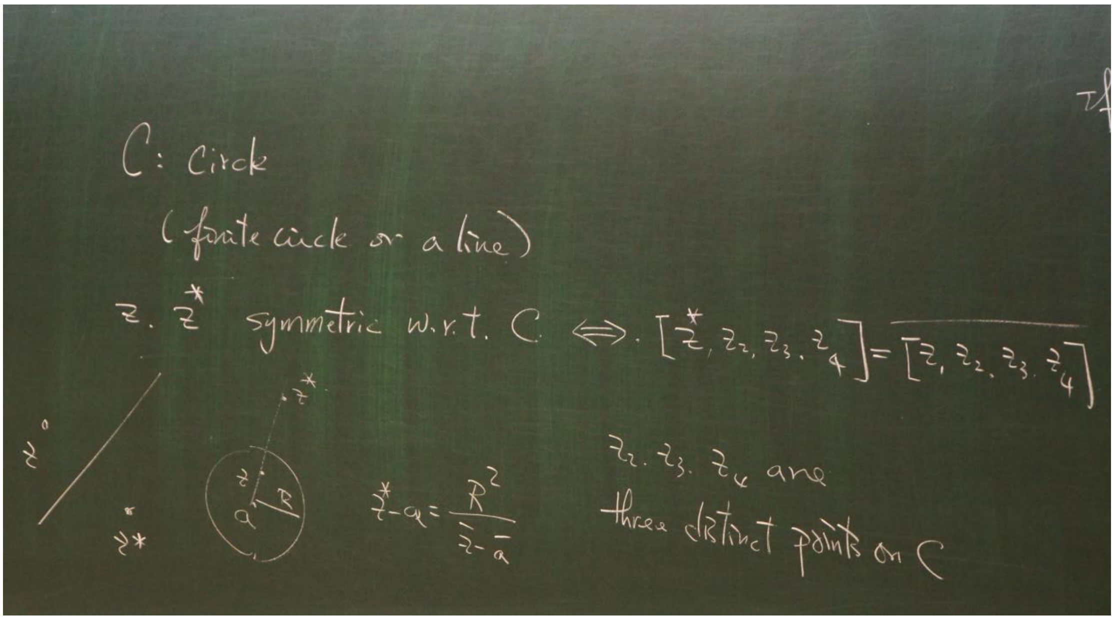
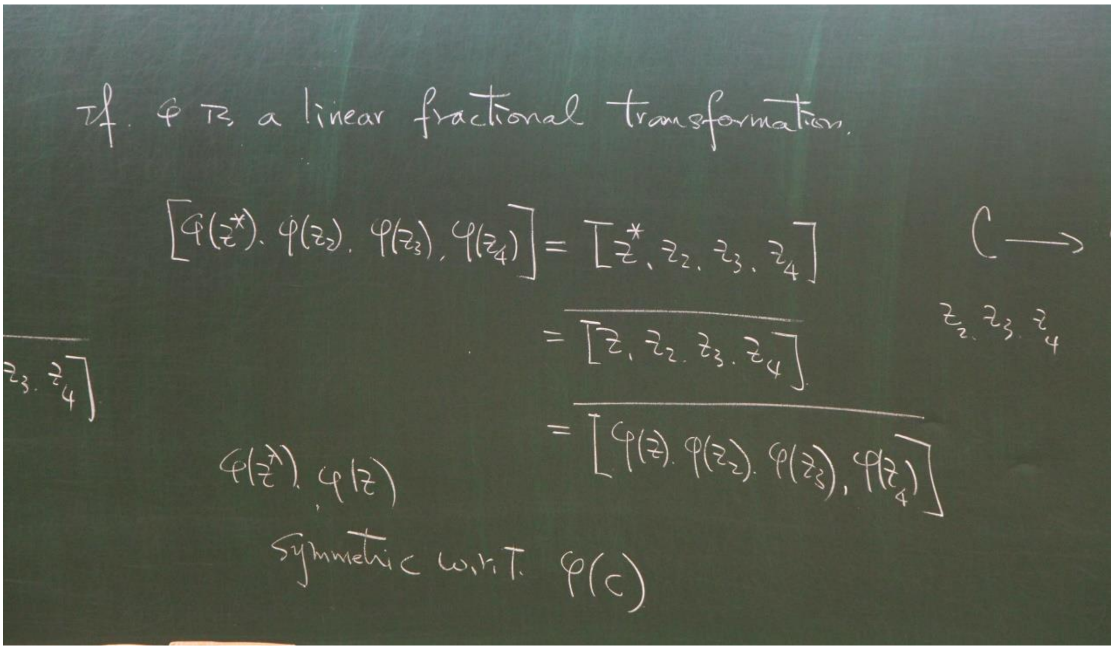
【10920 程守慶教授複變數函數論 / 第 27 堂版書】





If φ is a linear fractional transformation.

Circle

$C \longrightarrow \varphi(C)$

$z_2, z_3, z_4 \qquad \varphi(z_2), \varphi(z_3), \varphi(z_4)$

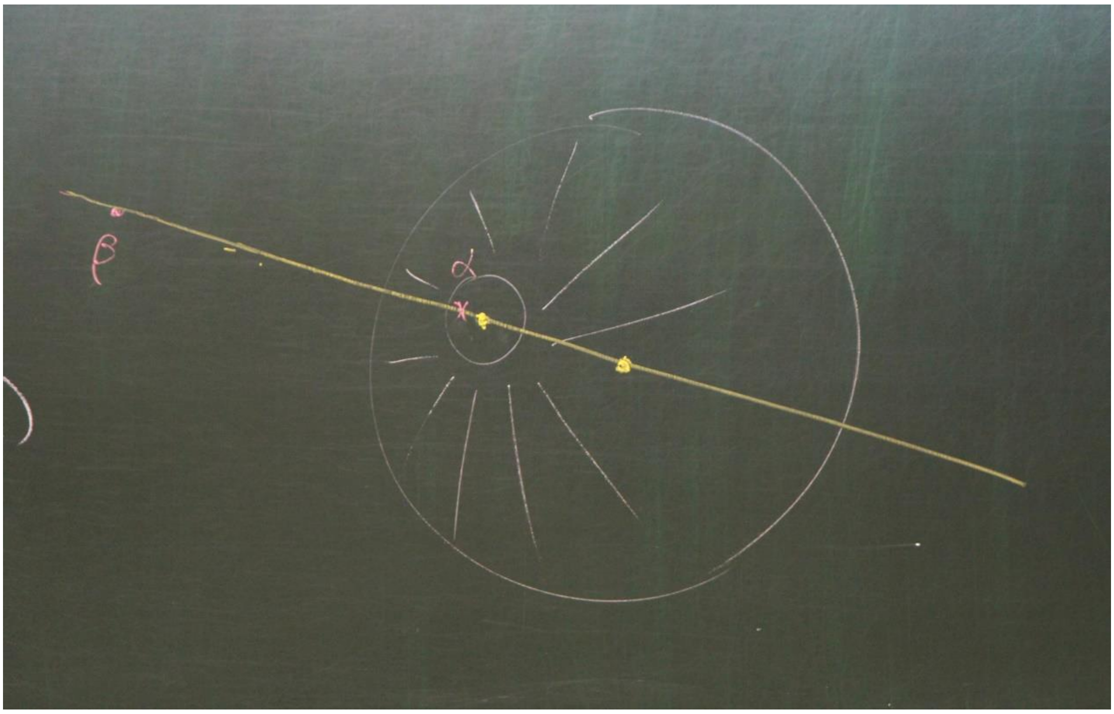
$$[\varphi(z^*), \varphi(z_2), \varphi(z_3), \varphi(z_4)] = [z^*, z_2, z_3, z_4]$$

$$= \overline{[z, z_2, z_3, z_4]}$$

$$= \overline{[\varphi(z), \varphi(z_2), \varphi(z_3), \varphi(z_4)]}$$

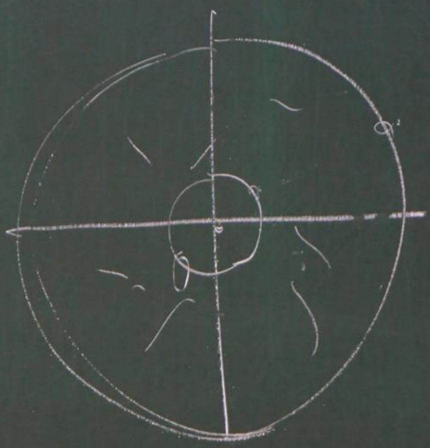
$\varphi(z^*), \varphi(z)$

symmetric w.r.t. $\varphi(C)$



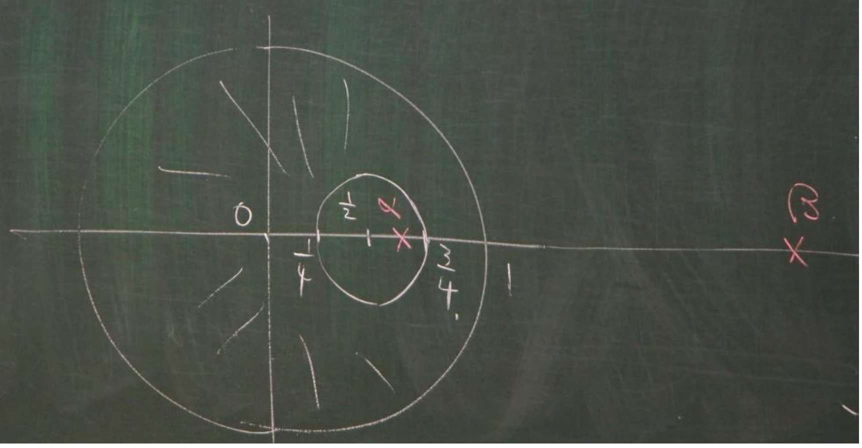
Consider

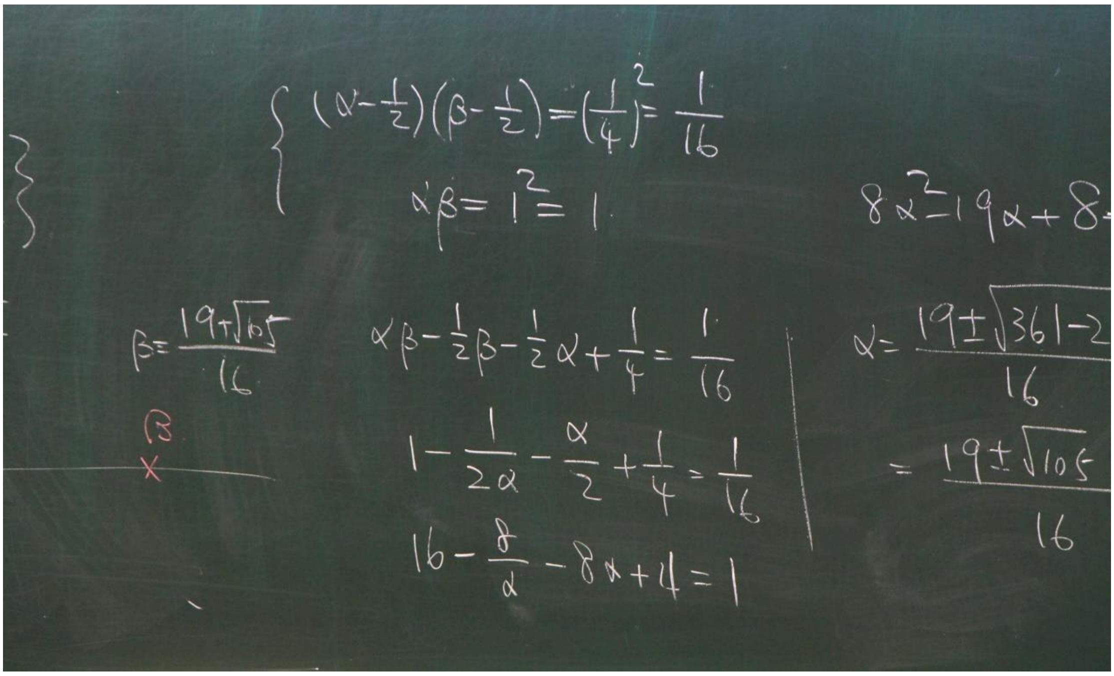
$$\varphi(z) = \frac{z-\alpha}{z-\beta}.$$

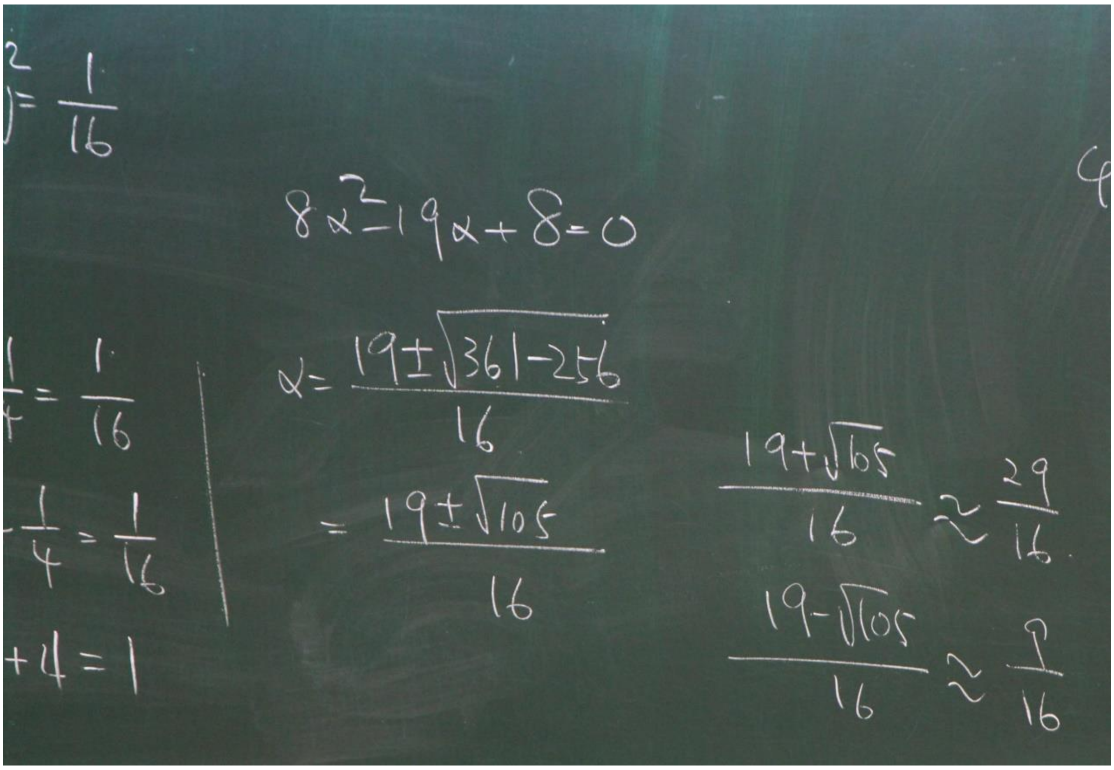


Ex. D: domain bounded by

$C_1 = \{ z \mid |z| = 1 \}$ and $C_2 = \{ z \mid |z - \frac{1}{2}| = \frac{1}{4} \}$.

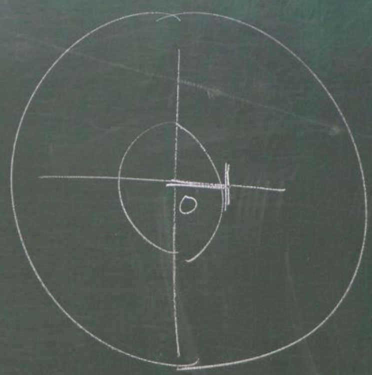



$(\alpha-\frac{1}{2})(\beta-\frac{1}{2})=(\frac{1}{4})^2=\frac{1}{16}$
$\alpha\beta=1^2=1$
$8\alpha^2-19\alpha+8=$
$\beta=\frac{19+\sqrt{105}}{16}$
β
$\alpha\beta-\frac{1}{2}\beta-\frac{1}{2}\alpha+\frac{1}{4}=\frac{1}{16}$
$1-\frac{1}{2\alpha}-\frac{\alpha}{2}+\frac{1}{4}=\frac{1}{16}$
$16-\frac{8}{\alpha}-8\alpha+4=1$
$\alpha=\frac{19\pm\sqrt{361-2}}{16}$
$=\frac{19\pm\sqrt{105}}{16}$


$8\alpha^2-19\alpha+8=0$
$\alpha=\frac{19\pm\sqrt{361-256}}{16}$
$=\frac{19\pm\sqrt{105}}{16}$
$\frac{19+\sqrt{105}}{16}\approx\frac{29}{16}$
$\frac{19-\sqrt{105}}{16}\approx\frac{9}{16}$

$z = \frac{1}{4}.$

$$\varphi(z) = \frac{z - \frac{19-\sqrt{105}}{16}}{z - \frac{19+\sqrt{105}}{16}}$$



$\frac{29}{16}$

$$\alpha = \frac{19-\sqrt{105}}{16} \longmapsto 0.$$

$\frac{9}{16}$

$$\beta = \frac{19+\sqrt{105}}{16} \longmapsto \infty$$

$z = \frac{1}{4}, \ \frac{3}{4}$

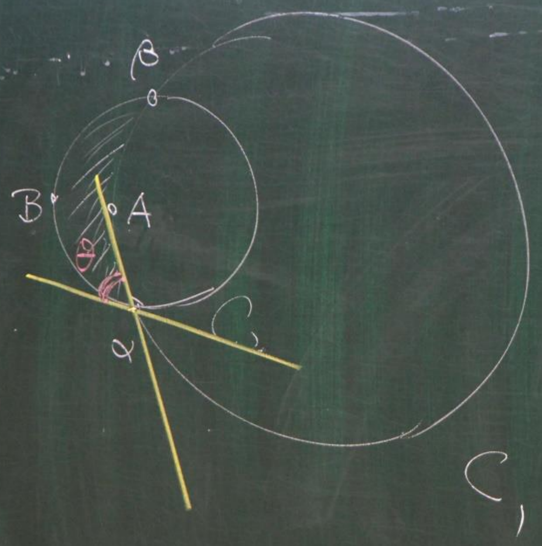
$$|\varphi(\tfrac{1}{4})| = \left| \frac{\frac{1}{4} - \frac{19-\sqrt{105}}{16}}{\frac{1}{4} - \frac{19+\sqrt{105}}{16}} \right| = \frac{|-15+\sqrt{105}|}{|-15-\sqrt{105}|} = \frac{|330-30\sqrt{105}|}{120} = \frac{|11-\sqrt{105}|}{4}$$

$$|\varphi(\tfrac{3}{4})| = \frac{\left|\frac{3}{4} - \frac{19-\sqrt{105}}{16}\right|}{\left|\frac{3}{4} - \frac{19+\sqrt{105}}{16}\right|} = \frac{|-7+\sqrt{105}|}{|+7+\sqrt{105}|} = \frac{|-154+14\sqrt{105}|}{56}$$

$$= \frac{|-77+7\sqrt{105}|}{28} = \frac{|-11+\sqrt{105}|}{4}$$

$$\varphi(1)=\left|\frac{1-\frac{19-\sqrt{105}}{16}}{1-\frac{19+\sqrt{105}}{16}}\right|=\frac{|-3+\sqrt{105}|}{3+\sqrt{105}}=\frac{|-114+6\sqrt{105}|}{96}=\frac{19-\sqrt{105}}{16}$$

$$\varphi(-1)=\left|\frac{-1-\frac{19-\sqrt{105}}{16}}{-1-\frac{19+\sqrt{105}}{16}}\right|=\frac{|-35+\sqrt{105}|}{|+35+\sqrt{105}|}=\frac{|-1330+70\sqrt{105}|}{1120}=\frac{19-\sqrt{105}}{16}$$



$$\varphi(z)=\frac{z-\alpha}{z-\beta}$$

$$\alpha \longmapsto 0$$

$$\beta \longmapsto \infty$$

$$\varphi'=\frac{z-\beta-(z-\alpha)}{(z-\beta)^2}=\frac{\alpha-}{(z-}$$

$\alpha = z$

$$\varphi(z) = \frac{z-\alpha}{z-\beta}$$

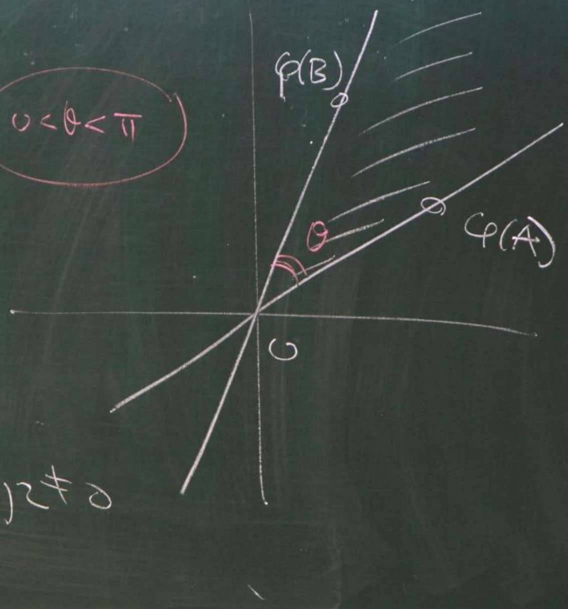
$0<\theta<\pi$

$\alpha \mapsto 0$

$\beta \mapsto \infty$

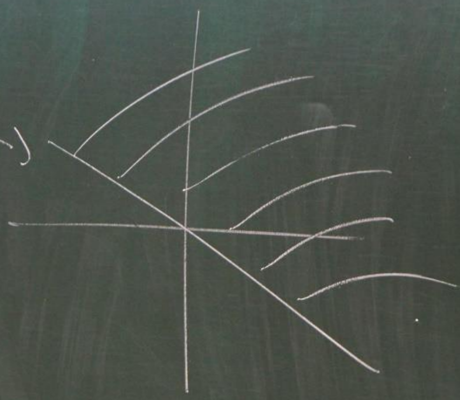
$$\varphi' = \frac{z-\beta-(z-\alpha)}{(z-\beta)^2} = \frac{\alpha-\beta}{(z-\beta)^2} \neq 0$$

$\alpha = z$



choose a holo. branch of $\log w$

$\xrightarrow{w^{\frac{\pi}{\theta}}}$



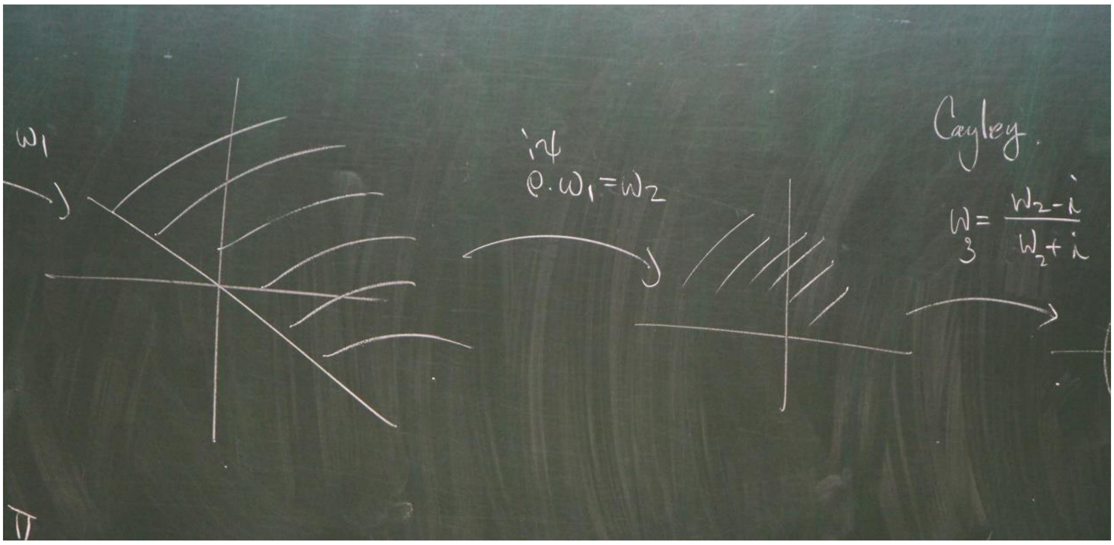
Consider

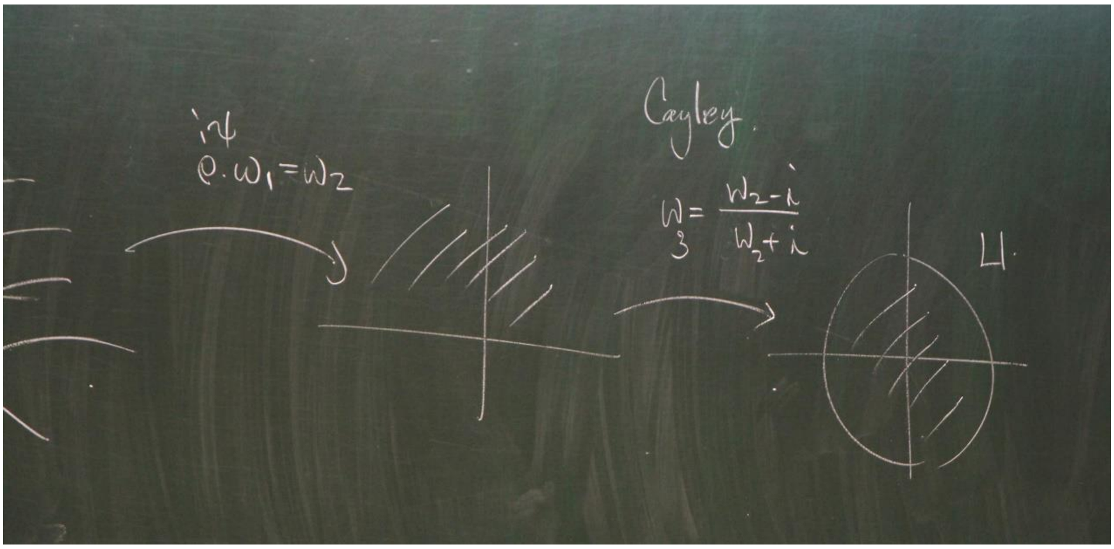
$$w^{\frac{\pi}{\theta}} \doteq \exp\left(\frac{\pi}{\theta}\log w\right)$$

$\gamma \mapsto \frac{\pi}{\theta}\cdot\gamma$

$\delta \mapsto \frac{\pi}{\theta}\cdot\delta$

$$\frac{\pi}{\theta}(\delta-\gamma) = \frac{\pi}{\theta}\cdot\theta = \pi$$


w_1
$e^{i\pi/4}\cdot w_1 = w_2$
Cayley.
$w_3 = \frac{w_2 - i}{w_2 + i}$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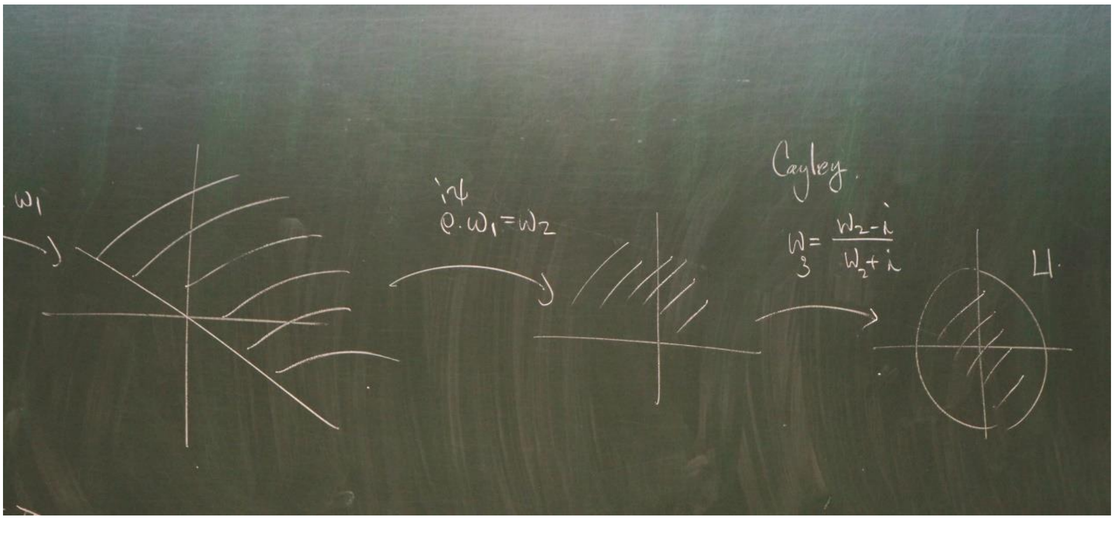
Cayley.
$w_3 = \frac{w_2 - i}{w_2 + i}$
U.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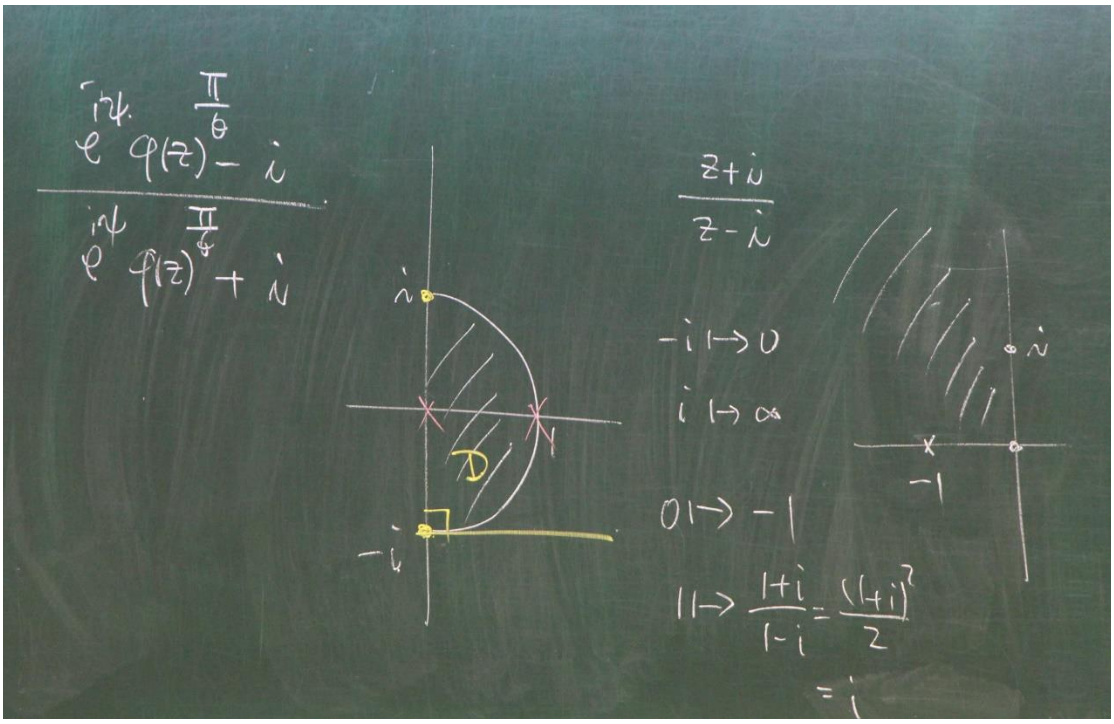
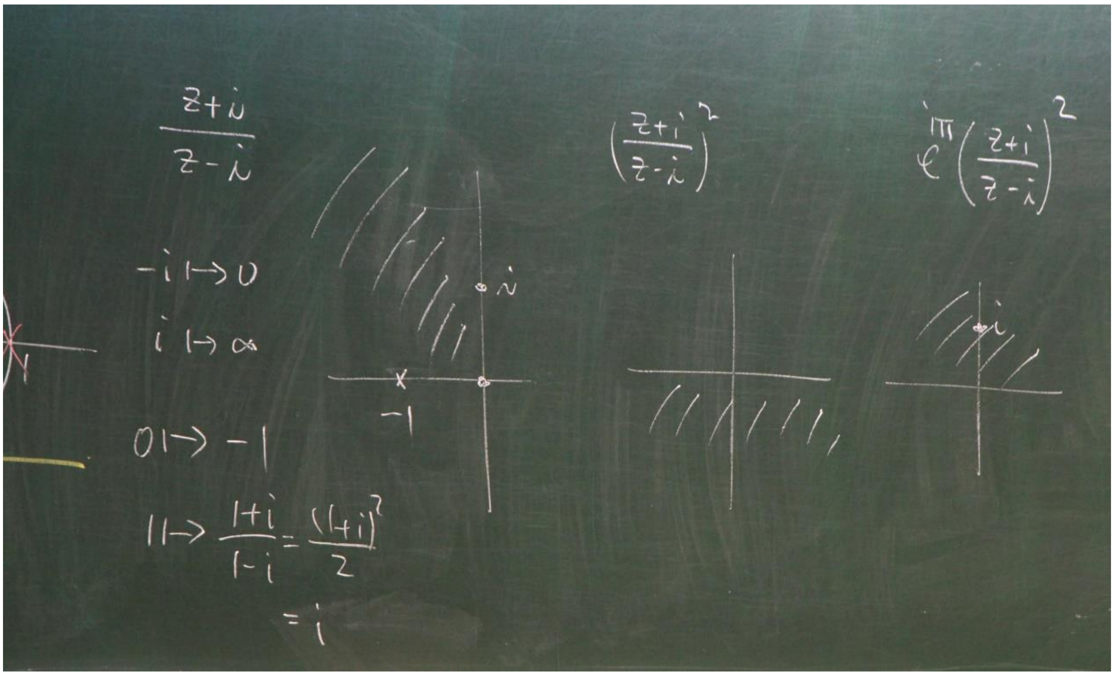

$\dfrac{e^{i\psi}\varphi(z)^{\frac{\pi}{\theta}} - i}{e^{i\psi}\varphi(z)^{\frac{\pi}{\theta}} + i}$
i
$-i$
1
D
$\dfrac{z+i}{z-i}$
$-i \mapsto 0$
$i \mapsto \infty$
$0 \mapsto -1$
$1 \mapsto \dfrac{1+i}{1-i} = \dfrac{(1+i)^2}{2}$
$= i$
-1
i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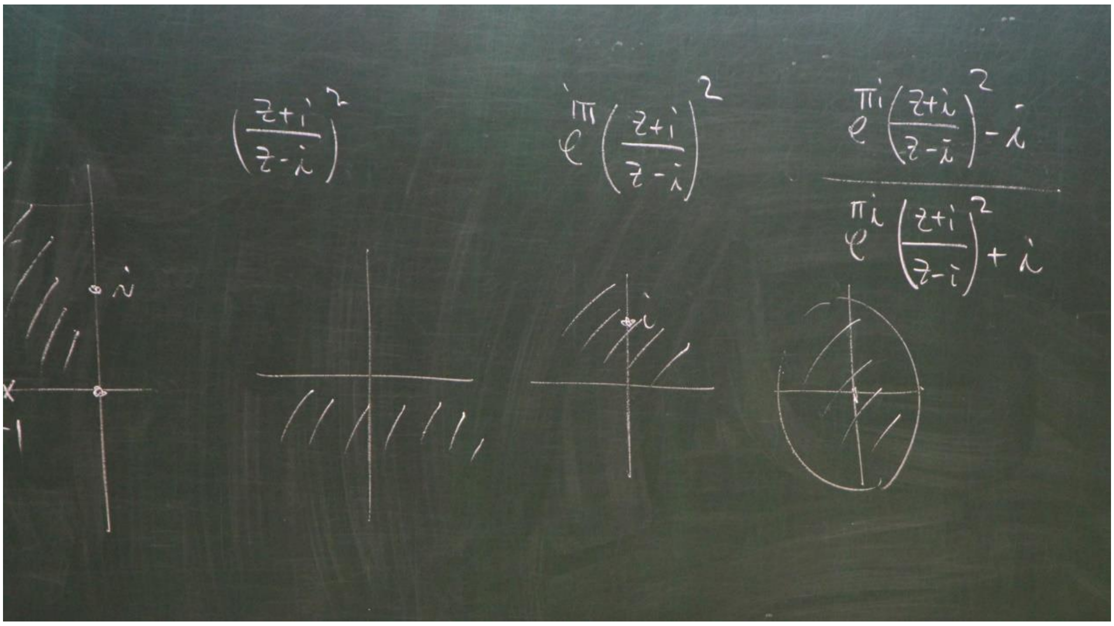
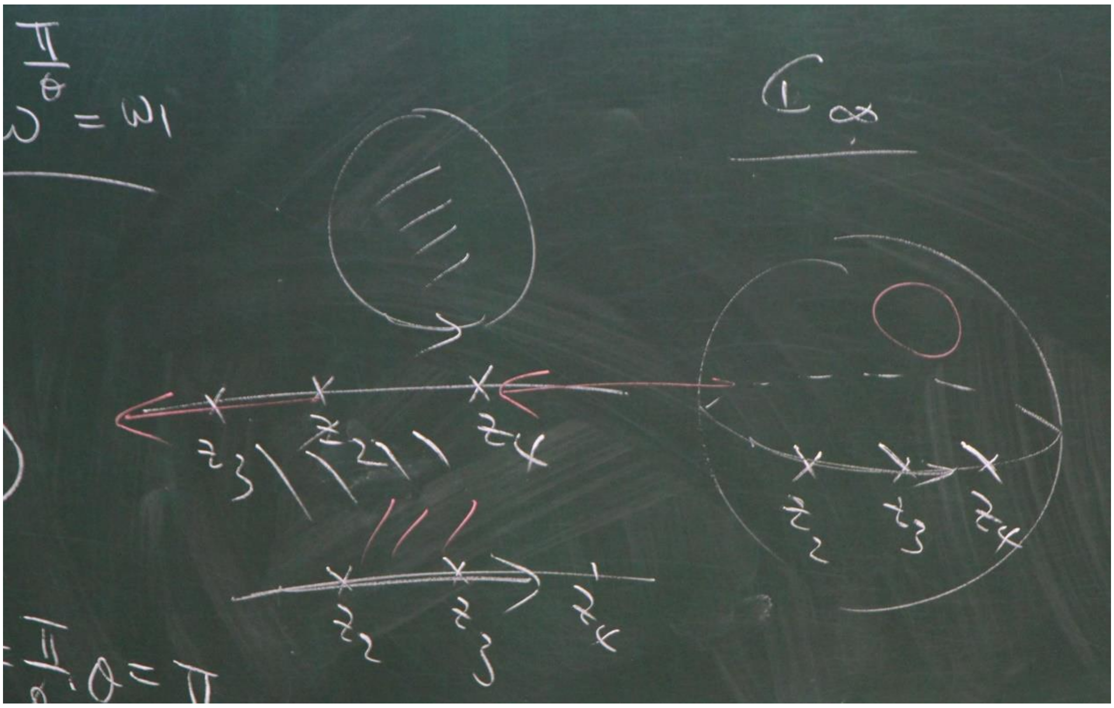
$1 \mapsto \dfrac{1+i}{1-i} = \dfrac{(1+i)^2}{2}$
$= i$
-1
i
$\left(\dfrac{z+i}{z-i}\right)^2$
$e^{i\pi}\left(\dfrac{z+i}{z-i}\right)^2$
i


$\left(\frac{z+i}{z-i}\right)^2$
$e^{\pi i}\left(\frac{z+i}{z-i}\right)^2$
$\frac{e^{\pi i}\left(\frac{z+i}{z-i}\right)^2-i}{e^{\pi i}\left(\frac{z+i}{z-i}\right)^2+i}$
i


$w^{\frac{\pi}{\theta}} = w_1$
$\mathbb{C}_\infty$
z_3 z_2 z_4
z_2 z_3 z_4
z_2 z_3 z_4'