Unit 3

Boolean Algebra (Continued)

Outline

- Multiplying out and factoring expressions
- Exclusive-OR and equivalence operations
- The consensus theorem
- Simplification of switching expression
- Proving the validity of equation

Multiplying Out and Factoring Expressions (1/2)

$$
\left.\begin{aligned}
x(y+z) &= xy+xz &\cdots(a)\\
x+yz &= (x+y)(x+z) &\cdots(b)
\end{aligned}\right\} Duality
$$

$$\overbrace{(x+\underbrace{y)(x'}+z)} = xz + yx' \cdots (c)$$

$$
\begin{aligned}
Pf: \quad for \quad (c) \quad & x=0 \quad LHS = y \quad RHS = y\\
& x=1 \quad LHS = z \quad RHS = z
\end{aligned}
$$

Examples

1. $(Q+AB')(C'D+Q') = QC'D + Q'AB'$
2. $(A+B+C')(A+B+D)(A+B+E)(A+D'+E)(A'+C)$

$= [(A+B)+C'DE](AC+A'D'+A'E)$

$= AC + ABC + A'BD' + A'BE + A'C'DE$

Multiplying Out

Multiplying Out and Factoring Expressions (2/2)

3.

$$
\begin{aligned}
& AC + A'BD' + A'BE + A'C'DE \\
= \;& AC + A'(BD' + BE + C'DE) \\
= \;& (A + BD' + BE + C'DE)(A' + C) \\
= \;& [\underbrace{A + C'DE}_{x} + \underbrace{B}_{y}(\underbrace{D' + E}_{z})](A' + C) \\
= \;& (A + C'DE + B)(A + C'DE + D' + E)(A' + C) \\
= \;& (A + B + C')(A + B + D)(A + B + E)(A + D' + E)(A' + C)
\end{aligned}
$$

Exclusive-OR & Equivalence Operations (1/4)

$\oplus$: *Exclusive–OR* $\quad$ $+$: *Inclusive–OR*

$$0 \oplus 0 = 0$$
$$0 \oplus 1 = 1$$
$$1 \oplus 0 = 1$$
$$1 \oplus 1 = 0$$

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

$X \oplus Y = 1$ *if* $X = 1$ *or* $Y = 1$ *not both*

$$X \oplus Y = XY' + X'Y$$

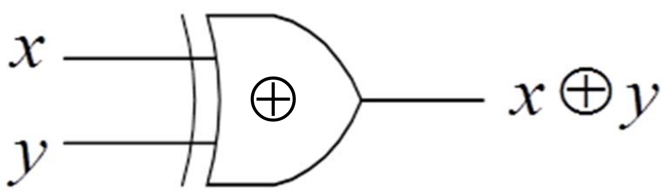
Properties :

$X \oplus 0 = X, \quad X \oplus 1 = X', \quad X \oplus Y = Y \oplus X$

$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$

$X(Y \oplus Z) = XY \oplus XZ$

$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$

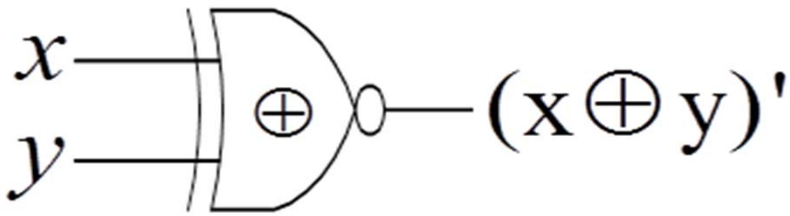


Exclusive-OR & Equivalence Operations (2/4)

$$(0 \equiv 0) = 1$$
$$(0 \equiv 1) = 0$$
$$(1 \equiv 0) = 0$$
$$(1 \equiv 1) = 1$$

X	Y	$X \equiv Y$
0	0	1
0	1	0
1	0	0
1	1	1

$$X \equiv Y = 1 \;\; if \;\; X = Y = 1 \;\; or \;\; X = Y = 0$$
$$\therefore (X \equiv Y) = XY + X'Y'$$



$$(X \oplus Y)' = (XY' + X'Y)' = (X + Y')(X' + Y) = XY + X'Y' = (X \equiv Y)$$

Exclusive-OR & Equivalence Operations (3/4)

$$\textit{Boolean Expression without } \oplus, \equiv :$$
$$X \oplus Y = XY' + X'Y$$
$$(X \equiv Y) = XY + X'Y'$$

$$\begin{aligned} F &= (A'B \equiv C) + (B \oplus AC') \\ &= [(A'B)C + (A'B)'C'] + [B(AC')' + B'AC'] \\ &= A'BC + AC' + B'C' + BA' + BC + AB'C' \\ &= AC' + B'C' + BA' + BC \\ &= C'(A + B') + B(A' + C) \end{aligned}$$

Exclusive-OR & Equivalence Operations (4/4)

$$(xy' + x'y)' = xy + x'y' \quad or$$
$$xy' + x'y = (xy + x'y')'$$

$$\begin{aligned} Ex. \quad A' \oplus B \oplus C &= (A'B' + AB) \oplus C \\ &= (A'B' + AB)C' + (A'B' + AB)'C \\ &= A'B'C' + ABC' + (AB' + A'B)C \\ &= A'B'C' + ABC' + AB'C + A'BC \end{aligned}$$

The Consensus Theorem

$$Consensus\ Theorem: xy + x'z + yz = xy + x'z$$

$$pf: \ xy + x'z + yz$$
$$= xy + x'z + (x + x')yz$$
$$= (xy + xyz) + (x'z + x'yz)$$
$$= xy(1 + z) + x'z(1 + y)$$
$$= xy + x'z$$

$$Dual\ \ form: (x + y)(x' + z)(y + z) = (x + y)(x' + z)$$

$$Ex: 1. \ a'b' + ac + bc' + b'c + ab = (a'b' + ac + bc')$$
$$2. \ (a + b + c')(a + b + d')(b + c + d')$$
$$= (a + b + c')(b + c + d')$$

Simplification of Switching Expression (1/5)

1. Combining terms

$$use \quad xy + xy' = x$$

Examples

1. $abc'd' + abcd' = abd'$
2. $ab'c + abc + a'bc = ab'c + abc + abc + a'bc = ac + bc$
3. $\underbrace{(a + bc)}_{Y}\underbrace{(d + e')}_{X} + \underbrace{a'(b' + c')}_{Y'}\underbrace{(d + e')}_{X} = d + e'$

Simplification of Switching Expression (2/5)

2. Eliminating terms

$$use \quad (a).\ x + xy = x$$

$$(b).\ Consensus\ Th's$$

$$xy + x'z + yz = xy + x'z$$

Examples

$$1.\ a'b + a'bc = a'b$$

$$2.\ a'bc' + bcd + a'bd = a'bc' + bcd$$

Simplification of Switching Expression (3/5)

3. Eliminating literals use $x + x'y = x + y$

Example

$$A'B + A'B'C'D' + ABCD' = A'(B + B'C'D') + ABCD' = A'(B + C'D') + ABCD'$$
$$= B(A' + ACD') + A'C'D' = B(A' + CD') + A'C'D' = A'B + BCD' + A'C'D'$$

4. Adding redundant terms

add xx', yz to $xy + x'z$, xy to x, multiply $(x + x')$

Example

$$wx + xy + x'z' + wy'z' = wx + xy + x'z' + wy'z' + wz'$$
$$= wx + xy + x'z' + wz' = wx + xy + x'z'$$

Simplification of Switching Expression (4/5)

Use all 4 methods

Example: SOP

$$\underbrace{A'B'C'D' + A'BC'D'}_{❶} + \underbrace{A'BD + A'BC'D}_{❷} + ABCD + ACD' + B'CD'$$

$$= A'C'D' + BD\underbrace{(A' + AC)}_{❸\ A'+C} + ACD' + B'CD'$$

$$= A'C'D' + A'BD + \underbrace{BCD + ACD'}_{+ABC\ ❹} + B'CD'$$

$$= A'C'D' + \underbrace{A'BD + \cancel{B}CD + \cancel{A}CD' + \overbrace{B'CD' + ABC}^{Consensus\ ACD'}}_{Consensus\ BCD}$$

$$= A'C'D' + A'BD + B'CD' + ABC$$

Simplification of Switching Expression (5/5)

Example: POS

$$\underbrace{(A'+B'+C')(A'+B'+C)}_{❶}(B'+C)(A+C)\underbrace{(A+B+C)}_{❷}$$

Use duals of the theorem

$$= (A'+B')\underbrace{(B'+C)}_{❸}(A+C)$$

$$= (A'+B')(A+C)$$

↑*Consensus Th.*

Proving the Validity of Equation (1/2)

Example 1.

prove $A'BD' + BCD + ABC' + AB'D$

$= BC'D' + AD + A'BC$ is valid

Methods:

1. Construct Truth Table for LHS and RHS

2. Simplify LHS and RHS

$$A'BD' + BCD + ABC' + AB'D + BC'D' + A'BC + ABD$$

$$A'BD' \quad A'BD' \quad BCD$$

$$ABC' \quad BCD \quad ABC'$$

$$AD + A'BD' + BCD + ABC' + BC'D' + A'BC$$

$$= BC'D' + AD + A'BC$$

Proving the Validity of Equation (2/2)

Example 2.

prove $A'BC'D + (A'+BC)(A+C'D') + BC'D + A'BC'$

$$= ABCD + A'C'D' + ABD + ABCD' + BC'D$$

$$LHS = (A'+BC)(A+C'D') + A'BC'D + BC'D + A'BC'$$

$$= (A'+BC)(A+C'D') + BC'D + A'BC'$$

$$= ABC + A'C'D' + BC'D + A'BC'$$

$$= ABC + A'C'D' + BC'D$$

$$RHS = ABC + A'C'D' + \underbrace{ABD}_{\text{consensus}} + BC'D$$

$$= ABC + A'C'D' + BC'D$$

Note

$$x+y=x+z \not\Rightarrow y=z$$

$$\because x=1, y=0, z=1 \Rightarrow 1+0=1+1 \Rightarrow 0=1$$

$$xy=xz \not\Rightarrow y=z$$

$$\because\ x=0,\ y=1,\ z=0,$$

$$\Rightarrow 0\cdot 1=0\cdot 0 \Rightarrow 0=1$$

True for ordinary algebra,

but not true for Boolean algebra

Supplement for P.13 Example

A’B’C’D’+A’BC’D’+A’BD+A’BC’D+ABCD+ACD’+B’CD’
--------- --------- (get common term A’C’D’, A’C’D’(B’+B))
------- -------- ------- (Get common term BD, BD (A’+A’C’+AC))
- ---- = A’

= A’C’D’+BD(A’+AC)+ACD’+B’CD’
------------ (expand)
= A’C’D’+A’BD+ABCD+ACD’+B’CD’
= A’C’D’+A’BD+(ABCD+ACD’+ABC)+B’CD’
------- ----- ^^^^ (Consensus theorem, take D, then combine ABCD+ABC)
= A’C’D’+A’BD+(ACD’+ABC+B’CD’)
^^^^ ------ ------ (Consensus ACD’, take B)
= A’C’D’+A’BD+ABC+B’CD’