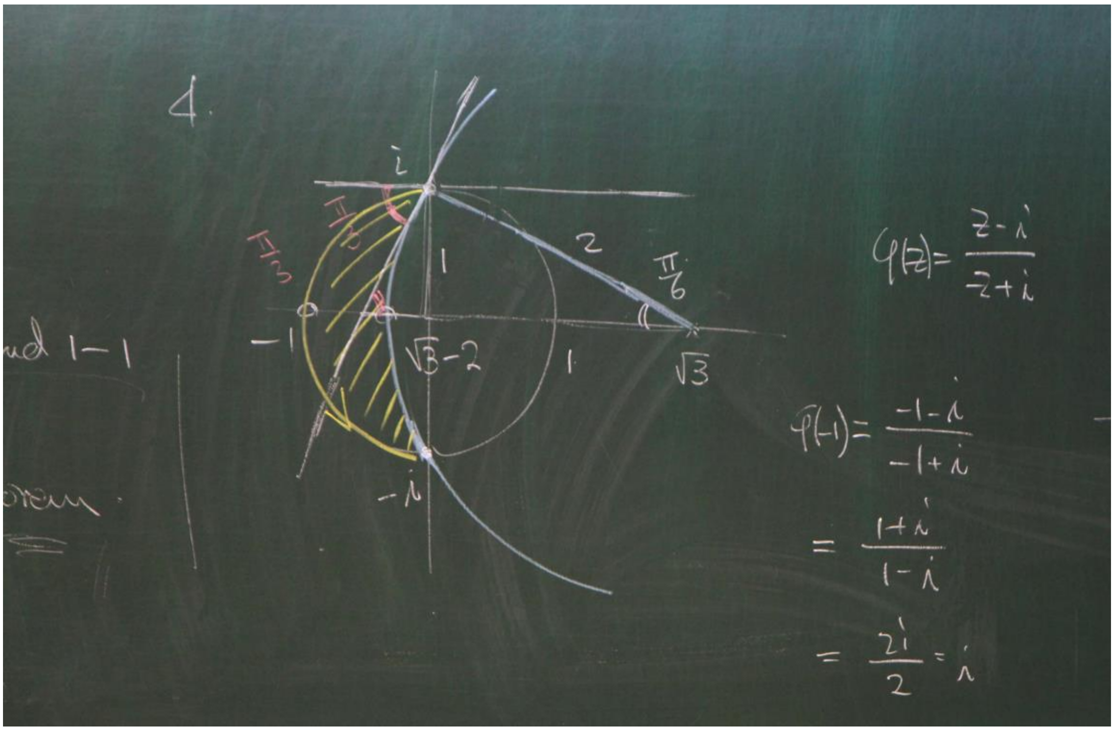
$\Rightarrow \exists: f \in \mathcal{O}(\Omega)$. bounded and 1-1

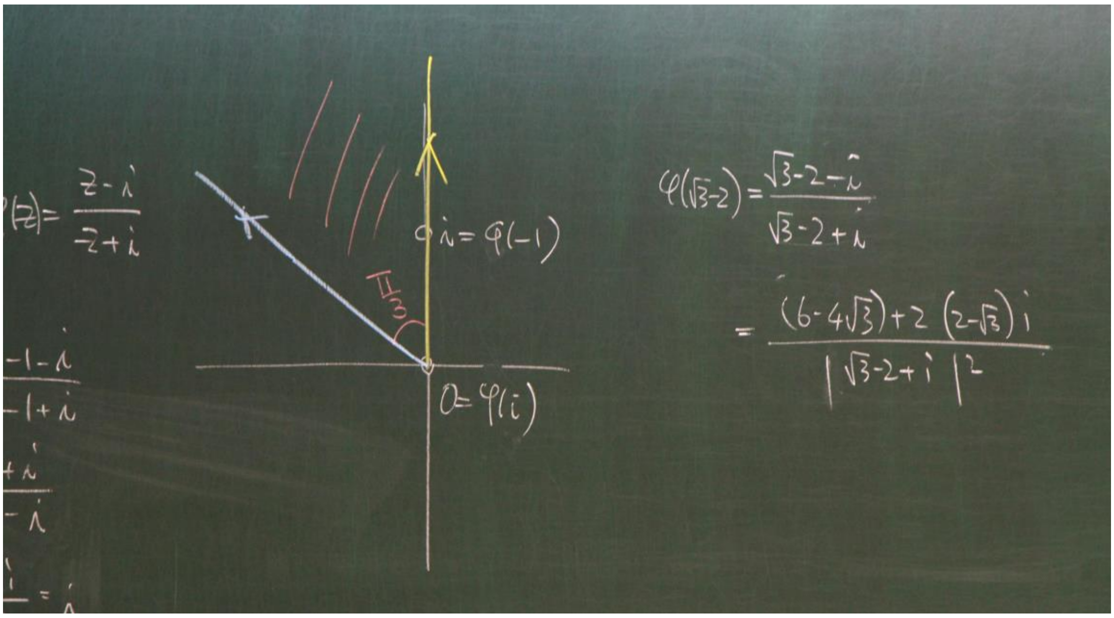
By Riemann mapping theorem.

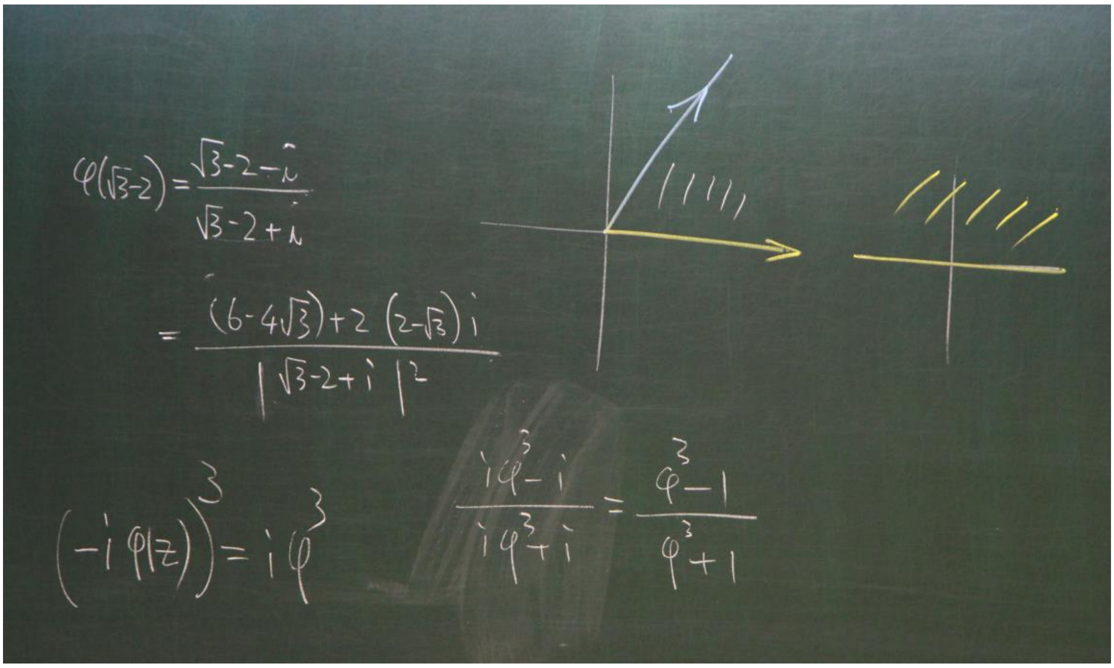
i, —

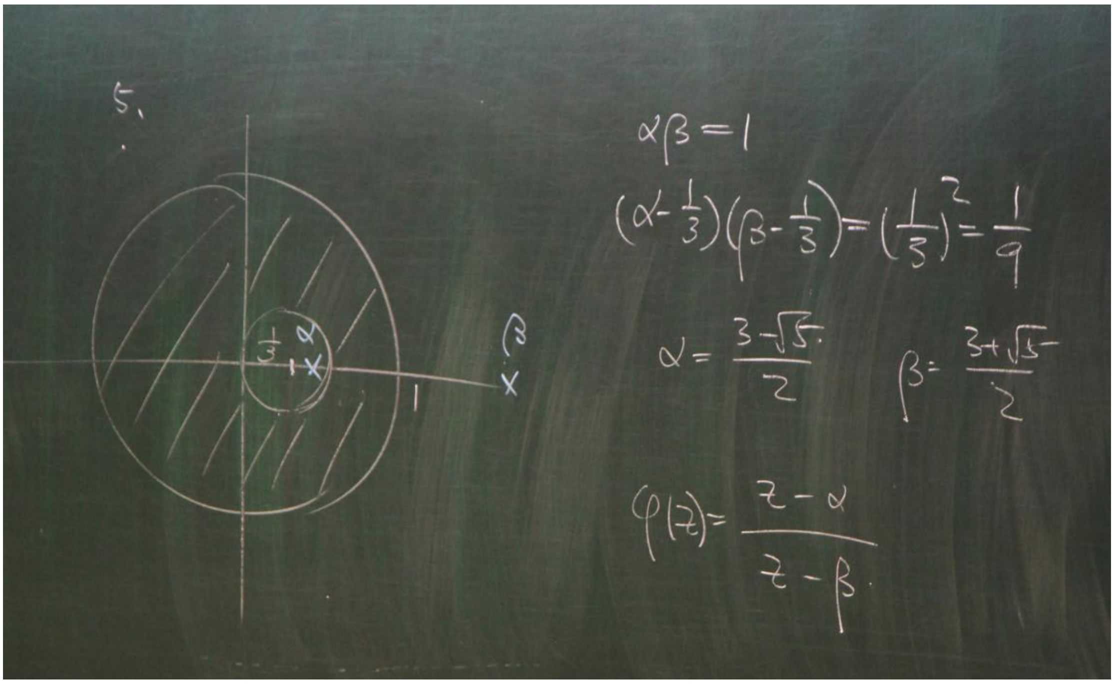
4.

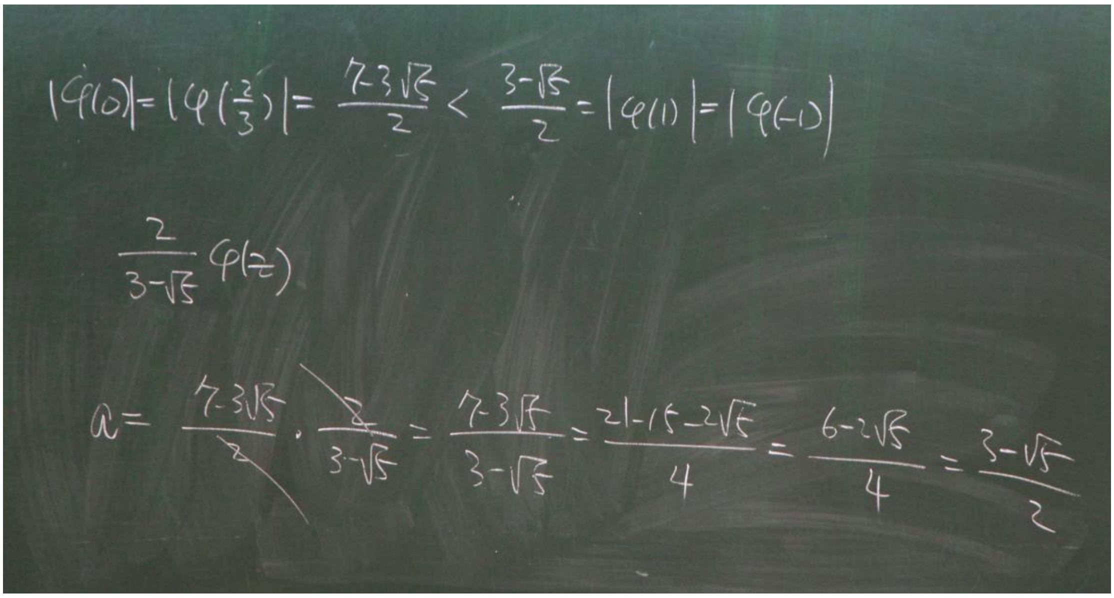



4.
i
π/3
π/6
1
2
π/6
−1
√3−2
1
√3
−i
nd 1−1
orem
φ(z) = (z − i)/(−z + i)
φ(−1) = (−1 − i)/(−1 + i)
= (1 + i)/(1 − i)
= 2i/2 = i


φ(z) = (z − i)/(−z + i)
i = φ(−1)
π/3
0 = φ(i)
φ(√3−2) = (√3 − 2 − i)/(√3 − 2 + i)
= ((6 − 4√3) + 2 (2 − √3) i)/|√3 − 2 + i|²


$\varphi(\sqrt{3}-2) = \dfrac{\sqrt{3}-2-i}{\sqrt{3}-2+i}$
$= \dfrac{(6-4\sqrt{3})+2(2-\sqrt{3})i}{|\sqrt{3}-2+i|^2}$
$(-i\varphi(z))^3 = i\varphi^3$
$\dfrac{i\varphi^3-i}{i\varphi^3+i} = \dfrac{\varphi^3-1}{\varphi^3+1}$


5.
$\frac{1}{3}$
α
1
β
$\alpha\beta = 1$
$(\alpha-\frac{1}{3})(\beta-\frac{1}{3}) = (\frac{1}{3})^2 = \frac{1}{9}$
$\alpha = \dfrac{3-\sqrt{5}}{2}$
$\beta = \dfrac{3+\sqrt{5}}{2}$
$\varphi(z) = \dfrac{z-\alpha}{z-\beta}$



$|\varphi(0)| = |\varphi(\frac{2}{3})| = \frac{7-3\sqrt{5}}{2} < \frac{3-\sqrt{5}}{2} = |\varphi(1)| = |\varphi(-1)|$
$\frac{2}{3-\sqrt{5}}\varphi(z)$
$a = \frac{7-3\sqrt{5}}{\cancel{2}} \cdot \frac{\cancel{2}}{3-\sqrt{5}} = \frac{7-3\sqrt{5}}{3-\sqrt{5}} = \frac{21-15-2\sqrt{5}}{4} = \frac{6-2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2}$