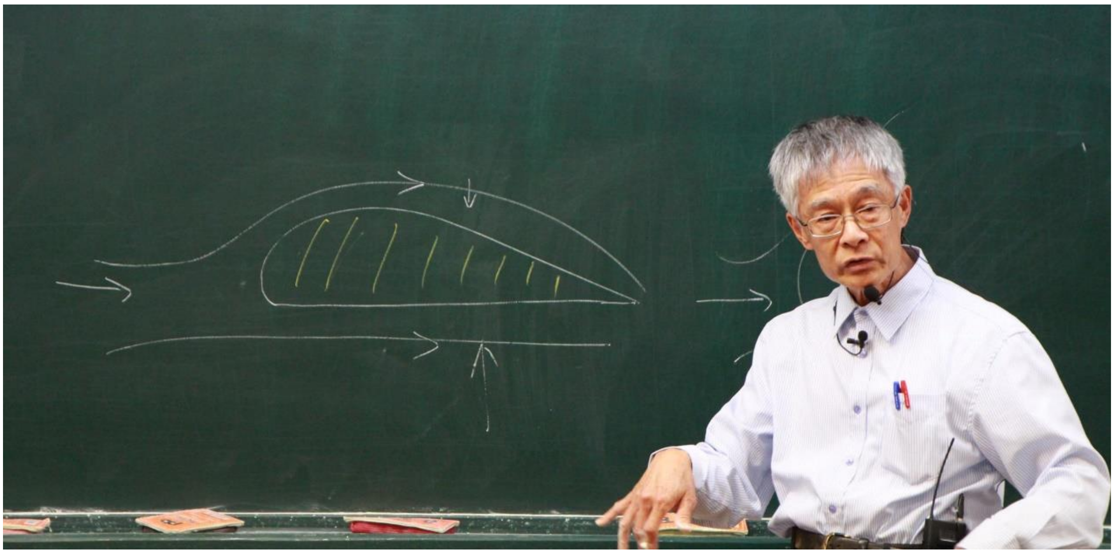
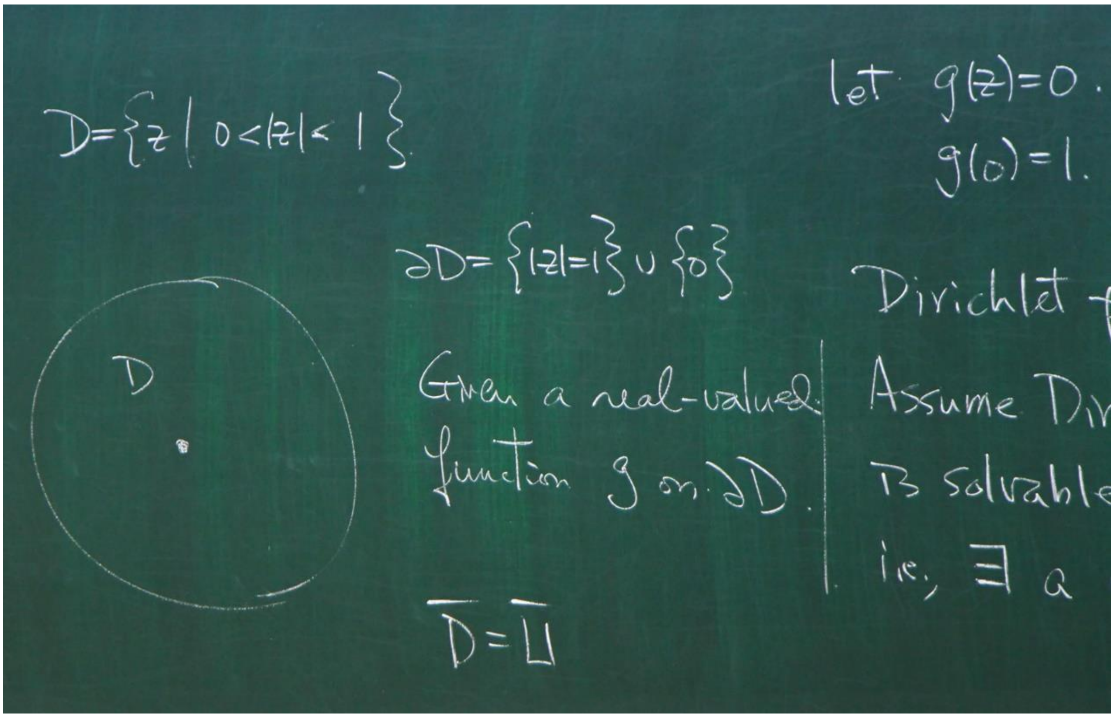
【10920 程守慶教授複變數函數論 / 第 17 堂版書】





$\cup\{0\}$

al-valued

on ∂D.

let $g(z)=0$. $|z|=1$

$g(0)=1$.

Dirichlet problem

Assume Dirichlet problem is solvable for this case.

i.e., $\exists$ a $u\in C(\overline{U})$ s.t.

(i) $u(z)=g(z)=0$, $|z|=1$

$u(0)=g(0)=1$,

(ii) $\Delta u=0$ on D

By the extension theorem

u is harmonic on $U=D\cup\{0\}$.

$\because$ $u(z)=0$ on $|z|=1$

$\Rightarrow$ $u(z)\equiv 0$ on $\overline{U}$

$\therefore u(0)=0\neq 1$.

tension theorem

armonic on $U=D\cup\{0\}$.

on $|z|=1$

on $\overline{U}$

$=0\neq 1$.

$g(z)=0$, $|z|=1$

$g(0)=1$,

on D

$\forall\, p\in\partial U$

If $\exists$ a line segment L.

P is one of the endpoint of L

s.t. $L\setminus\{p\}\subseteq \mathbb{C}\setminus D$.

nsion theorem

moric on $U=D\cup\{0\}$.

on $|z|=1$

on $\overline{U}$

$=0\neq 1$.

$=0$, $|z|=1$

$(0)=1$,

on D

$\forall\, p\in\partial U$

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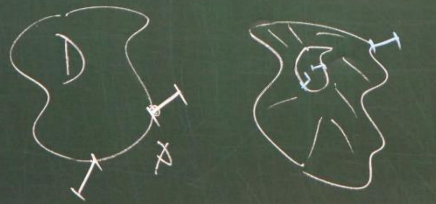
s.t. $L\subseteq \mathbb{C}\setminus D$



Perron's method

Subharmonic

L

n's method

armonic

Thm (Harnack)

real-valued

$D \subseteq \mathbb{C}$ domain. u_n = harmonic on D.

(i) If u_n uniformly converges on compact subsets to u, then u is also harmonic on D.

(ii) If $u_1 \le u_2 \le u_3 \le \cdots \le u_n \le \cdots$, $\forall z \in D$

then either u_n converges uniformly on compact subsets

or. $u_n(z)$ diverges to $+\infty$ for every $z \in D$.

L

n's method

armonic

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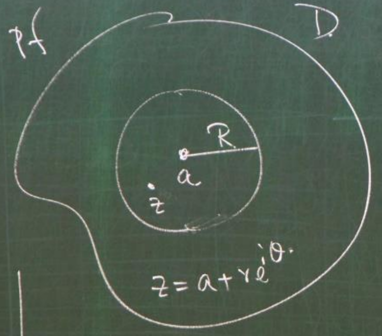
Pf

Pf



$\overline{B}(a;R) \subseteq D.$

$$u_n(z) = \frac{1}{2\pi}\int_0^{2\pi} \frac{R^2 - r^2}{R^2 - 2rR\cos(\theta - t) + r^2}\, u_n(a + Re^{it})\, dt$$

$$\downarrow$$

$$u(z) = \frac{1}{2\pi}\int_0^{2\pi} \frac{R^2 - r^2}{R^2 - 2rR\cos(\theta - t) + r^2}\, u(a + Re^{it})\, dt$$

iii, May assume $u_1(z) \geq 0$ $\qquad 0 < r < R$

If not, replace $u_n(z)$ by $u_n(z) - u_1(z)$.

$$\frac{R^2 - r^2}{R^2 + 2rR + r^2} \leq \frac{R^2 - r^2}{R^2 - 2rR\cos(\theta - t) + r^2} \leq \frac{R^2 - r^2}{R^2 - 2rR + r^2} = \frac{R^2 - r^2}{(R - r)^2}.$$

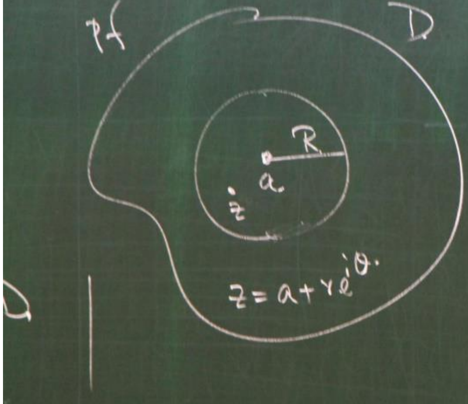
$$= \frac{R^2 - r^2}{(R + r)^2}$$

$$\boxed{\frac{R - r}{R + r} \leq \frac{R^2 - r^2}{R^2 - 2rR\cos(\theta - t) + r^2} \leq \frac{R + r}{R - r}}$$

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{R - r}{R + r}\, u_n(a + Re^{it})\, dt \leq \frac{1}{2\pi}\int_0^{2\pi} \frac{R^2 - r^2}{R^2 - 2rR\cos(\theta - t) + r^2}\, u_n(a + Re^{it})\, dt \leq \frac{1}{2\pi}\int_0^{2\pi} \frac{R + r}{R - r}\, u_n(a + Re^{it})\, dt.$$

$$\boxed{\frac{R - r}{R + r}\, u_n(a) \leq u_n(a + re^{i\theta}) \leq \frac{R + r}{R - r}\, u_n(a). \quad \text{Harnack inequality.}}$$

pf

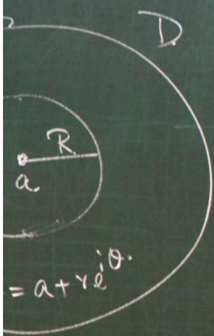


Set $A = \{ z \in D \mid u_n(z) \text{ converges} \}$

$B = D \setminus A$

$A \cup B = D$. $A \cap B = \phi$.

claim: A. B. both are open subsets.



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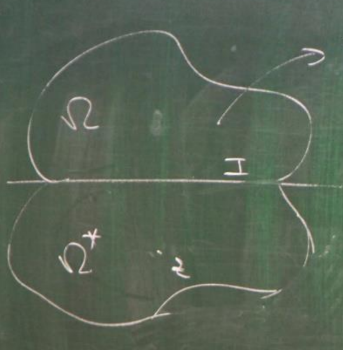
① $A = \phi$. $B = D$

② $B = \phi$. $\boxed{A = D}$

If $a \in A$ A: open.

If $a \in B$ B: open



$f(z) \in \mathcal{O}(\Omega)$

$f \in C(\Omega \cup I)$

$f|_I$: real.

$$F(z) = \begin{cases} f(z) & z \in \Omega \\ f(z) & z \in I \\ \overline{f(\bar{z})} & z \in \Omega^* \end{cases}$$

$F \in \mathcal{O}(\Omega \cup \Omega^* \cup I)$

Schwarz reflection principle for harmonic functions.

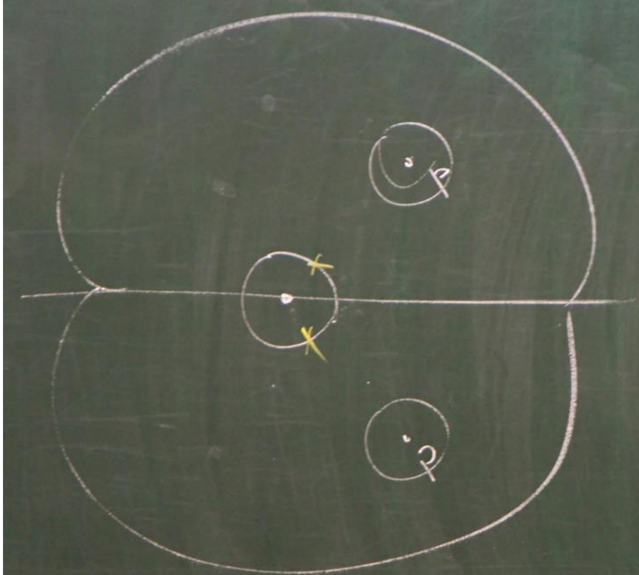
$u : \Omega \to \mathbb{R}$ harmonic. Define

$u \in C(\Omega \cup I)$

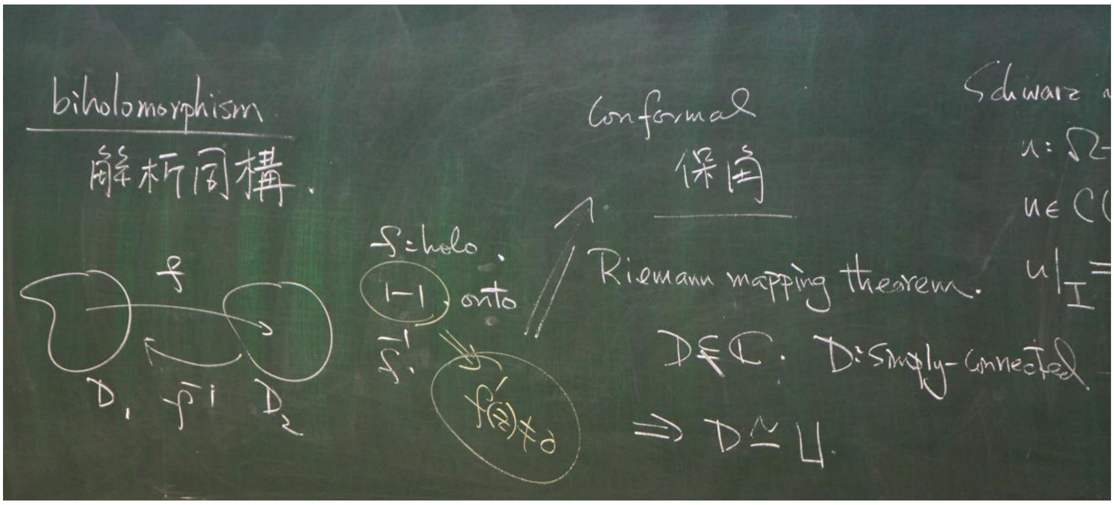
$u|_I = 0$.

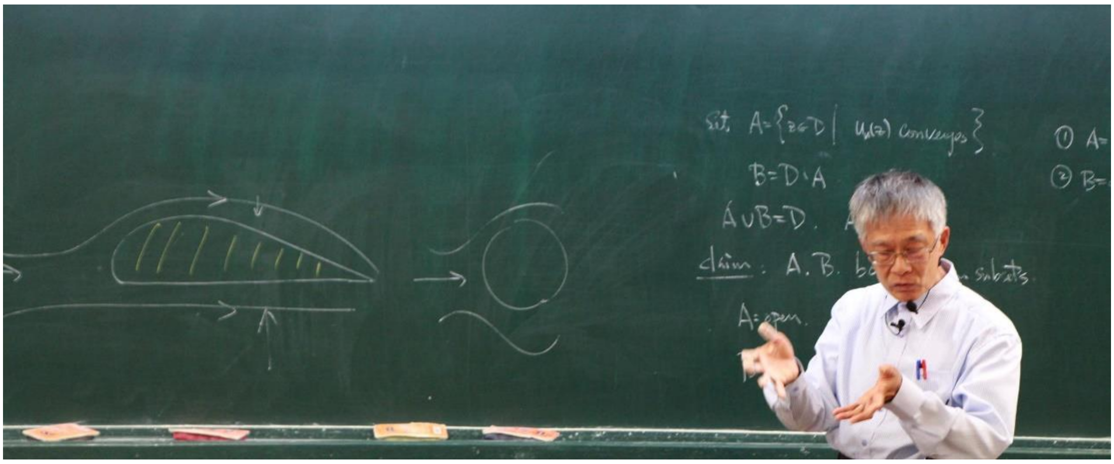
$$h(z) = \begin{cases} u(z), & z \in \Omega \\ 0, & z \in I \\ -u(\bar{z}), & z \in \Omega^* \end{cases}$$

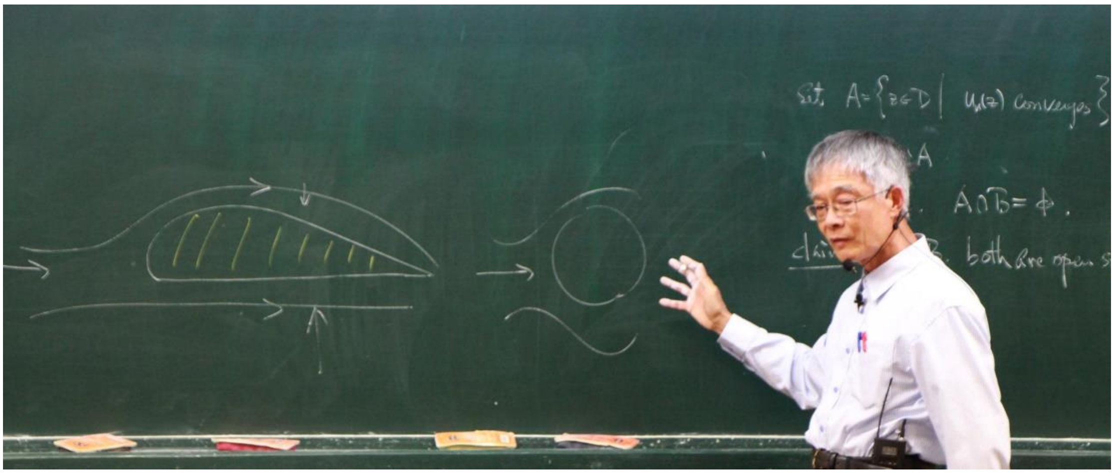
$\Rightarrow$ h is harmonic on $\Omega \cup \Omega^* \cup I$

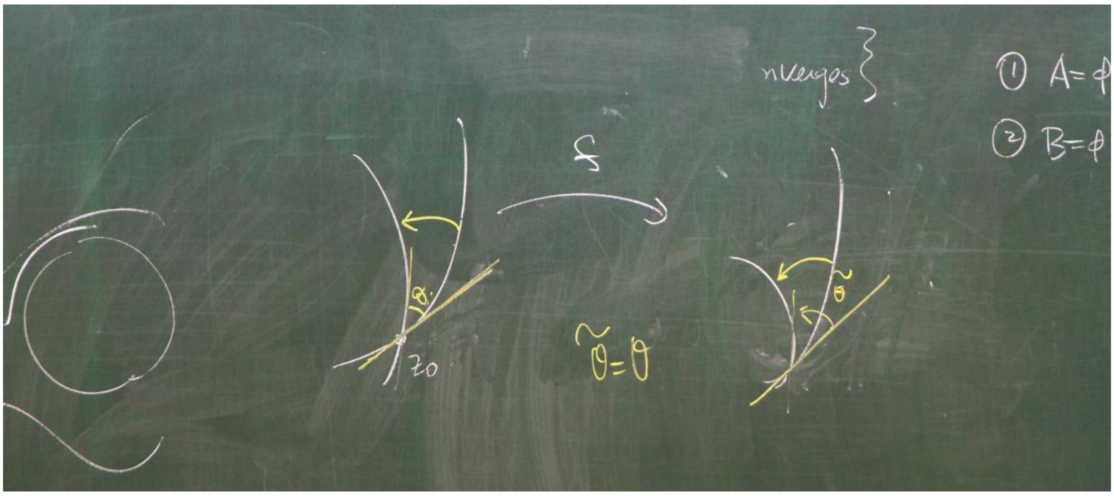


h: continuous.


biholomorphism
解析同構.
f
D_1 f^{-1} D_2
f: holo.
1-1, onto
f^{-1}:
$f'(z) \neq 0$
Conformal
保角
Riemann mapping theorem.
$D \subsetneq \mathbb{C}$. D: simply-connected
$\Rightarrow D \cong \mathbb{U}$.
Schwarz


Set $A = \{z \in D \mid u_n(z)$ converges$\}$
$B = D \setminus A$
$A \cup B = D$.
claim: A, B. both are open subsets
A: open.


Set $A = \{z \in D \mid u_n(z)$ converges$\}$
$A \cap B = \phi$.
both are open


nvegos
① A=ϕ
② B=ϕ
f
θ
z0
θ̃=θ
θ̃