Unit 6

Quine-McClusky Method

Outline

- Determination of prime implicants
- The prime implicant chart
- Petrick's method
- Simplification of incompletely specified functions

Overview (1/2)

- A systematic simplification procedure

- **Input**: minterm expansion
  Output: a minimum sum of products
  Step:
  – 1. Generate all prime implicants
    Eliminate as many literals as possible from each term by systematically applying the theorem
    $XY+XY'=X$
  – 2. Find the minimum solution
    Use a prime implicant chart to select a minimum set of prime implicants which contain a minimum number of literals

Overview (2/2)

- **Example**: $F(a,b,c)=a'b'c'+ab'c'+ab'c+abc$

All implicants:

$a'b'c', ab'c', ab'c, abc, ab', b'c', ac$

Prime implicants:

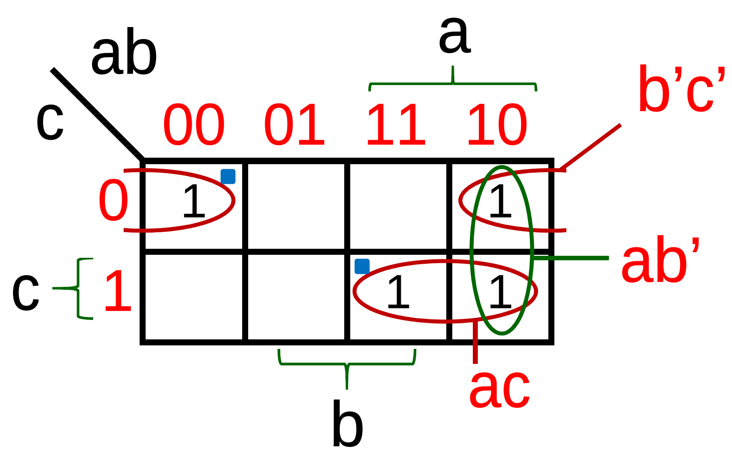
$ab', b'c', ac$

Essential prime implicants:

$b'c', ac$

Minimum sum of products:

$F(a,b,c) = b'c'+ac$



Determination of Prime Implicants (1/5)

- **Example:** Find all of the prime implicants of the function

$$f(a,b,c,d) = \Sigma m(0,1,2,5,6,7,8,9,10,14)$$

	Column I		Column II		Column III	
group 0	0	0000	0, 1	000-	0, 1, 8, 9	-00-
group 1	1	0001	0, 2	00-0	0, 2, 8, 10	-0-0
	2	0010	0, 8	-000	~~0, 8, 1, 9~~	~~-00-~~
	8	1000	1, 5	0-01	~~0, 8, 2, 10~~	~~-0-0~~
group 2	5	0101	1, 9	-001	2, 6, 10, 14	--10
	6	0110	2, 6	0-10	~~2, 10, 6, 14~~	~~--10~~
	9	1001	2, 10	-010		
	10	1010	8, 9	100-		
group 3	7	0111	8, 10	10-0		
	14	1110	5, 7	01-1		
			6, 7	011-		
			6, 14	-110		
			10, 14	1-10		

Determination of Prime Implicants (2/5)

	Column I		Column II		Column III	
group 0	0	0000 √	0, 1	000- √	0, 1, 8, 9	-00- P4
group 1	1	0001 √	0, 2	00-0 √	0, 2, 8, 10	-0-0 P5
	2	0010 √	0, 8	-000 √	~~0, 8, 1, 9~~	~~-00-~~
	8	1000 √	1, 5	0-01 P1	~~0, 8, 2, 10~~	~~-0-0~~
group 2	5	0101 √	1, 9	-001 √	2, 6, 10, 14	--10 P6
	6	0110 √	2, 6	0-10 √	~~2, 10, 6, 14~~	~~--10~~
	9	1001 √	2, 10	-010 √		
	10	1010 √	8, 9	100- √		
group 3	7	0111 √	8, 10	10-0 √		
	14	1110 √	5, 7	01-1 P2		
			6, 7	011- P3		
			6, 14	-110 √		
			10, 14	1-10 √		

All of the prime implicants:

P1 = {1,5} = 0-01 = *a'c'd*

P2 = {5,7} = 01-1 = *a'bd*

P3 = {6,7} = 011- = *a'bc*

P4 = {0,1,8,9} = -00- = *b'c'*

P5 = {0,2,8,10} = -0-0 = *b'd'*

P6 = {2,6,10,14} = --10 = *cd'*

Determination of Prime Implicants (3/5)

- Find *all* of the prime implicants
  - (1) Represent each minterm by a binary code
  - (2) Find the decimal number for each binary code
  - (3) Define the number of 1's in binary number as the *index* of the number.
    - (3-1) Group all the binary numbers of the same index into a group
    - (3-2) List all the groups in a column in the index ascending order
    - (3-3) Within each group, the binary number are listed in the ascending order of their decimal-number equivalent

Determination of Prime Implicants (4/5)

- (4) Start with the terms in the set of lowest index; compare them with those, if any, in the set whose index is 1 greater, and *eliminate all redundant variables* by XY+XY'=X
- (5) Check off all the terms *that entered into the combinations*. The ones that are left are prime implicants
- (6) Repeat step (4) and (5) until no further reduction is possible

Determination of Prime Implicants (5/5)

$f = a'c'd$	$+\ a'bd$	$+\ a'bc$	$+\ cd'$	$+\ b'd'$	$+\ b'c'$
(1, 5)	**(5, 7)**	**(6, 7)**	**(2, 6, 10, 14)**	**(0, 2, 8, 10)**	**(0, 1, 8, 9)**
P1	**P2**	**P3**	**P4**	**P5**	**P6**

Minimum form ???

$$f = a'bd + cd' + b'c'$$

The Prime Implicant Chart (1/7)

- Example:

Prime Implicant Table

			0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9)	*b'c'*	*P6*	×	×					×	$\otimes$		
(0, 2, 8, 10)	*b'd'*	*P5*	×		×				×		×	
(2, 6, 10, 14)	*cd'*	*P4*			×		×				×	$\otimes$
(1, 5)	*a'c'd*	*P1*		×		×						
(5, 7)	*a'bd*	*P2*				×		×				
(6, 7)	*a'bc*	*P3*					×	×				

The Prime Implicant Chart (2/7)

Prime Implicant Table

			0	1	2	5	6	7	8	9	10	14
~~(0, 1, 8, 9)*~~	~~b'c'~~	~~P6*~~	X	X					X	⊗		
(0, 2, 8, 10)	b'd'	P5	X		X				X		X	
~~(2, 6, 10, 14)*~~	~~cd'~~	~~P4*~~			X		X				X	⊗
(1, 5)	a'c'd	P1		X		X						
(5, 7)	a'bd	P2				X		X				
(6, 7)	a'bc	P3					X	X				

			5	7
(1, 5)	a'c'd	P1	X	
~~(5, 7)~~	~~a'bd~~	~~P2~~	X	X
(6, 7)	a'bc	P3		X

No secondary essential term.

Include the essential prime implicants in the minimal sum;
The minimal sum is:
f(a,b,c,d)=b'c'+cd'+a'bd

The Prime Implicant Chart (3/7)

- Construct the Prime Implicant Table (Chart) and find the Essential Prime Implicants of the function
  - (1) Construct the prime implicant table
    - (1-1) Each column carries a decimal number at the top which correspond to the one of the *minterm* in the given function
    - (1-2) The column are assigned by such a number in *ascending order*
    - (1-3) Each row corresponds to one of the prime implicants, P1, P2, …

The Prime Implicant Chart (4/7)

- (2) Make a cross under each decimal number that is a term contained in the prime implicant represented by that row
- (3) Find *all* the columns *that contain a single cross* and circle them; place an asterisk * at the left of those rows in which you circle a cross

  The rows marked with an asterisk are the essential prime implicants

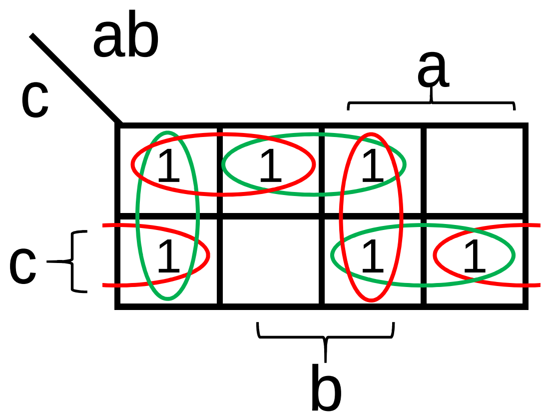
The Prime Implicant Chart (5/7)

- **Example with a cyclic prime implicant table**

Sol: Find all of the prime implicants

$$F = \Sigma\, m(0,1,2,5,6,7)$$

0	000 √	0, 1	00-	P1
1	001 √	0, 2	0-0	P2
2	010 √	1, 5	-01	P3
5	101 √	2, 6	-10	P4
6	110 √	5, 7	1-1	P5
7	111 √	6, 7	11-	P6



The Prime Implicant Chart (6/7)

Select P1 first

				0	1	2	5	6	7
① →	(0, 1)	$a'b'$	P1	X	X				
	(0, 2)	$a'c'$	P2	X		X			
	(1, 5)	$b'c$	P3		X		X		
② →	(2, 6)	bc'	P4			X		X	
③ →	(5, 7)	ac	P5				X		X
	(6, 7)	ab	P6					X	X

The minimum sum of products $F=a'b'+bc'+ac$

The Prime Implicant Chart (7/7)

Select P2 first

			0	1	2	5	6	7
(0, 1)	*a'b'*	*P1*	×	×				
(0, 2)	*a'c'*	*P2*	×		×			
(1, 5)	*b'c*	*P3*		×		×		
(2, 6)	*bc'*	*P4*			×		×	
(5, 7)	*ac*	*P5*				×		×
(6, 7)	*ab*	*P6*					×	×

The minimum sum of products $F = a'c' + b'c + ab$

The minimum sum of product is <u>not unique</u>

Petrick's Method (1/6)

- A technique for determining all minimum sum-of-products solutions from a prime implicant table
- Before applying Petrick's method, all essential prime implicants and minterms they cover should be removed from the table

Petrick's Method (2/6)

- **Example**: $F=\Sigma m(0,1,2,5,6,7)$

			0	1	2	5	6	7
P_1	(0, 1)	$a'b'$	×	×				
P_2	(0, 2)	$a'c'$	×		×			
P_3	(1, 5)	$b'c$		×		×		
P_4	(2, 6)	bc'			×		×	
P_5	(5, 7)	ac				×		×
P_6	(6, 7)	ab					×	×

Petrick's Method (3/6)

- In order to cover *minterm* 0, we must choose P_1 or P_2
  - the expression P_1+P_2 must be true

cover	0	$\Rightarrow$	P_1+P_2
	1	$\Rightarrow$	P_1+P_3
	2	$\Rightarrow$	P_2+P_4
	5	$\Rightarrow$	P_3+P_5
	6	$\Rightarrow$	P_4+P_6
	7	$\Rightarrow$	P_5+P_6

Petrick's Method (4/6)

Using $(X+Y)(X+Z)=X+YZ$ and the distributive law

$$P=(P_1+P_2)(P_1+P_3)(P_2+P_4)(P_3+P_5)(P_4+P_6)(P_5+P_6) = 1$$

$$\begin{aligned} P &= (P_1+P_2P_3)(P_4+P_2P_6)(P_5+P_3P_6) \\ &= (P_1P_4+P_1P_2P_6+P_2P_3P_4+P_2P_3P_6)(P_5+P_3P_6) \\ &= P_1P_4P_5+P_1P_2P_5P_6+P_2P_3P_4P_5+P_2P_3P_5P_6+P_1P_3P_4P_6 \\ &\quad + P_1P_2P_3P_6+P_2P_3P_4P_6+P_2P_3P_6 \end{aligned}$$

Petrick's Method (5/6)

Use $X + XY = X$ to delete redundant terms from P

$$P = P_1P_4P_5+P_1P_2P_5P_6+P_2P_3P_4P_5+\underline{P_2P_3}P_5\underline{P_6}$$

$$+P_1P_3P_4P_6+P_1\underline{P_2P_3P_6}+\underline{P_2}P_3P_4\underline{P_6}+\underline{P_2P_3P_6}$$

$$= P_1P_4P_5+ P_1P_2P_5P_6+ P_2P_3P_4P_5+P_1P_3P_4P_6+\underline{P_2P_3P_6}$$

3 implicants 4 4 4 3

Two minimum solutions:

$$F = P_1+P_4+P_5 = a'b'+bc'+ac$$

$$F = P_2+P_3+P_6 = a'c'+b'c+ab$$

Petrick's Method (6/6)

- Petrick's Method
  - 1. Label the rows of the table, P_1, P_2, …
  - 2. Form a logic function $P(P_1,P_2,\ldots)$, which is true when all of the minterms in the table have been covered
  - 3. Reduce P to a minimum sum of products using $(X+Y)(X+Z)=X+YZ$ and $X+XY=X$
  - 4. Select one solution that has minimum number of prime implicant, minimum number of literals

Simplification of Incompletely Specified Functions (1/3)

- Modify the Quine-McCluskey procedure
  - Finding the Prime Implicants
    - Treat the don't care terms as if they were required minterms
  - Forming the Prime Implicant Table
    - The don't cares are not listed at the top of the table

Simplification of Incompletely Specified Functions (2/3)

- **Example:** Simplify $F(A,B,C,D)= \Sigma m(2,3,7,9,11,13)+\Sigma d(1,10,15)$

Sol: Treat the don’t cares (1, 10, 15) as required minterms

●	1	0001 √	(1, 3)	00-1 √	(1, 3, 9, 11)	-0-1
	2	0010 √	(1, 9)	-001 √	(2, 3, 10, 11)	-01-
	3	0011 √	(2, 3)	001- √	(3, 7, 11, 15)	--11
	9	1001 √	(2, 10)	-010 √	(9, 11, 13, 15)	1--1
●	10	1010 √	(3, 7)	0-11 √		
	7	0111 √	(3, 11)	-011 √		
	11	1011 √	(9, 11)	10-1 √		
	13	1101 √	(9, 13)	1-01 √		
●	15	1111 √	(10, 11)	101- √		
			(7, 15)	-111 √		
			(11, 15)	1-11 √		
			(13, 15)	11-1 √		

Simplification of Incompletely Specified Functions (3/3)

- The don't cares are not listed at the top of the table

	2	3	7	9	11	13
(1, 3, 9, 11)		X		X	X	
*(2, 3, 10, 11)	⊗	X			X	
*(3, 7, 11, 15)		X	⊗		X	
*(9, 11, 13, 15)				X	X	⊗

$F = B'C + CD + AD$

*essential prime implicants