



1. $u(x,y) = x^3y - xy^3$ $f(z) = (x^3y - xy^3) + i(\frac{3}{2}x^2y^2 - \frac{1}{4}x^4 - \frac{1}{4}y^4)$

$u_x = v_y = 3x^2y - y^3$

$u_y = -v_x = x^3 - 3xy^2$ $-\frac{i}{4}z^4$

$v = \int u_x dy = \frac{3}{2}x^2y^2 - \frac{1}{4}y^4 + \phi(x)$

$u_x = -x^3 + 3xy^2 = 3xy^2 + \phi'(x)$

$\phi'(x) = -x^3, \phi(x) = -\frac{1}{4}x^4 + c$

2. 3. $\lambda > 1$

4) $f(z) = \lambda - z - e^{-z} = 0$

has exactly one sol. in the right half plane

$g(z) = \lambda - z$

$h(z) = e^{-z}$

$R \gg \lambda$

I. $z = iy, |e^{-iy}| = 1$
 $|\lambda - iy| \geq \lambda > 1$

II. $z = Re^{i\theta} = R\cos\theta + iR\sin\theta$
 $|e^{-z}| = e^{-R\cos\theta} \leq 1$
 $|\lambda - z| \geq |\lambda| - |z| = \lambda - R > 1$

$f(z) = 2+z$
 $g(z) = -e^z$

$|f(iy)| = |2+iy| \geq 2$
 $|g(iy)| = |e^{iy}| = 1$
 $f(z) + g(z) = 2+z - e^z$
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$2+x - e^x = h(x)$
 $h(0) = 2+0-1 = 1$
 $\lim_{x \rightarrow \infty} (2+x - e^x) = -\infty$
 $h(c) = 0, 0 < c < \infty$

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4. $f(z) = \frac{1}{z}$ $\alpha > p < 2$

$\iint \left| \frac{1}{z} \right|^p dx dy$
 $= \iint \left(\frac{1}{r} \right)^p r dr d\theta$
 $= 2\pi \int_0^1 r^{1-p} dr$
 $= \frac{2\pi}{2-p} \left(r^{2-p} \right) \Big|_0^1 = \frac{2\pi}{2-p} < \infty$

$\frac{1}{z}$ $\alpha < p < 2$ $D = \{z \neq 0\}$

$\frac{1}{z} \int dx dy$

$\frac{1}{r} \int r dr d\theta$

$r^{-p} dr$

$\left(\frac{1}{r} \right)^p \Big|_0^{\infty} = \frac{2\pi}{2-p} < \infty$

$P[u] = h$

$|z| = r$

$q = u-h$ or $h-u$

$\boxed{\varepsilon > 0}$ $\varphi_\varepsilon(z) = \varphi(z) + \varepsilon \frac{|z|}{r}$

≤ 0

$\varphi_\varepsilon(z) \geq 0$

6.

iR

Γ

≤ 1

$-iR$

$= R - \lambda > 1$

$|z|=2$
 $|z|^2=4$
 x^2+y^2

$|z|=2, \quad g=10+x-2y^2$
 $g(1)=8$
 $10+x-2y^2$
 $= 6+4+x-2y^2$
 $= 6+x^2+y^2+x-2y^2$
 $= 6+x+x^2-y^2$