

2018 Spring CS 210002 電路與電子學一 (Circuits and Electronics (I))

1.1: Semiconductor materials and properties

1.1.1: Intrinsic semiconductor

Device: Diode, BJT, FET

C: 2-4

Si: 2-8-4

Ge: 2-8-18-4

What is semiconductor ?

Sn: ...

Pb: ...

Conductor (metal)

Insulator

Valence electron (價電子)

Free electrons

Covalence bond (共價鍵)

$\sim 10^{22}/\text{cm}^3$

$\sim 10^{0-1}/\text{cm}^3$

$\sim 10^{10}/\text{cm}^3$

Bandgap energy E_g :

III

IV

V

B (硼)

C (碳)

$1\text{eV} = 1.6 \times 10^{-19}$ Joules

Al (鋁)

Si (矽)

P (磷)

Electron \longleftrightarrow hole

Ga (鎳)

Ge (鍺)

As (砷)

電子 電洞

Sn (錫)

(e^-) (h^+)

Pb (鉛)

$e^- h^+$ pair (電子電洞對)

n : 電子濃度 negative

p : 電洞濃度 positive

Insulator \longrightarrow Semi. \longrightarrow Conductor

Elemental semi : Si, Ge

Compound semi : GaAs (砷化鎳)

$$n = p = ni(T)$$

$$ni(T) \text{ (intrinsic carrier concentration)}$$

$$= BT^{3/2} e^{(-E_g / 2kT)}$$

For Si

$$B = 5.23 \times 10^{15} \text{ cm}^{-3} \text{K}^{-3/2}$$

$$k = 86 \times 10^{-6} \text{ eV/K (Boltzmann's Const)}$$

$$T = \text{絕對溫度 K}$$

$$E_g = 1.1 \text{ eV}$$

$$ni(T) = n = p = 1.5 \times 10^{10} / \text{cm}^3 \text{ at } T=300\text{K}$$

Semi. has two current components

e^- flow } the same direction
 h^+ flow }

Conductor has only one e^- flow

1.1.2: Extrinsic semiconductor

doping (摻雜)

why doping ?

impurities : $\begin{cases} \text{III : B} \\ \text{V : P, As} \end{cases}$

How ?

How many ?

Effect : 導電係數 \uparrow

V impurity : donate electron, $n \uparrow \uparrow$
 Donor (施體 or 施子)
 N_d : donor impurity concentration

$$n \cong N_d \gg ni$$

n-type semi.

Mass - Action law (質量作用定律)

$$n \times p = ni(T)^2$$

$$n \gg p$$

e^- : majority carrier (多數載子)

h^+ : minority carrier (少數載子)

III impurity : accept electron, p ↑↑
 Acceptor (受體 or 受子)
 Na : acceptor impurity concentration

$$p \cong Na \gg ni$$

p-type semi.

$$p \gg n$$

e^- : minority carrier

h^+ : majority carrier

charge neutrality (電中性)

doping changes current components in semiconductor

1.1.3: Drift and diffusion current

Total current in semi.

$$= e^- \text{ current} + h^+ \text{ current}$$

Two mechanisms: Drift and Diffusion

Drift current ($e^- + h^+$)

$$\text{Volt} \rightarrow E \rightarrow F \rightarrow \text{carrier moving}(Vd)$$

$$\text{Volt} \uparrow \rightarrow E \uparrow \rightarrow F \uparrow \rightarrow Vd \uparrow \rightarrow I \uparrow$$

= ohm's law

Drift velocity

Mobility (移動率) μ

$$V_d = \mu \cdot E \quad [\mu] = \left[\frac{\text{cm}}{\text{s}} \right] / \left[\frac{\text{V}}{\text{cm}} \right] = \frac{\text{cm}^2}{\text{V} \cdot \text{s}}$$

$$V_{dn} = -\mu_n \cdot E \quad \text{for } e^-, \mu_n = 1350 \text{ cm}^2/\text{V} \cdot \text{s}$$

$$V_{dp} = \mu_p \cdot E \quad \text{for } h^+, \mu_p = 480 \text{ cm}^2/\text{V} \cdot \text{s}$$

$$\begin{aligned} J &= J_n + J_p = qn\mu_n E + qp\mu_p E \\ &= (qn\mu_n + qp\mu_p) \cdot E = \sigma \cdot E \end{aligned}$$

$$\sigma(\text{Conductivity}) = \frac{J}{E} = qn\mu_n + qp\mu_p$$

$$[\sigma] = \frac{1}{\Omega \cdot \text{cm}}$$

$$R = \rho \cdot \frac{L}{A} \quad I = \frac{V}{R} \quad \rho \equiv \text{resistivity}$$

$$J \cdot A = \frac{V}{R} = \frac{V}{\rho \cdot \frac{L}{A}} = E \left(\frac{1}{\rho} \right) \cdot A$$

$$J = E \left(\frac{1}{\rho} \right) = \sigma \cdot E$$

$$\Rightarrow \frac{1}{\rho} = \sigma \quad \rho = \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$$

$$[\rho] = \left[\frac{1}{\sigma} \right] = \Omega \cdot \text{cm}$$

$$J_n = -qnV, \quad V_{dn} = -\mu_n \cdot E$$

$$J_n = -qn(-\mu_n \cdot E) = qn\mu_n \cdot E$$

$$[J_n] = \left[\frac{A}{\text{cm}^2} \right] = \frac{\text{col}}{\text{s} \cdot \text{cm}^2}$$

Impurity scattering

$$T \uparrow \quad \mu \uparrow$$

Lattice scattering

$$T \uparrow \quad \mu \downarrow$$

$$J_p = qp(\mu_p \cdot E) = qp\mu_p \cdot E$$

Intrinsic semi. $\sigma = qn_i(\mu_n + \mu_p)$

$T \uparrow \quad n_i \uparrow \quad \mu \downarrow \Rightarrow \sigma \uparrow$

n-type semi. $\sigma = qn\mu_n = qN_d\mu_n$

$T \uparrow \quad \sigma \downarrow$

p-type semi. $\sigma = qp\mu_p = qN_a\mu_p$

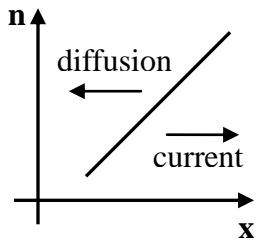
$T \uparrow \quad \sigma \downarrow$

Conductor $T \uparrow \quad \sigma \downarrow$

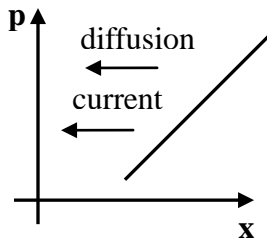
Diffusion current does not exist in conductor

Why diffusion?

Probability



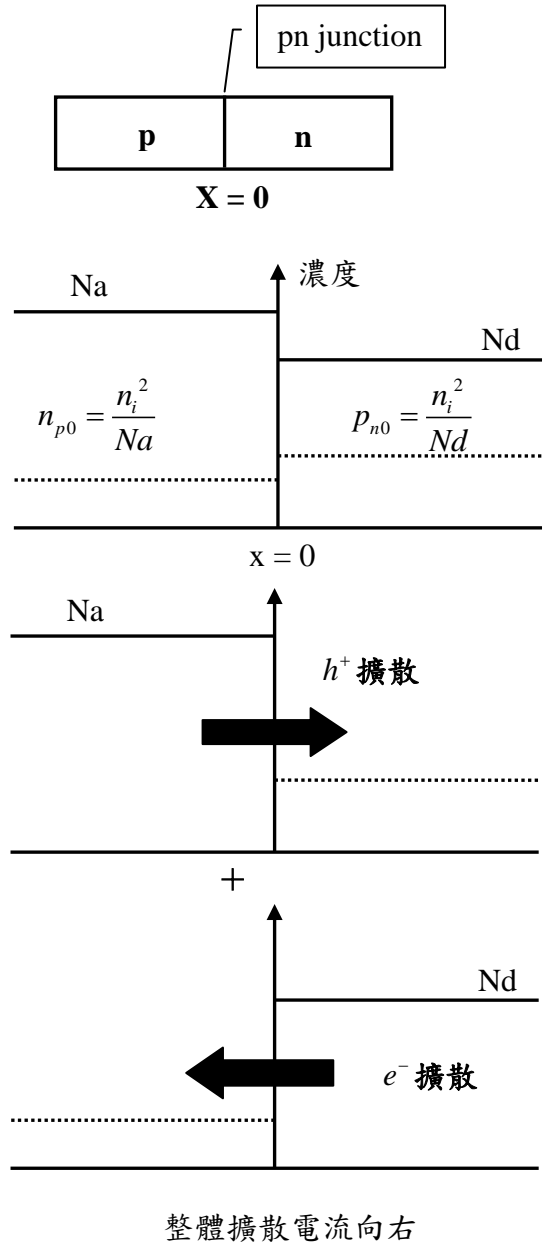
電子



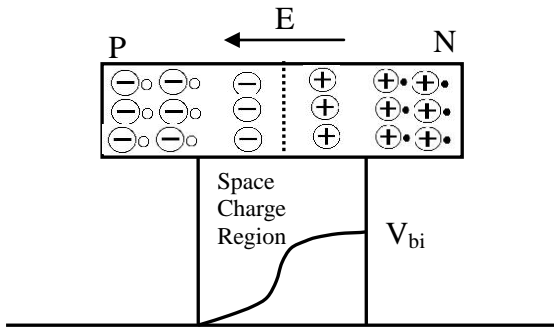
電洞

1.2: The PN junction

1.2.1: The equilibrium pn junction (open circuit pn junction)



No current !!



$$I_R = 0$$

Reverse-bias saturation current I_s

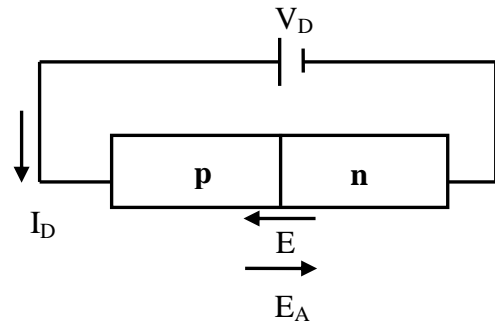
space-charge region or depletion region
(空間電荷區) (空乏區)

Built-in potential barrier (內建位障)

$$V_{bi} = V_T \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

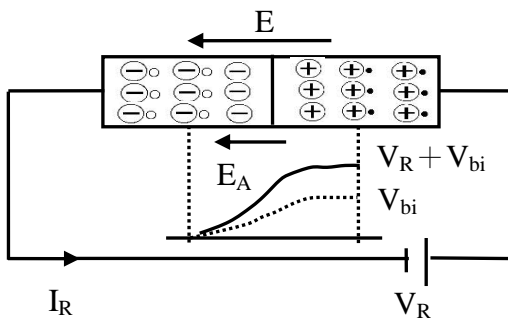
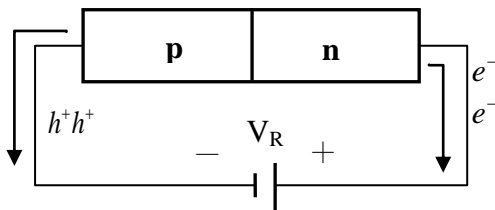
$$V_T = \frac{KT}{q} \text{ (Thermal voltage)} = \frac{T}{11600}$$

1.2.3: Forward-biased pn junction

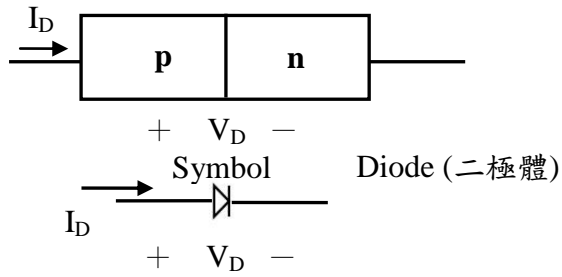


$$I_D \neq 0$$

1.2.2 Reverse-biased pn junction



1.2.4: Ideal current-voltage relationship



$$I_D = I_s (e^{V_D/nV_T} - 1)$$

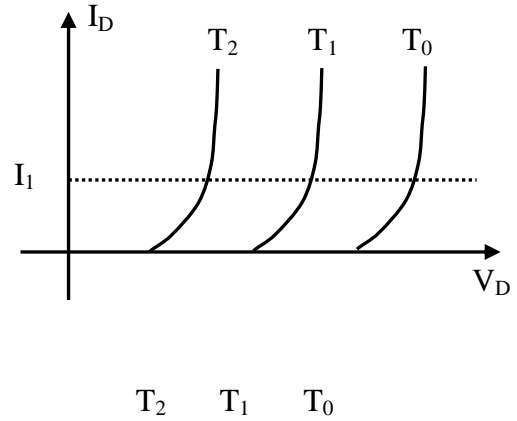
I_s : reverse-bias saturation current or scale current $\cong (10^{-15} \sim 10^{-9})$

n : ideality factor (1 or 2)

V_T : thermal voltage (熱電壓)

Cut-in voltage (切入電壓)

$$V_D \Rightarrow -2mv/^\circ C$$



breakdown (崩潰)

Avalanche breakdown (疊增崩潰)

Diode : 電壓控制開關

ON : 大電流 (外部電路決定)

OFF : 微小電流 = $-I_s$

Called 整流器 (rectifier)

$$I_s(t_2) = I_s(t_1) \times 2^{\frac{t_2-t_1}{10}}$$

Zener breakdown (齊納崩潰)

$$I_D = I_S (e^{V_D/nV_T} - 1) \quad \text{----- (1)}$$

$$V_{DD} = I_D \times R + V_D \quad \text{----- (2)}$$

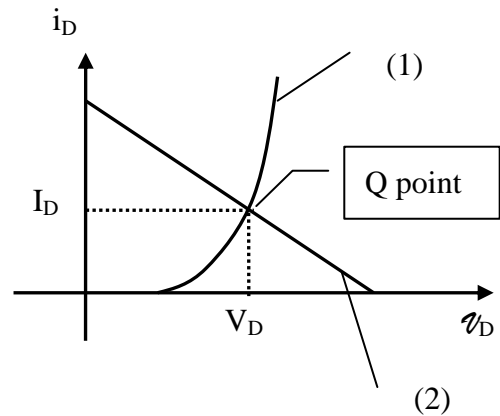
solve V_D ? I_D ?

1. Graphical analysis technique (圖解法)

zener : $T \uparrow$, breakdown voltage \downarrow

avalanche : $T \uparrow$, breakdown voltage \uparrow

PIV: (peak inverse voltage)



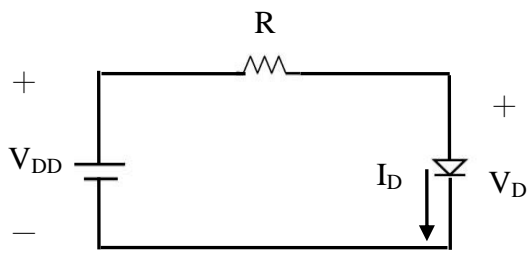
Zener diode : 工作於 breakdown region

1.3: Diodes circuit, DC analysis and models

DC 直流

AC 交流

Model



2. Iteration analysis (疊代法)

assume $V_D = 0.7V$

by (2) $I_D = \frac{V_{DD} - V_D}{R} \quad \text{----- (3)}$

by (1) $V_D = nV_T \ln\left(\frac{I_D}{I_S}\right) \quad \text{----- (4)}$

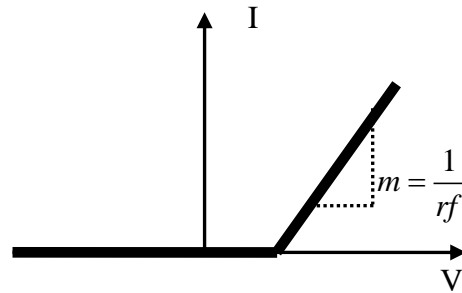
$V_D = 0.7$ 帶入(3), 求出 I_D

此 I_D 再代入(4), 求出 V_D (if 與 0.7 不符)

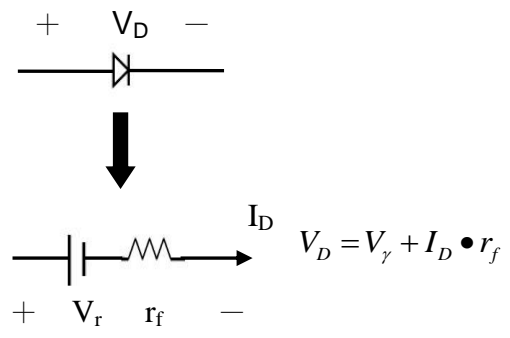
再代入(3).....直到 V_D fixed.

物理意義

b. battery-plus-resistance model



Forward



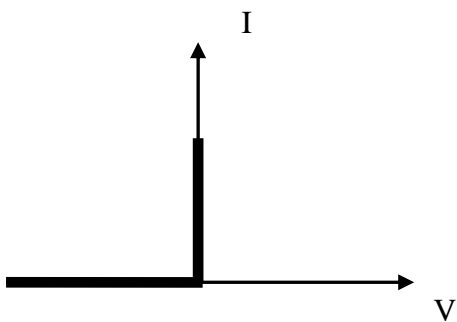
3. SPICE

Tool supports

4. Approximation (近似法)

Piecewise linear model, easy and fast

a. ideal diode model

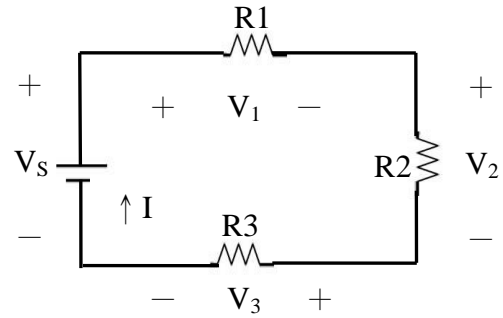
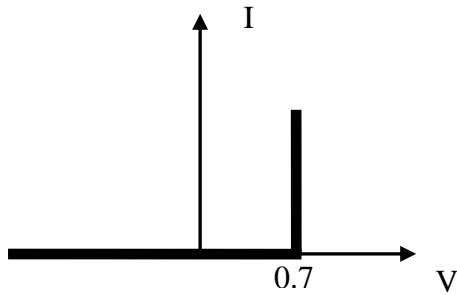


Forward $V_D = 0$ $I_D > 0$ (*short*)

Reverse $V_D < 0$ $I_D = 0$ (*open*)

Reverse (open)

c. constant-voltage drop model



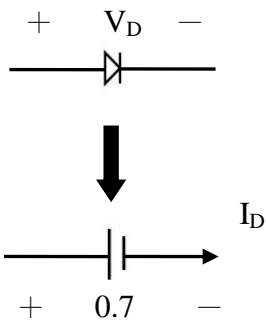
Conservation of Energy

$$IV_S - IV_1 - IV_2 - IV_3 = 0, I \neq 0$$

$$V_S - V_1 - V_2 - V_3 = 0$$

$$V_S = V_1 + V_2 + V_3$$

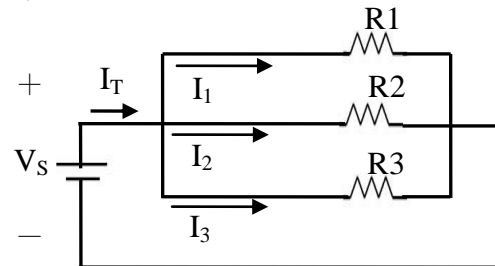
Forward



Resistors in series

Kirchhoff's Current Law (KCL)

流進 = 流出
節點電流和 = 0



Reverse (open)

$$I_T - I_1 - I_2 - I_3 = 0$$

$$I_T = I_1 + I_2 + I_3$$

Tradeoff

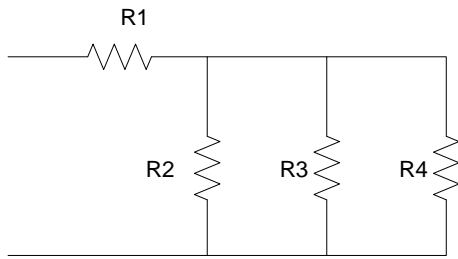
Resistors in parallel

基本電路理論

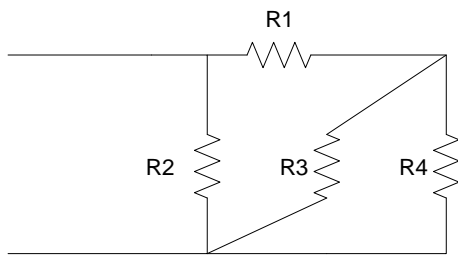
Kirchhoff's Voltage Law (KVL)

電壓昇 = 電壓降
迴路電壓和為 0

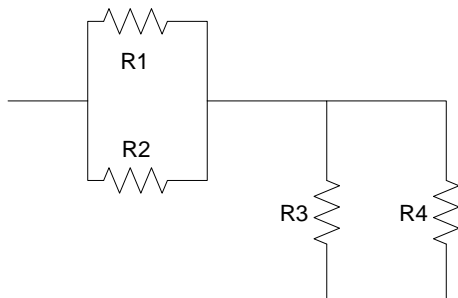
串並聯等效電路



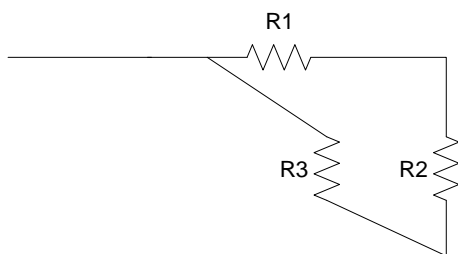
$$R = R_1 + (R_2 \parallel R_3 \parallel R_4)$$



$$R = R_2 \parallel (R_1 + R_3 \parallel R_4)$$

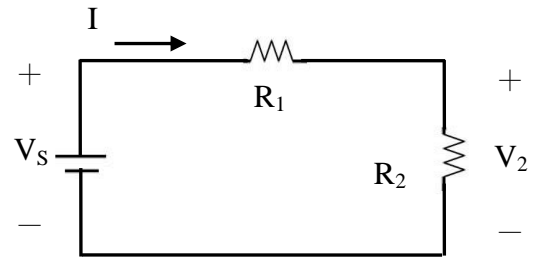


$$R = (R_1 \parallel R_2) + (R_3 \parallel R_4)$$



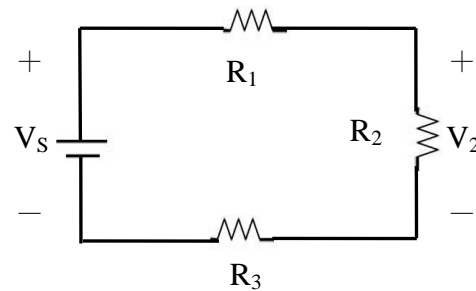
$$R = R_3 \parallel (R_1 + R_2)$$

分壓定律



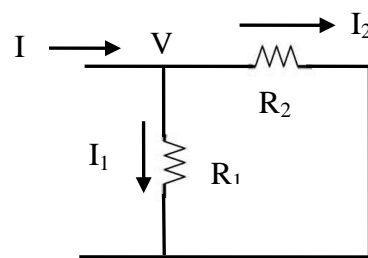
$$I = \frac{V_s}{R_1 + R_2} \quad (\text{Ohm's law})$$

$$V_2 = I \cdot R_2 = \frac{V_s}{R_1 + R_2} \cdot R_2 = V_s \cdot \frac{R_2}{R_1 + R_2}$$



$$V_2 = V_s \cdot \frac{R_2}{R_1 + R_2 + R_3}$$

分流定律



$$I_1 = \frac{V}{R_1} \quad I_2 = \frac{V}{R_2}$$

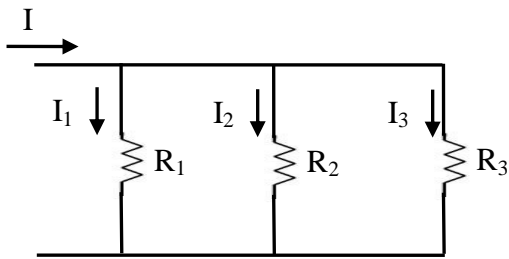
$$V = I_1 \cdot R_1 = I_2 \cdot R_2 \quad I_1 = \frac{R_2}{R_1} \cdot I_2$$

$$I = I_1 + I_2 = \frac{R_2}{R_1} \cdot I_2 + I_2$$

$$= \left(\frac{R_2 + R_1}{R_1} \right) \cdot I_2$$

$$I_2 = \frac{R_1}{R_1 + R_2} \cdot I$$

$$I_1 = \frac{R_2}{R_1 + R_2} \cdot I$$



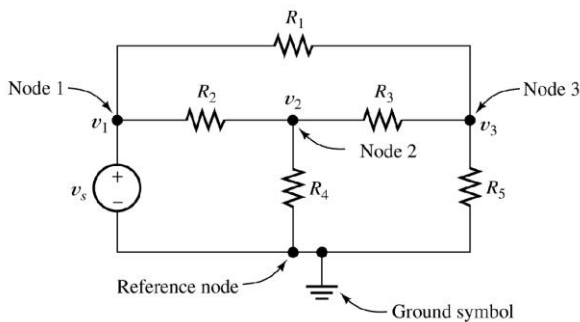
$$I_1 = \frac{R_2 \parallel R_3}{R_1 + (R_2 \parallel R_3)} \cdot I$$

Where $R_2 \parallel R_3 = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{R_2 \cdot R_3}{R_2 + R_3}$

Node Analysis (節點分析法)

- Steps :
- 1 決定參考點(及電流方向)
 - 2 指定節點電壓
 - 3 列出每個節點的 KCL 方程式
 - 4 解聯立方程式

Ex :

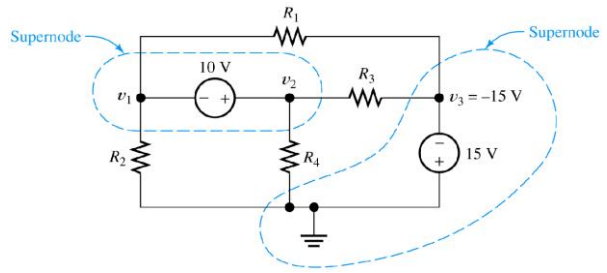


$$\frac{V_1 - V_2}{R_2} + \frac{V_3 - V_2}{R_3} - \frac{V_2}{R_4} = 0$$

$$\frac{V_1 - V_3}{R_1} = \frac{V_3 - V_2}{R_3} + \frac{V_2}{R_5}$$

$V_1 = V_3$ (可少一個變數)

Ex :



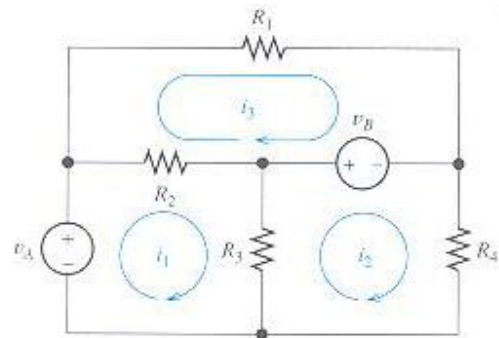
$$V_1 + 10 = V_2$$

$$\frac{V_1}{R_2} + \frac{V_2}{R_4} + \frac{V_1 - (-15)}{R_1} + \frac{V_2 - (-15)}{R_3} = 0$$

Mesh Analysis (網目分析法)

- Steps :
- 1 決定網目電流方向
 - 2 指定網目電流
 - 3 列出每個網目的 KVL 方程式
 - 4 解聯立方程式

Ex :

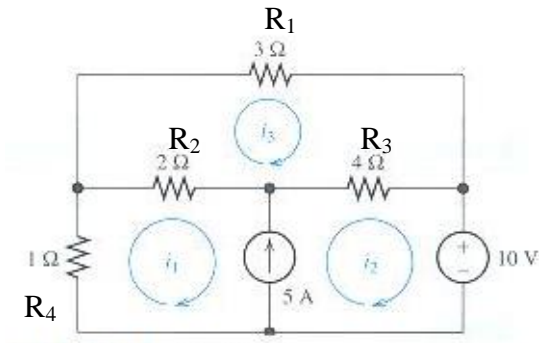


$$V_A = R_2(i_1 - i_3) + R_3(i_1 - i_2)$$

$$V_B = R_3(i_1 - i_2) - R_4 i_2$$

$$V_B = R_2(i_3 - i_1) + R_1 i_3$$

Ex :



$$10 = R_3(i_3 - i_2) + R_2(i_3 - i_1) + R_4(-i_1)$$

$$R_1 i_3 + R_3(i_3 - i_2) + R_2(i_3 - i_1) = 0$$

$$i_2 - i_1 = 5$$

Superposition Theorem (重疊定理)
欲處理電路中有多個電壓源或電流源時

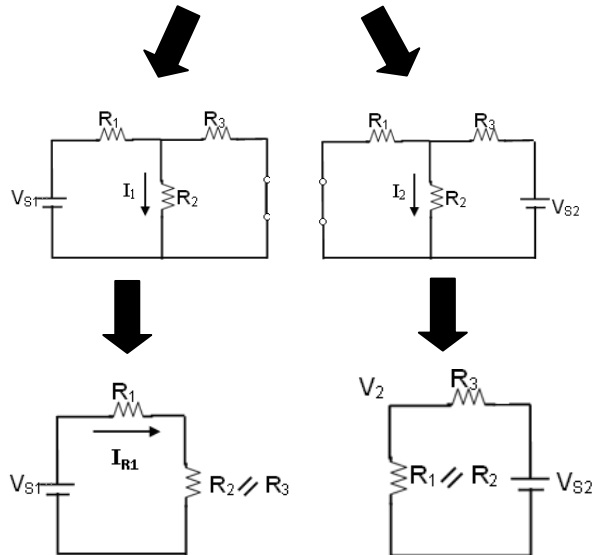
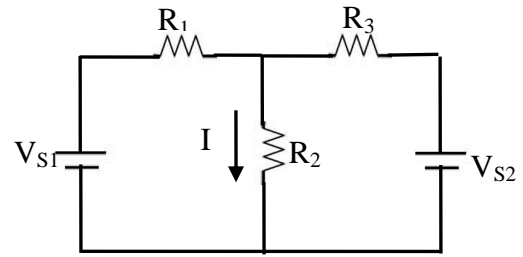
Steps : 1 一次考慮一個電源的作用
“弄死”其他電源

$\begin{cases} V \rightarrow \text{short} \\ I \rightarrow \text{open} \end{cases}$

2 解電路

3 將結果加起來

Ex :



$$I_{R1} = \frac{V_{S1}}{R_1 + (R_2 \parallel R_3)}$$

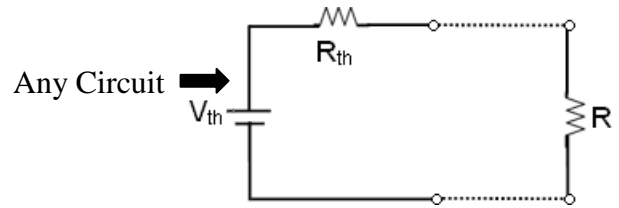
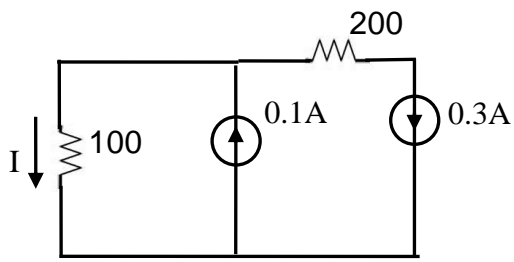
$$I_1 = I_{R1} \cdot \frac{R_3}{R_2 + R_3} \quad (\text{by 分流})$$

$$V_2 = V_{S2} \cdot \frac{R_1 \parallel R_2}{(R_1 \parallel R_2) + R_3} \quad (\text{by 分壓})$$

$$I_2 = \frac{V_2}{R_2}$$

$$I = I_1 + I_2$$

Ex :

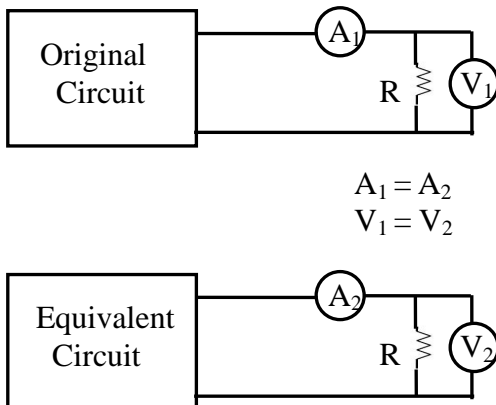


V_{th} : open circuit voltage
 R_{th} : terminal resistance
 (弄死電源)

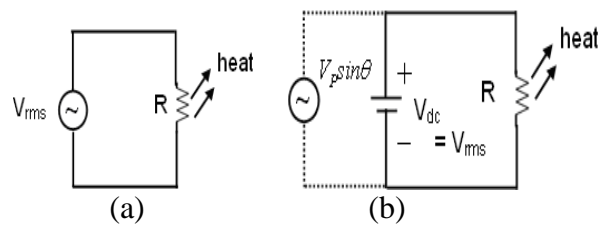
$I = -0.2 A$

Thevenin's Theorem (戴維寧定理)

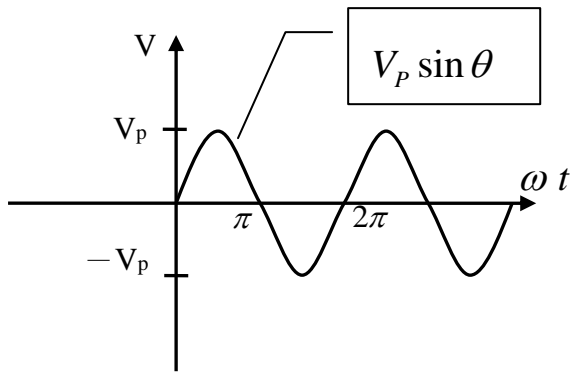
Terminal Equivalence:



Root Mean Square (RMS) value
 (均方根值) or Effective value (有效值)
 ≡ The rms value of a sine wave is equal to the DC voltage that produces the same amount of heat as the sinusoidal voltage



Same amount of heat as in (a)



2.1: Rectifier circuit (整流電路)

V_p : peak value

V_{p-p} : peak to peak value

$$V_{avg} = 0$$

$$V_p = \sqrt{2} V_{rms}$$

$$V_p \sin \theta = V_p \sin \omega t = V_p \sin(2\pi f)t$$

ω : angular frequency = $2\pi f$
(角頻率)

f : Hertz ($1/S$) 頻率

T : 週期 (s)

$$f = \frac{1}{T}$$

Half-wave 半波

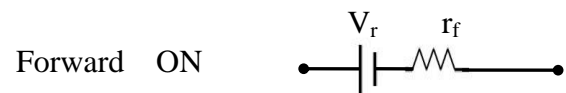
Full-wave 全波

Input : 交流(AC)訊號 (極性不固定)

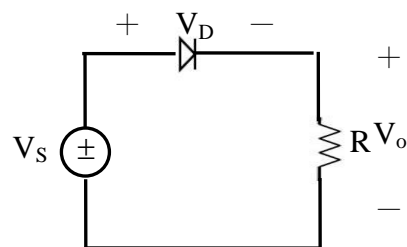
Output : 直流(DC)訊號 (極性固定)

Diode “單向導通”

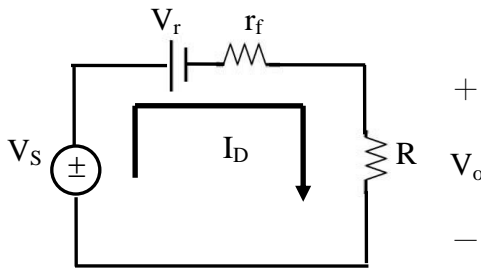
Diode model



2.1.1: Half-wave rectifier (半波整流電路)



Forward ($V_S \geq V_r$)



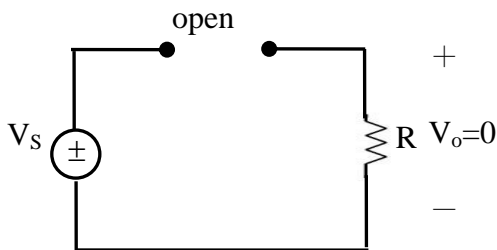
$$I_D = \frac{V_S - V_r}{r_f + R}$$

$$V_o = I_D \cdot R = (V_S - V_r) \frac{R}{r_f + R}$$

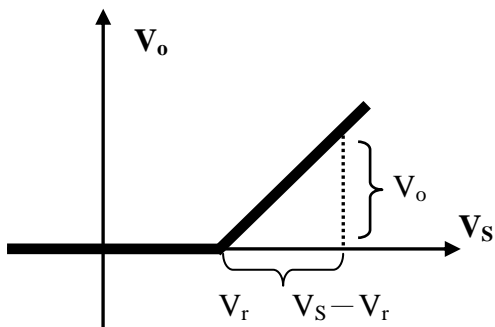
if $r_f = 0$, $V_o = V_S - V_r$

if $r_f = 0$, $V_r = 0$, $V_o = V_S$

Reverse ($V_S < V_r$)



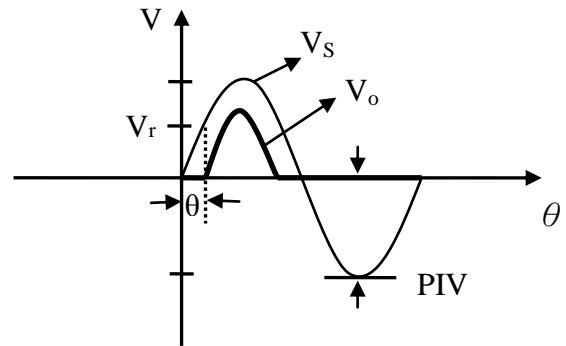
Voltage Transfer Characteristic (VTC)
(電壓轉換特性曲線)



$$\frac{V_o}{V_S - V_r} = \frac{R}{r_f + R} = m$$

$m = 1$ if $r_f \ll R$
or $r_f = 0$

V_o waveform with V_S



$$V_r = V_p \sin \theta, \quad \theta = \sin^{-1}\left(\frac{V_r}{V_p}\right) \equiv \text{cut-in angle}$$

(切入角)

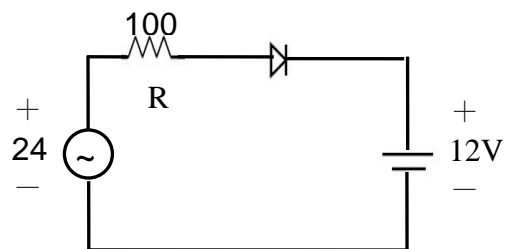
V_D waveform with V_S

$$\text{PIV} = V_p$$

Half-wave rectifier:

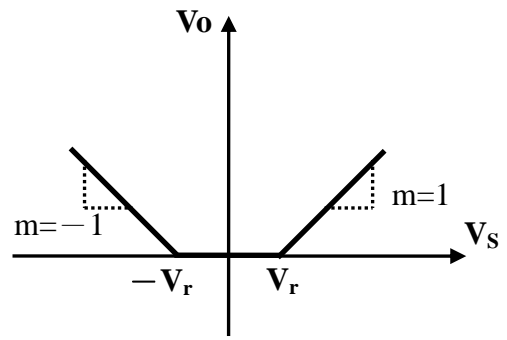
1. 不是 $t=0$ 即有 V_o (cut-in angle)
2. peak value 不到 V_p
3. on state 不足 half period

Ex:

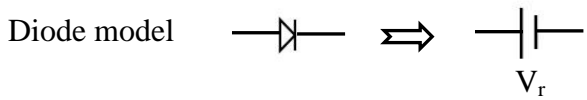
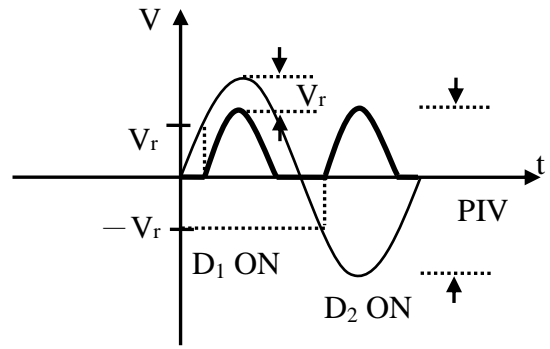
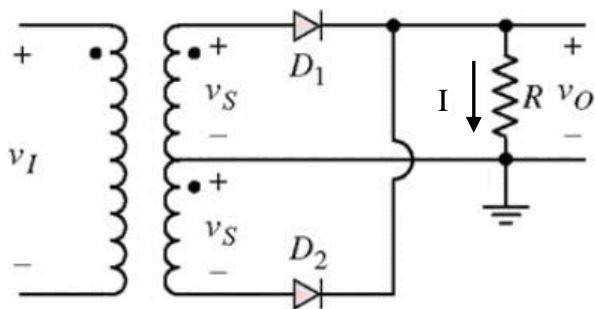


$$V_r = 0.6V \quad r_f = 0$$

VTC



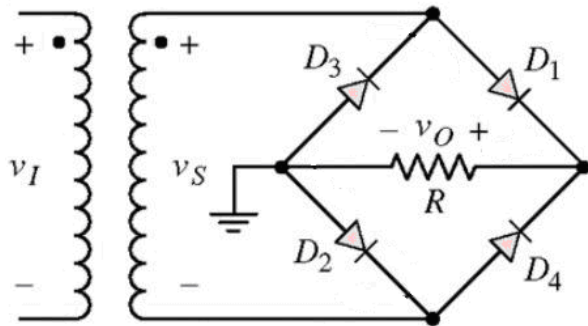
2.1.2: Full-wave rectifier (全波)



Overhead

1. 線圈 2 倍，Diode 多一個
2. $PIV = 2V_P - V_r \approx$ 半波整流的 2 倍

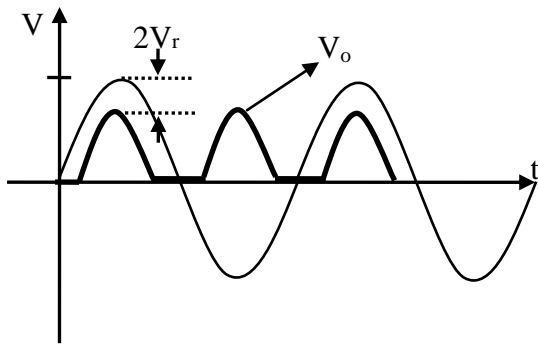
Bridge rectifier (橋式)



Ex:

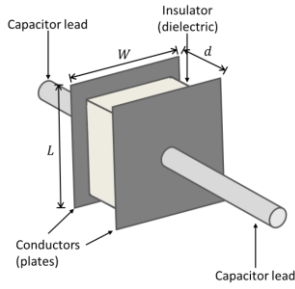
$V_I = 120 \text{ V (rms)}$, given $V_T = 0.7$
 desired $V_o = 9 \text{ V (peak)}$

find $\frac{N_1}{N_2}$ & PIV in both Full-wave
 rectifiers

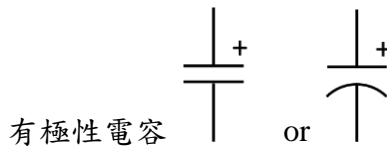
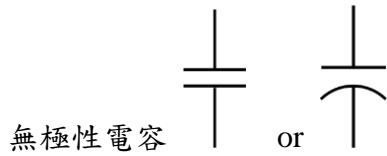


$$PIV = V_P - V_r$$

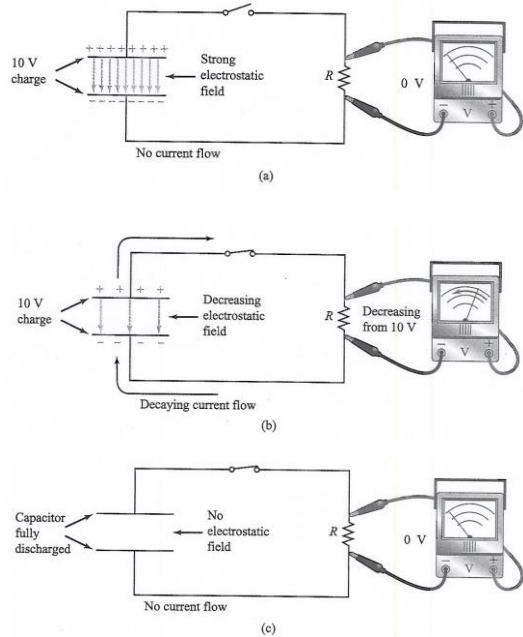
Capacitance (電容)
Capacitor (電容器)



Symbol



Discharging process



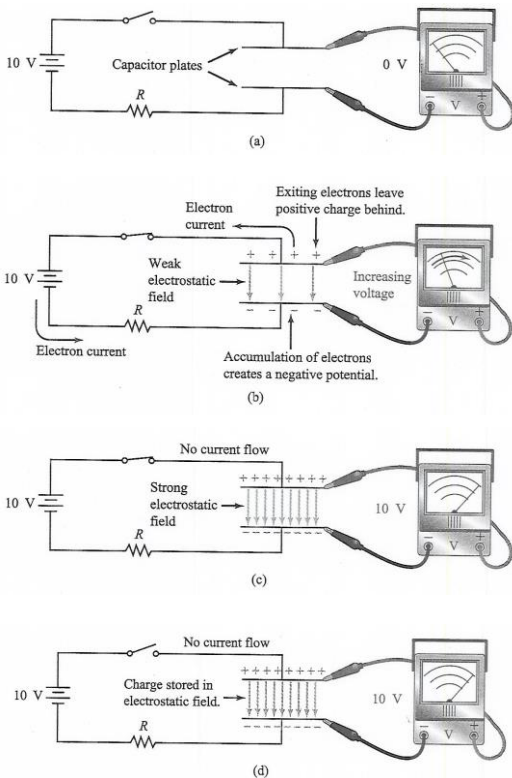
Capacitance computation for parallel-plate capacitors

$$\text{Area } A = W \times L$$

$$C = \frac{\epsilon A}{d}$$

Unit: Farad (法拉) = Coulomb/volt

Charging process



ϵ is the dielectric constant of the material (permittivity 介電常數)

$$\epsilon = \epsilon_r \epsilon_0$$

$$\epsilon_0 \cong 8.85 \times 10^{-12} \text{ F/m for vacuum}$$

ϵ_r is the relative dielectric constant

Materials	ϵ_r
Air	1.0
Mica	7.0
Quartz	4.3
Water	78.5

Ex:

Compute the capacitance of a parallel-plate capacitor having rectangular plates 10 cm by 20 cm separated by a distance of 0.1 mm. The dielectric is air.

Sol:

$$C = \frac{\epsilon A}{d} = \frac{8.85 \times 10^{-12} \times 0.02}{10^{-4}} = 1770 \times 10^{-12} \text{ F}$$

microfarads =

picofarads =

The voltage-current relationship

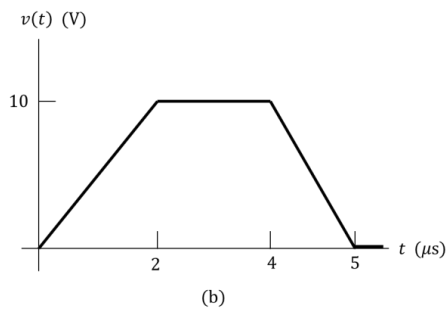
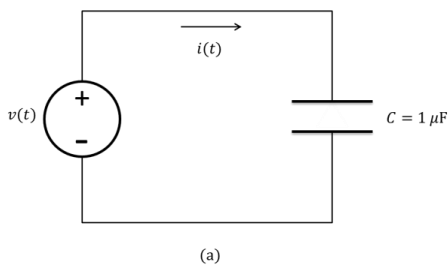
$$q(t) = Cv(t)$$

$$i(t) = \frac{dq(t)}{dt}$$

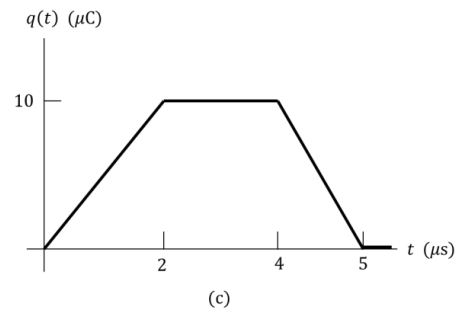
$$i(t) = C \frac{dv(t)}{dt}$$

Ex:

Given $v(t)$ as Fig. (b), determine the $i(t)$ for the circuit in Fig. (a).



$$q(t) = Cv(t) = 10^{-6}v(t)$$

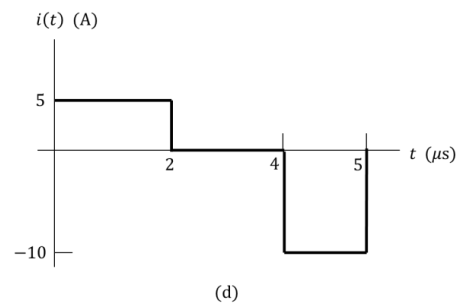


$$i(t) = C \frac{dv(t)}{dt} = 10^{-6} \frac{dv(t)}{dt}$$

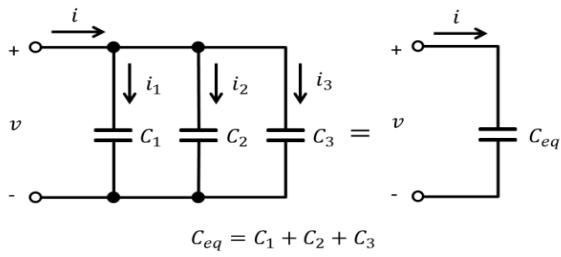
t between 0 and 2 μs

$t = 2$ and 4 μs

between $t = 4$ and 5 μs



Capacitances in parallel



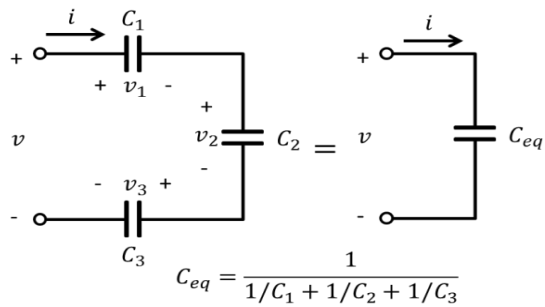
$$i_1 = C_1 \frac{dv}{dt} \quad i_2 = C_2 \frac{dv}{dt}$$

$$i_3 = C_3 \frac{dv}{dt}$$

By KCL

$$C_{eq} = C_1 + C_2 + C_3$$

Capacitances in series



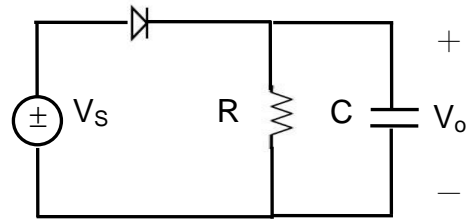
$$v_1 = \frac{q}{C_1} \quad v_2 = \frac{q}{C_2}$$

$$v_3 = \frac{q}{C_3}$$

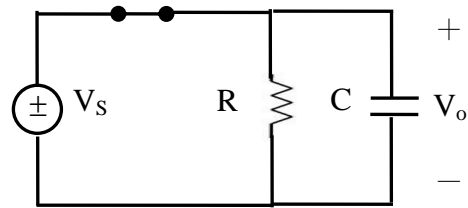
By KVL

$$v = v_1 + v_2 + v_3$$

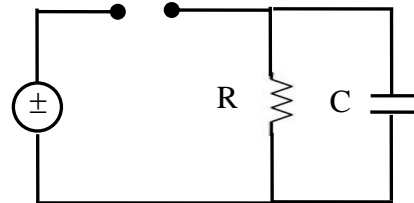
2.1.3: Filter (濾波器)

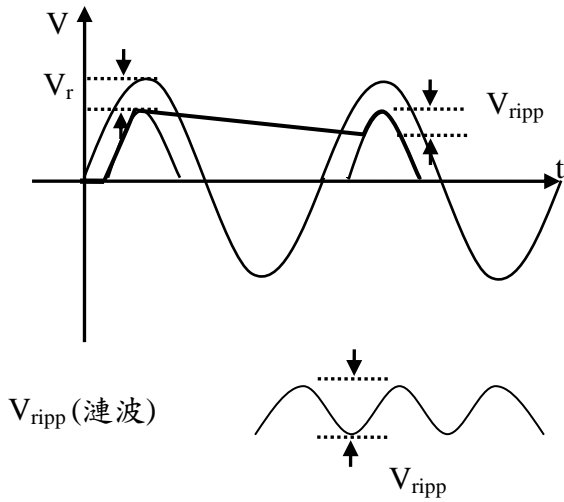


Positive cycle (電容充電)

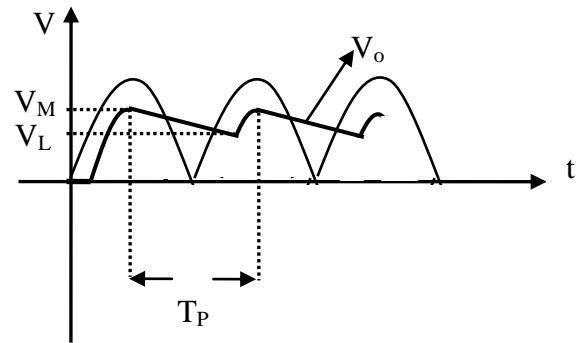


Negative cycle (電容放電)





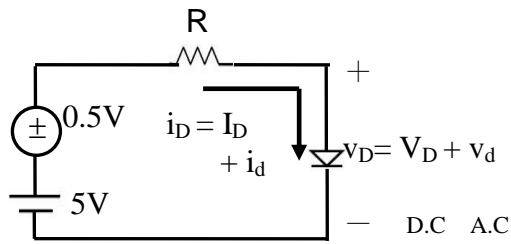
Quantify the ripple voltage



$$\begin{aligned}
 PIV : \text{ half - wave} &= V_p \quad (\text{no filter}) \\
 &= 2V_p - V_r - \frac{1}{2}V_{ripp} \quad (\text{with filter}) \\
 \text{Full - wave} &= 2V_p - V_r \quad (\text{no filter}) \\
 &= 2V_p - V_r \quad (\text{with filter})
 \end{aligned}$$

1.4: Diode AC equivalent circuits

Ex:



$$\begin{aligned}
 i_D &= I_S (e^{v_D/nV_T} - 1) = I_S (e^{(V_D+v_d)/nV_T} - 1) \\
 &\cong I_S e^{(V_D+v_d)/nV_T} \\
 &= I_S e^{V_D/nV_T} \bullet e^{v_d/nV_T} \\
 &= I_D \bullet e^{v_d/nV_T} \doteq I_D (1 + \frac{v_d}{nV_T}) \\
 &= I_D + I_D \bullet \frac{v_d}{nV_T} = I_D + i_d
 \end{aligned}$$

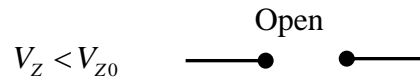
$$r_d = \frac{nV_T}{I_D} \equiv \frac{v_d}{i_d}$$

交流時 Diode models(模型化)成一個電阻 r_d

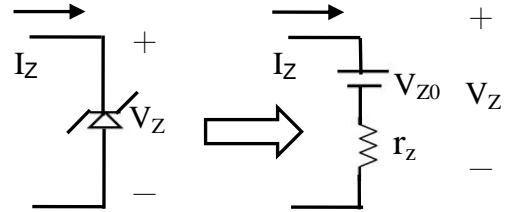
Voltage regulator (穩壓)

1.5.5: Zener diode (breakdown diode)
(齊納 or 崩潰二極體)

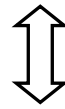
D.C model



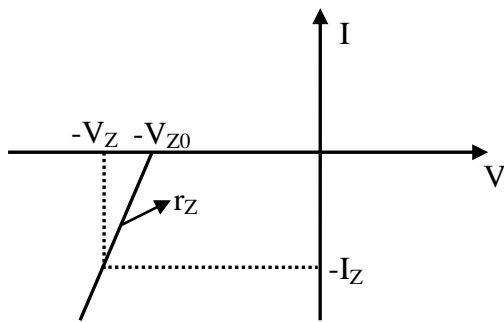
$V_Z \geq V_{Z0}$



$V_Z = V_{Z0} + I_Z \bullet r_Z$



$V_D = V_r + I_D \bullet r_f$



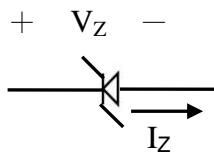
$V_Z = V_{Z0} + I_Z \bullet r_z$

I_{ZK} : knee current

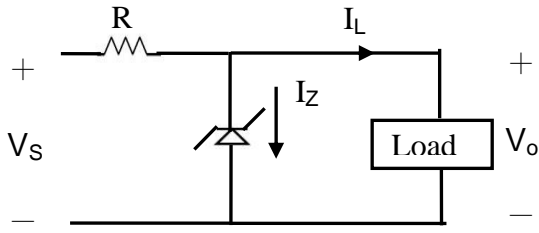
Maxpower : $P = IV (I_{Zmax})$

$I_{ZK} < I_Z < I_{Zmax}$

Symbol



2.2: Zener regulator



$$V_o = \frac{R}{R+r_Z} \cdot V_{Z0} + \frac{r_z}{R+r_z} \cdot V_S - \frac{R \cdot r_z}{R+r_z} \cdot I_L$$

V_o 受哪些因素影響？

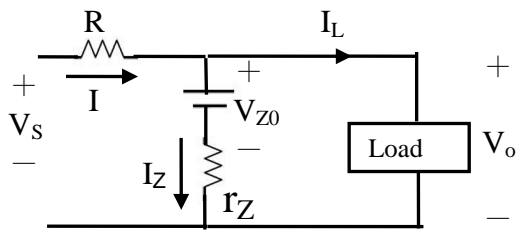
1. 輸入改變 (V_S b)
 2. 負載改變 (I_L b)
- ⇒ $V_o(V_S, I_L)$

$$\begin{aligned} \Delta V_o &= \frac{R}{R+r_Z} \cdot \Delta V_{Z0} + \frac{r_z}{R+r_z} \cdot \Delta V_S - (R \parallel r_z) \Delta I_L \\ &= \frac{r_z}{R+r_z} \cdot \Delta V_S - (R \parallel r_z) \Delta I_L \end{aligned}$$

$$\left. \frac{\Delta V_o}{\Delta V_S} \right|_{\Delta I_L=0} = + \frac{r_z}{R+r_z}$$

$$\text{line regulation} \equiv \left. \frac{\Delta V_o}{\Delta V_S} \right|_{\Delta I_L=0}$$

$$\text{load regulation} \equiv \left. \frac{\Delta V_o}{\Delta I_L} \right|_{\Delta V_S=0}$$



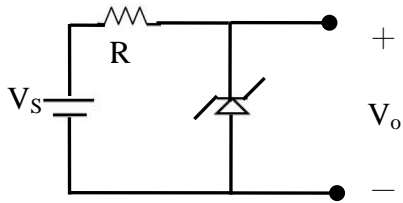
$$\left. \frac{\Delta V_o}{\Delta I_L} \right|_{\Delta V_S=0} = -(R \parallel r_z)$$

$$\begin{aligned} V_o &= V_{Z0} + I_Z \cdot r_z \\ &= V_{Z0} + (I - I_L) \cdot r_z \quad \dots\dots\dots (1) \end{aligned}$$

$$I = \frac{V_S - V_o}{R} \quad \dots\dots\dots (2)$$

Ex:

Given zener diode $V_Z = 6.8\text{V}$ at $I_{ZT} = 5\text{mA}$
 $r_z = 20\ \Omega$, $I_{zk} = 0.2\ \text{mA}$



(a) $V_S = 10\ \text{V}$, $R = 0.5\ \text{k}\Omega$, $V_o = ?$

(b) $\Delta V_S = \pm 1\ \text{V}$, $V_S = 10 \pm 1\ \text{V}$, $\Delta V_o = ?$
line regulation = ?

(d) $V_S = 10\text{V}$, $R_L = 2\text{k}\Omega \rightarrow 0.5\text{k}\Omega$, $V_o = ?$

(c) $V_S = 10$, $\Delta V_S = 0$, $R = 0.5\text{k}\Omega$, $R_L = \infty \rightarrow 2\text{k}\Omega$

$\Delta V_o = ?$ *load regulat*

$\Delta I_L = ?$

(e) $R_L = \infty \rightarrow 2k \rightarrow R_{L\min} \rightarrow 0.5k$
 Zener 有作用 | Zener 無作用

求 $R_{L\min}$ ($V_S = 10V, \Delta V_S = \pm 1V$) ?

$$R_{\min} = \frac{V_{S\max} - V_{O\max}}{I_{Z\max} + I_{L\min}}$$

$$R_{\max} = \frac{V_{S\min} - V_{O\min}}{I_{Z\min} + I_{L\max}}$$

$$V_S = 10 \pm 0.1V \quad V_{Z0} = 6.7V$$

$$r_z = 20, \quad I_{ZK} \text{ or } I_{Z\min} = 0.2mA$$

$$I_{Z\max} = 10mA, \quad I_L = 0 \sim 5mA$$

(f) $[R_{\min} \sim R_{\max}]$

$$R_{\min} = 0.32k\Omega$$

$$R_{\max} = 0.615k\Omega$$

Ex :

Given zener diode $V_Z = 7.5V$ at $I_{ZT} = 20mA$

$r_z = 10\Omega$, $I_{Zmin} = 5mA$, $V_S = 20 \pm 5V$

$I_L = 0 \sim 15mA$

(a) $R_{max} = ?$

(b) $V_o|_{at R_{max}} = ?$

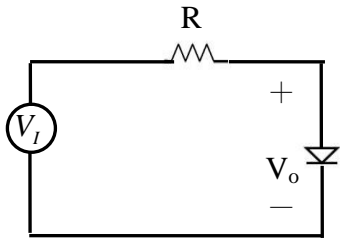
(c) $\Delta V_{o,max} (due\ to\ V_S\ b,\ I_L\ b)|_{at\ R_{max}} = ?$

$$\begin{aligned} \text{(d) percent regulation} &= \frac{V_{o,max} - V_{o,min}}{V_{o(nominal)}} \\ &= \frac{\Delta V_{o,max}}{V_{o(nom)}} \times 100\ \% \end{aligned}$$

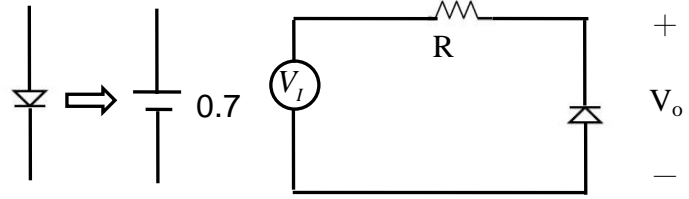
2.3.1: Clipping circuit (截波電流) or Clipper or limiting circuit

= 將 output 波形限制在某一點以上或以下的電路

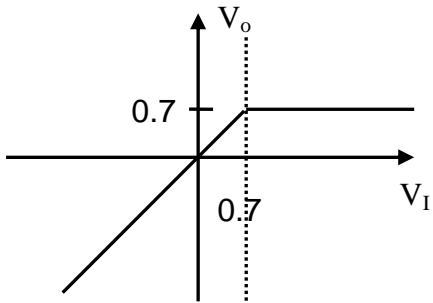
電路



電路

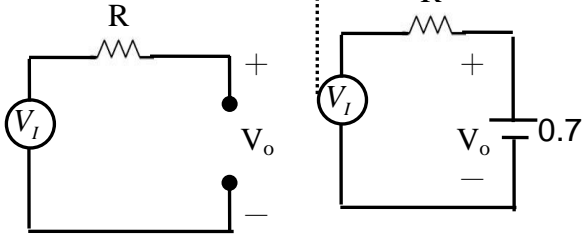


VTC

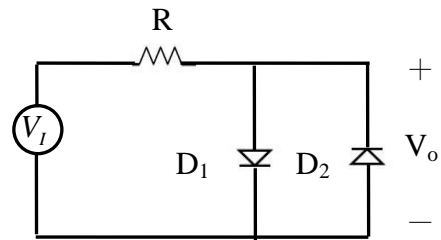


VTC

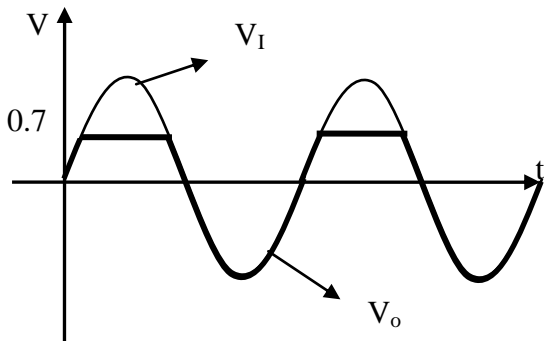
V_o 波形



電路 (double limiting clipper)

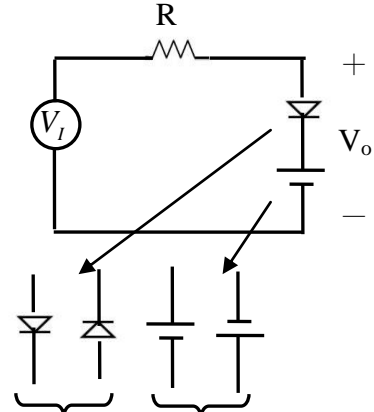


V_o 波形

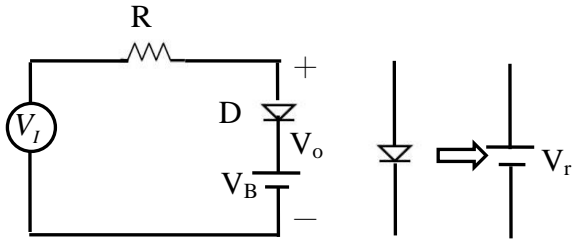


VTC

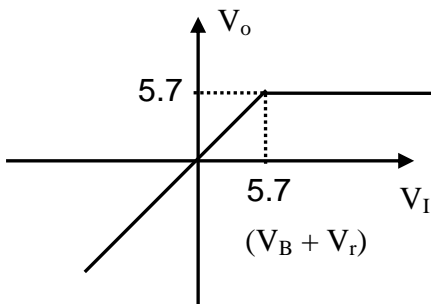
V_o 波形



電路

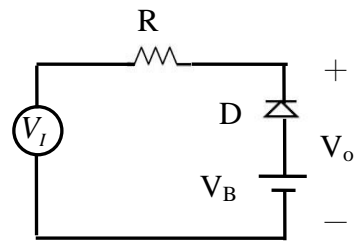


VTC

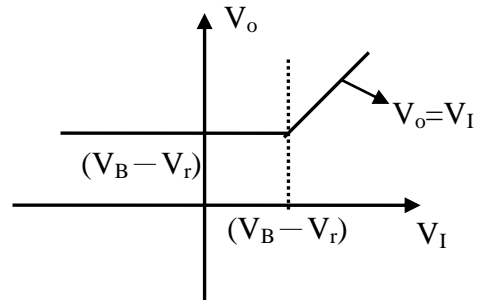


- (1) $V_I = 0$ D_{off} $V_o = V_I$
- (2) $V_I < 0$ D_{off} $V_o = V_I$
- (3) $V_I = V_B + V_r$ $D_{off \rightarrow ON}$
- (4) $V_I > V_B + V_r$ D_{ON} $V_o = V_B + V_r$

電路



VTC

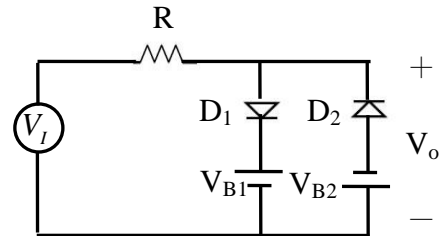


- $$V_I = 0 \quad D_{ON} \quad V_o = V_B - V_r$$
- $$V_I < 0 \quad D_{ON} \quad V_o = V_B - V_r$$
- $$V_I = V_B - V_r \quad D_{ON \rightarrow off}$$
- $$V_I > V_B - V_r \quad D_{off} \quad V_o = V_I$$

V_o 波形

V_o 波形

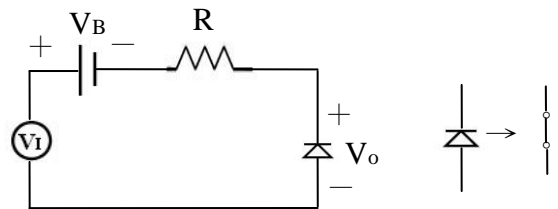
電路



VTC

V_o 波形

電路



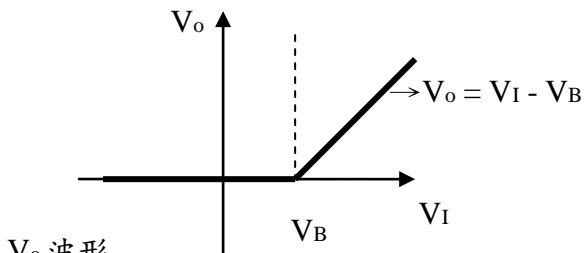
$$V_I = 0 \quad D_{\text{on}} \quad V_o = 0$$

$$V_I < 0 \quad D_{\text{on}} \quad V_o = 0$$

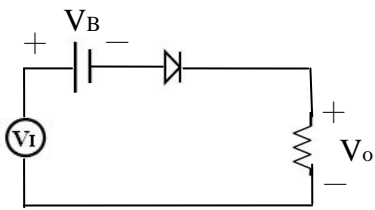
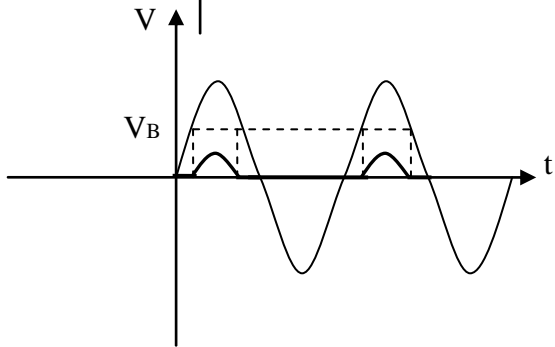
$$V_I = V_B \quad D_{\text{on}} \rightarrow \text{off}$$

$$V_I > V_B \quad D_{\text{off}} \quad V_o = -V_B + V_I$$

VTC

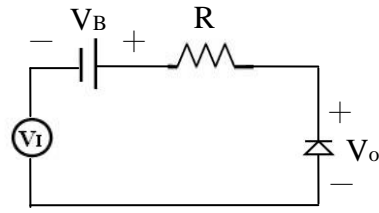


V_o 波形



Same as above

電路



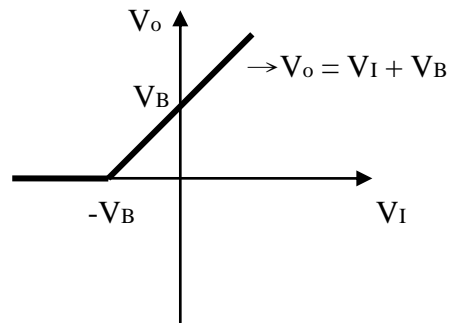
$$V_I = 0 \quad D_{\text{off}} \quad V_o = V_B + V_I = V_B$$

$$V_I > 0 \quad D_{\text{off}} \quad V_o = V_B + V_I$$

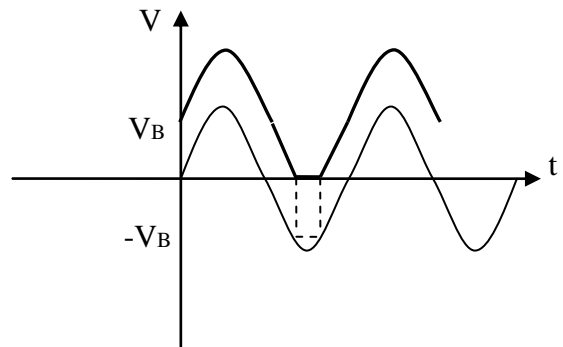
$$V_I = -V_B \quad D_{\text{off}} \rightarrow \text{on}$$

$$V_I < -V_B \quad D_{\text{on}} \quad V_o = 0$$

VTC



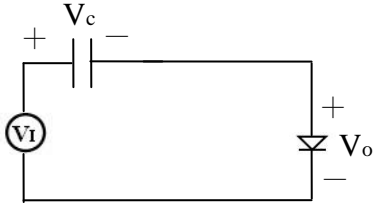
V_o 波形



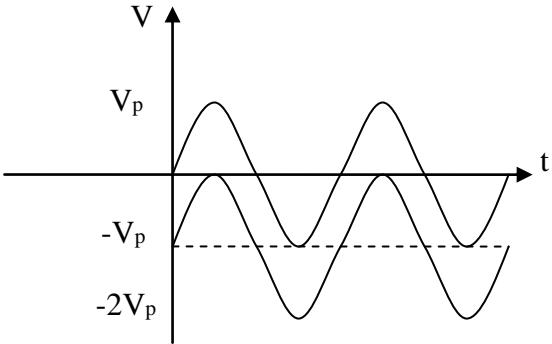
2.3.2: Clamping circuit (箝位電路)
or clamper

電路

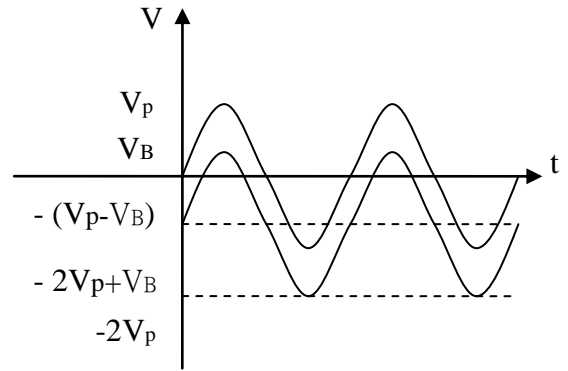
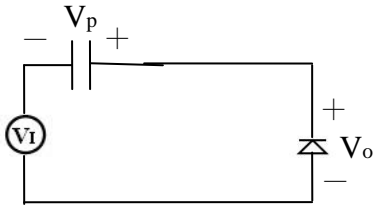
Peak Detector



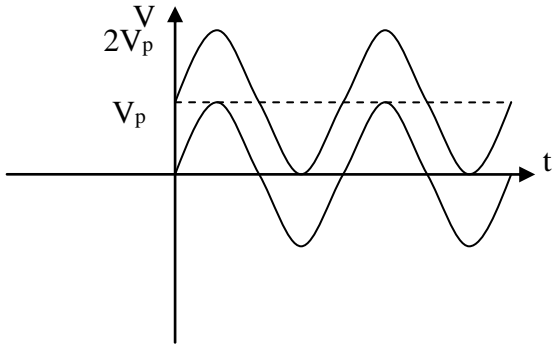
V_o 波形



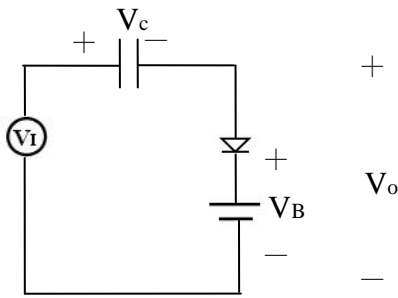
電路



V_o 波形



電路



Don 時 $V_c \max = V_p - V_B$

且維持固定值

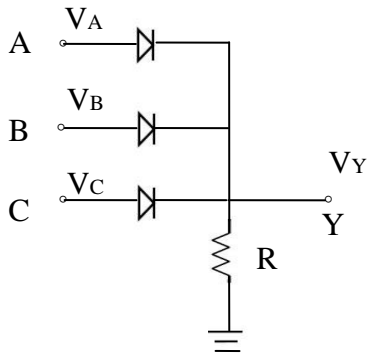
$$\Rightarrow V_o = -V_c + V_I$$

$$= -(V_p - V_B) + V_I$$

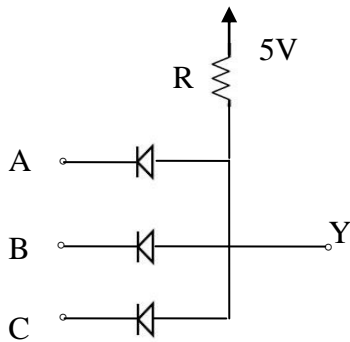
$$V_I = V_p \quad V_o = V_B$$

$$V_I = -V_p \quad V_o = -2V_p + V_B$$

2.4.2: Logic circuit

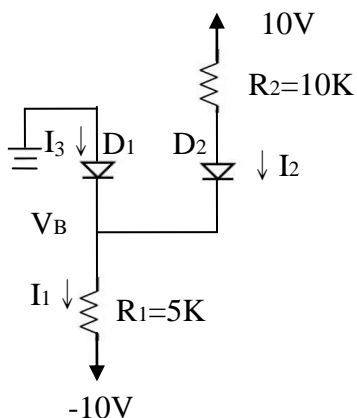


$Y = A+B+C$ (OR gate)



$Y = A \cdot B \cdot C$ (AND gate)

2.4.1: Multiple-diode circuit



Assume D_1, D_2 為 ideal diode, 求 $I_1, I_2, V_B = ?$

D_1, D_2 不是 ON, 就是 OFF, 4 種組合

ON \rightarrow short

OFF \rightarrow open

解法步驟

① 先假設 Diode 的 state

ON \rightarrow short

OFF \rightarrow open

② 依照假設的情形, 來解電路

③ 驗證結果是否與假設相符

if 假設 ON 要有 $I_D > 0, V_D > 0$ 的結果;

otherwise 改設為 OFF;

if 假設 OFF 要有 $I_D = 0, V_D < 0$ 的結果;

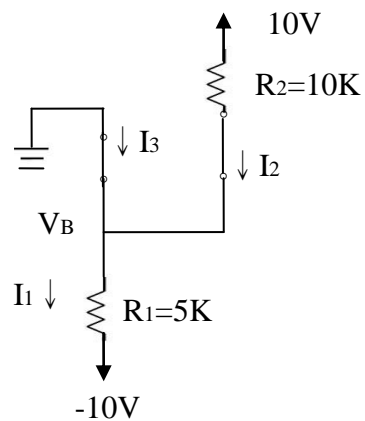
otherwise 改設為 ON;

④ 修正假設, 再解一次②

Sol:

Assume D_1, D_2 均為 ON

$D_1, D_2 \Rightarrow$ short



$V_B = 0$

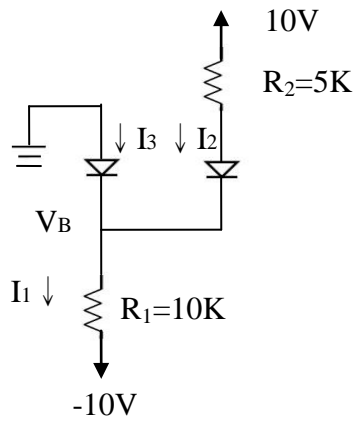
$I_1 = [0 - (-10)] / 5K = 2mA$

$I_2 = (10 - 0) / 10K = 1mA$

$I_3 = I_1 - I_2 = 1mA$

D_1, D_2 的電流均 > 0 , 猜對

Ex



5.1: Field-Effect Transistors (FETs)

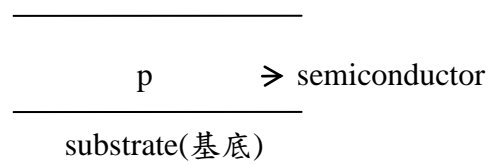
場效電晶體

MOSFETs (Metal-Oxide-Semiconductor)

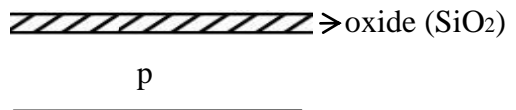
MOS：由結構而來

FET：由作用機制而來

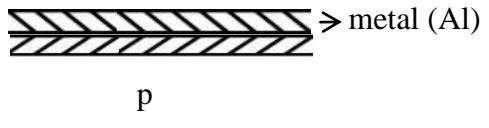
①



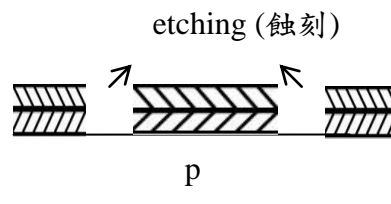
②



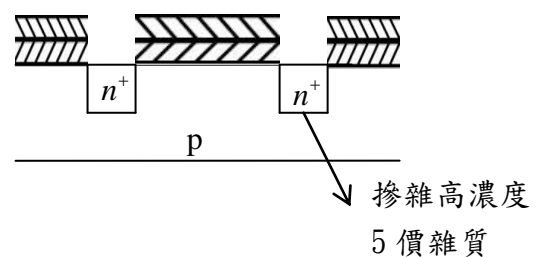
③



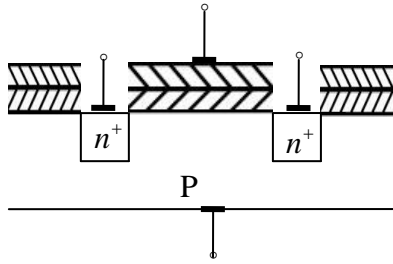
④



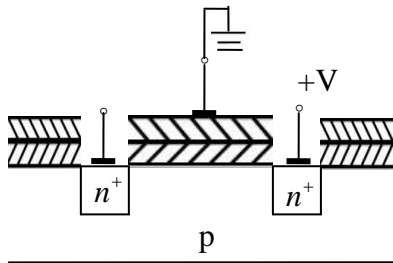
⑤



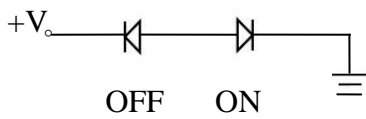
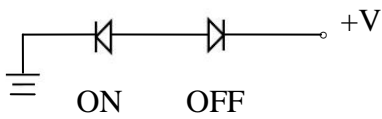
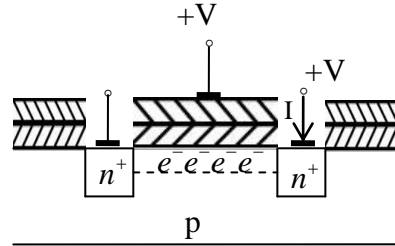
⑥



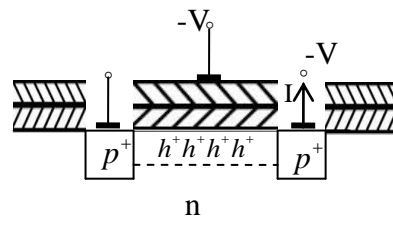
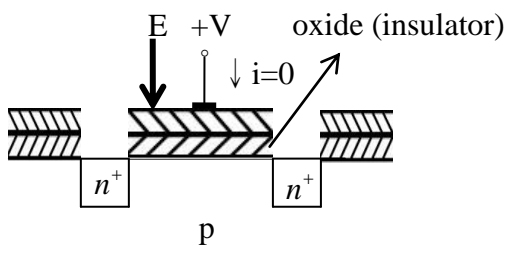
How does MOSFET work?



“electron inversion layer” (電子反轉層) or n channel (通道)

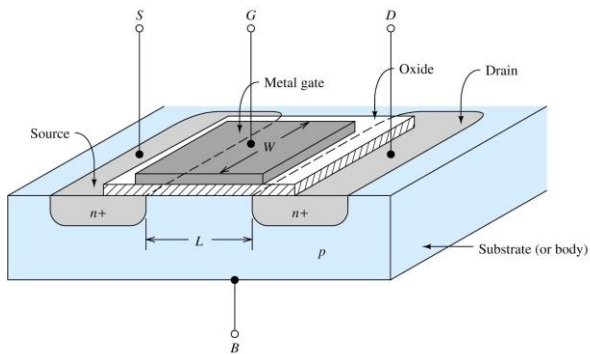


N channel MOS (NMOS)

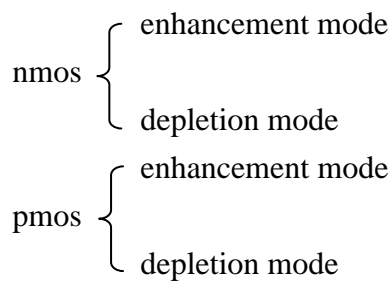
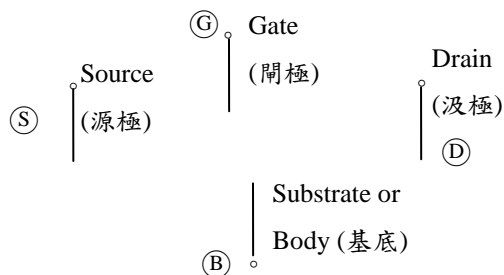


“hole inversion layer” (電洞反轉層) or p channel

P channel MOS (PMOS)



Terminal 命名



enhancement mode:

外加 gate 電壓,使 channel 形成

depletion mode:

通道已形成,外加 gate 電壓,改變 channel 厚度

V_{DS} 的極性

NMOS, $V_{DS} > 0$ (將電子拉出來)

PMOS, $V_{DS} < 0$ (將電洞拉出來)

S 提供 carrier

nmos s 提供 e^-

pmos s 提供 h^+

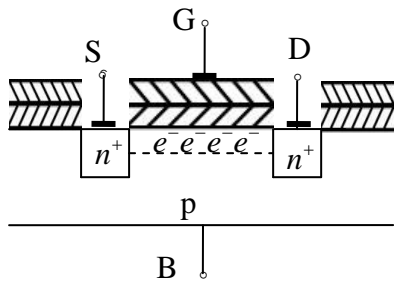
B 的極性

NMOS B 接(-)

PMOS B 接(+)

$V_{GS} > 0$ for nmos 讓反轉層形成

Enhancement n-type MOSFET
(Enhancement NMOS)



Gate 電壓：①正負要對②大小要夠
才能產生 channel

$V_{TN} \equiv V_{GS}$ 使得 channel 剛好產生
↙
nmos (Threshold Voltage 臨界電壓)

$$\frac{\epsilon_{ox}}{t_{ox}} \equiv C_{ox} \text{ (單位面積的氧化層電容)}$$

$$[C_{ox}] = F/cm^2$$

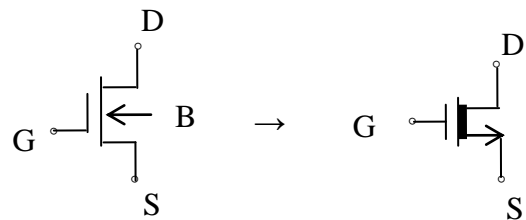
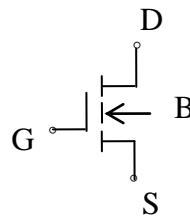
MOS Symbol

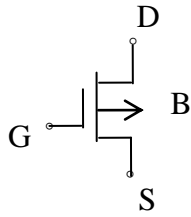
$V_{TN} = ?$ 1, 2, or 3V
影響 V_{TN} 大小的因素

① t_{ox} (Oxide thickness)

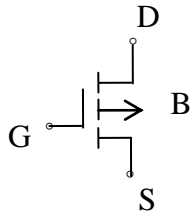
② ϵ_{ox} (Oxide permittivity)

③ N_A in substrate





① $V_{GS} < V_{TN}$ $i_D = 0, i_S = 0$
cutoff (截止區)



② $V_{GS} \geq V_{TN}$ 有通道產生
再外加 V_{DS} 會有電流

ⓐ V_{DS} 很小

I-V 特性曲線

I: $i_D = ?$ $i_S = ?$ $i_G = ?$

V: $V_{GS} = ?$ $V_{DS} = ?$

$$i_G = 0$$

$$i_D = i_S$$

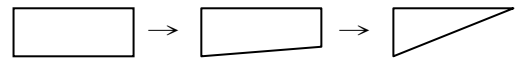
$$i_D = f(V_{GS}, V_{DS})$$

$$= f((V_{GS} - V_{TN}), V_{DS})$$

$$V_{\text{eff}} \equiv V_{GS} - V_{TN}$$

$$i_D = ?$$

$$\textcircled{b} \quad V_{DS} \uparrow \uparrow \quad V_{DS} \leq V_{GS} - V_{TN}$$



pinched-off (夹止)

$$i_D = \mu_n C_{ox} \frac{w}{L} (V_{GS} - V_{TN}) V_{DS}$$

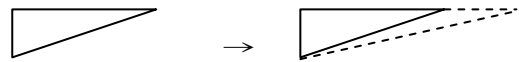
$$r_{DS} = \frac{V_{DS}}{i_D} = 1 / \mu_n C_{ox} \frac{w}{L} (V_{GS} - V_{TN})$$

$$\mu_n C_{ox} \equiv \kappa_n'$$

$$i_D = \kappa_n' \frac{w}{L} [(V_{GS} - V_{TN}) V_{DS} - \frac{1}{2} V_{DS}^2]$$

© $V_{DS} \geq V_{GS} - V_{TN}$

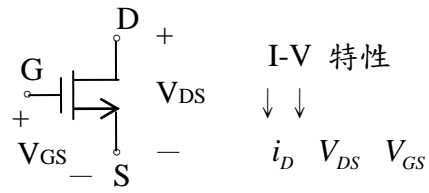
$$\begin{aligned} i_D &= \kappa_n' \frac{w}{L} [(V_{GS} - V_{TN})V_{DS} - \frac{1}{2}V_{DS}^2] \\ &= \frac{1}{2} \mu_n C_{ox} \frac{w}{L} [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2] \\ &= \kappa_n [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2] \end{aligned}$$



$$\kappa_n = \frac{1}{2} \mu_n C_{ox} \frac{w}{L} \quad (\text{conduction parameter})$$

MOS 工作在 triode region (三極體區) or non-saturation region

Summary



$$i_D = \kappa_n (V_{GS} - V_{TN})^2$$

MOS 工作在 saturation region (飽和區)

① $V_{GS} < V_{TN}$ Q : cutoff

$$i_D = i_S = i_G = 0$$

② $V_{GS} \geq V_{TN}$ Q : conducting $\left\{ \begin{array}{l} \text{a) triode} \\ \text{b) sat.} \end{array} \right.$

a) $V_{DS} \leq V_{GS} - V_{TN}$

$$i_D = \kappa_n [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

ⓑ $V_{DS} \geq V_{GS} - V_{TN}$

$$i_D = \kappa_n (V_{GS} - V_{TN})^2$$

Ex:

enhancement nmos $V_{TN} = 1.2V, V_{GS} = 2V$

$$\mu_n = 500 \text{ cm}^2 / V \cdot S, \quad \epsilon_{ox} =$$

$$(3.9)(8.85 \times 10^{-14}) \text{ F/cm}$$

$$t_{ox} = 450 \text{ \AA}, W = 100 \text{ \mu m}, L = 7 \text{ \mu m} \quad \text{求 } i_D$$

when (a) $V_{DS} = 0.4V$ (b) $V_{DS} = 1V$ (c) $V_{DS} = 5V$

Sol:

$$\kappa_n = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} =$$

$$\frac{1}{2} \cdot 500 \cdot \frac{(3.9)(8.85 \times 10^{-14}) \cdot 100}{450 \times 10^{-10} \times 10^2} \cdot \frac{100}{7} =$$

$$0.274 \text{ mA/V}^2$$

(a) $V_{DS} = 0.4 < (2-1.2) = 0.8V$, at triode region

$$i_D =$$

$$= 0.132 \text{ mA}$$

(b) $V_{DS} = 1 > 0.8$, at sat region

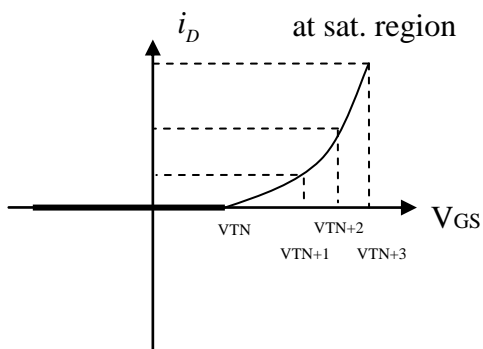
$$i_D =$$

$$= 0.175 \text{ mA}$$

(c) $V_{DS} = 5 > 0.8$, at sat region

$$i_D = 0.175 \text{ mA}$$

Ex:



非理想 I-V 特性曲線

Early Effect (厄立效應) or

Channel length modulation

(通道長度調變)

Output resistance $r_0 \equiv \left(\frac{\partial i_D}{\partial V_{DS}}\right)^{-1} \Big|_{V_{GS} = \text{const}}$

$$r_0 = [\lambda \kappa_n (V_{GS} - V_{TN})^2]^{-1}$$

Ex:

nmos enhancement device, $V_{TN} = 0.8V$

$$\kappa_n = 0.1mA/V^2, V_{GS} = 2.5V, \lambda = 0.02V^{-1}$$

$V_{DS} = 2V$, 求 i_D 及 r_0

$$i_D = \kappa_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$$

$$\lambda = \frac{1}{V_A}, V_A \equiv \text{Early voltage}$$

Subthreshold conduction (次臨界導通)

3. Oxide breakdown

Breakdown

1. pn junction breakdown

Body Effect (基體效應)

$V_{SB} \neq 0$ 影響 V_{TN} 之大小

2. punch-through

Ex:

$$V_{TN0}=1V, \quad \gamma=0.35V^{\frac{1}{2}}, \quad \phi_f=0.35V$$

求 V_{TN} at (a) $V_{SB} = 0V$ (b) $V_{SB} = 1V$

$$V_{TN} = V_{TN0} + \gamma [\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}]$$

Reason:

溫度效應

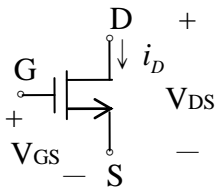
$T \uparrow, i_D \downarrow$

Negative feedback (負回授)

I-V characteristics for other types of MOS

nmos	{	enhancement
		depletion
pmos	{	enhancement
		depletion

“enmos”



n channel V_{TN} (吸電子
形成
channel)

$\Rightarrow V_{TN}(+)$
 V_{DS} (把通道的電子吸
出去, i_D 相反方向)
 $\Rightarrow V_{DS}(+)$

$$\lambda = \frac{1}{V_A} (+)$$

$V_{GS} < V_{TN}$, cutoff, (原來的 channel 消失
了) $i_D = 0$

$V_{GS} > V_{TN}$, conducting, (V_{GS} 可 (+) 可 (-))

① $V_{DS} \leq V_{GS} - V_{TN}$ (triode region)

$$i_D = \kappa_n [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

② $V_{DS} \geq V_{GS} - V_{TN}$ (sat. region)

$$i_D = \kappa_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$$

$V_{GS} < V_{TN}$, cutoff, $i_D = 0$

$V_{GS} > V_{TN}$, conducting

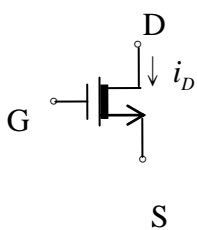
① $V_{DS} \leq V_{GS} - V_{TN}$ (triode region)

$$i_D = \kappa_n [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

② $V_{DS} \geq V_{GS} - V_{TN}$ (sat. region)

$$i_D = \kappa_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$$

“dnmos”



$V_{GS} = 0$ 時, 已有 channel
存在, V_{TN} 為使 channel
剛要形成的電壓 (V_{GS})
 $\Rightarrow V_{GS}$ 要加 (-) 的, 將
n channel 的 e^- 趕走

$\Rightarrow V_{TN}(-)$

$\Rightarrow V_{DS}(+)$ (吸引電子)

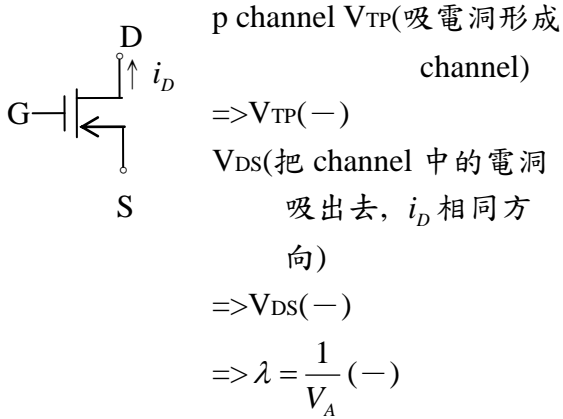
$$\Rightarrow \lambda = \frac{1}{V_A} (+)$$

$V_{GS} = 0$, i_D 稱 I_{DSS}

Ex:

Ex:

“epmos”



$V_{GS} > V_{TP}$, cutoff, $i_D = 0$
 (不夠負)

$V_{GS} < V_{TP}$, conducting

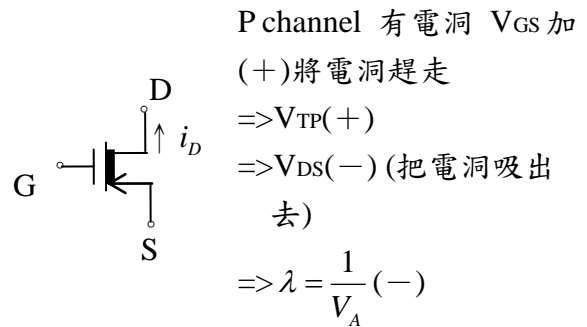
① $V_{DS} \geq V_{GS} - V_{TP}$ (triode region)

$$i_D = \kappa_p [2(V_{GS} - V_{TP})V_{DS} - V_{DS}^2]$$

② $V_{DS} \leq V_{GS} - V_{TP}$ (sat. region)

$$i_D = \kappa_p (V_{GS} - V_{TP})^2 (1 + \lambda V_{DS})$$

“dpmos”



$V_{GS} > V_{TP}$. 原有的 channel 消失了, $i_D = 0$,
 cutoff

$V_{GS} < V_{TP}$, conducting

① $V_{DS} \geq V_{GS} - V_{TP}$ (triode region)

$$i_D = \kappa_p [2(V_{GS} - V_{TP})V_{DS} - V_{DS}^2]$$

② $V_{DS} \leq V_{GS} - V_{TP}$ (sat. region)

$$i_D = \kappa_p (V_{GS} - V_{TP})^2 (1 + \lambda V_{DS})$$

Summary

四種 mos, i_D 公式均相同, 差別是判斷方式, nmos 和 pmos 相反

① cutoff 與 conducting 相反

$$V_{GS} \begin{matrix} \geq \\ \leq \end{matrix} V_T \text{ 之關係}$$

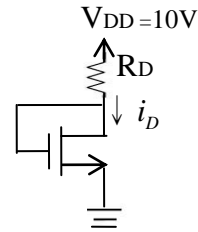
② triode 與 sat region 相反

$$V_{DS} \begin{matrix} \geq \\ \leq \end{matrix} (V_{GS} - V_T) \text{ 之關係}$$

Ex:

MOS DC Analysis

Ex1:



enmos

$i_D = 0.4\text{mA}$ 求 $R_D = ?$

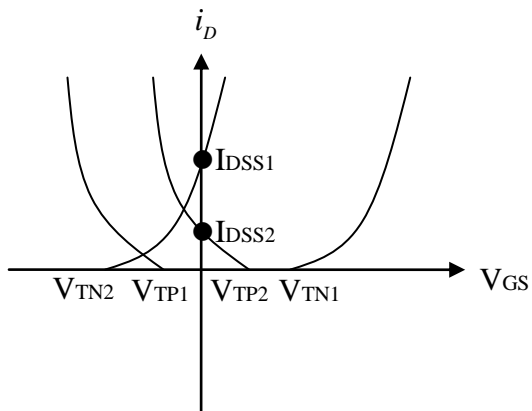
Given $V_{TN} = 2\text{V}$

$$\kappa_n' = 20\mu\text{A}/\text{V}^2$$

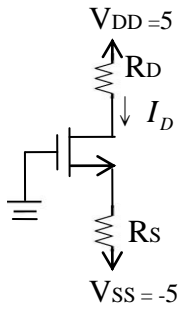
$$W = 100\mu\text{m}, L = 10\mu\text{m}$$

$$\lambda = 0$$

Sol:



Ex2:



enmos

$I_D = 0.4\text{mA}$, $V_D = 1$

求 $R_D = ?$ $R_S = ?$

Given $V_{TN} = 2\text{V}$,

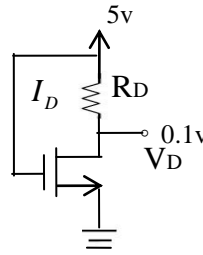
$\kappa_n' = 20\ \mu\text{A}/\text{V}^2$

$L = 10\ \mu\text{m}$, $W = 400\ \mu\text{m}$

$\lambda = 0$

Sol:

Ex3:



enmos

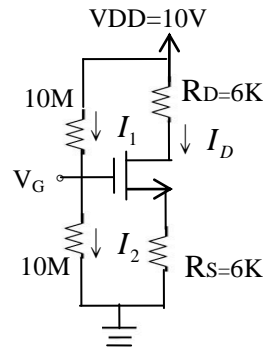
$V_D = 0.1\text{V}$, 求 R_D

Given $V_{TN} = 1$

$\kappa_n = 0.5\ \mu\text{A}/\text{V}^2$

Sol:

Ex4:



分析此電路

(求出所有電壓, 電流)

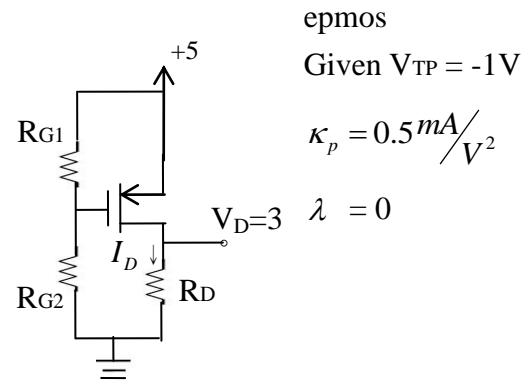
Given $V_{TN} = 1\text{V}$

$\kappa_n = 0.5\ \mu\text{A}/\text{V}^2$

$\lambda = 0$

Sol:

Ex5:

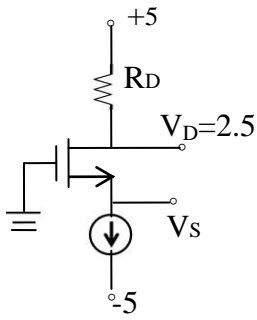


(a) design R_{G1} , R_{G2} , R_D such that Q at sat. region, and $I_D = 0.5mA$, $V_D = 3V$

(b) 使 Q 在 sat. region 的 $R_{Dmax} = ?$

Sol:

Ex6:



enmos

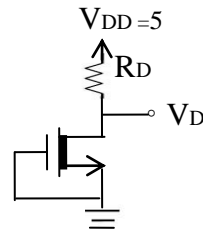
Given $V_{TN} = 0.8V$

$$\kappa_n' = 80 \mu A/V^2$$

$W/L = 3$, design R_D such that $I_D = 250 \mu A$
 $V_D = 2.5V$. What's $V_S = ?$

Sol:

Ex7:



dnmos

given $\kappa_n' = 20 \mu A/V^2$

$V_{TN} = -1V$

$\lambda = 0$

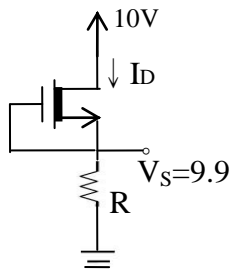
(a) 求 $W/L = ?$ such that

$I_D = 100 \mu A$ at sat.

(b) R_D range? in (a)

Sol:

Ex8:



dnmos

given $\kappa_n = 0.5 \text{ mA/V}^2$

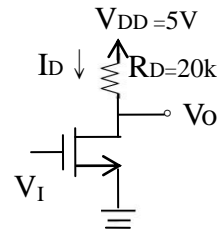
$V_{TN} = -1 \text{ V}$

求 R such that

$V_S = 9.9 \text{ V}$

Sol:

Ex9:



enmos

$\kappa_n = 0.1 \text{ mA/V}^2$

$V_{TN} = 0.8 \text{ V}$

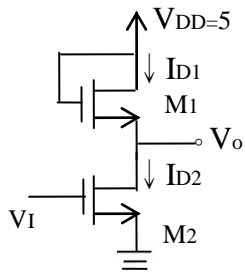
求 V_O, I_D if

(a) $V_I = 0$

(b) $V_I = 5 \text{ V}$

Sol:

Ex10:



$$V_{TN1} = V_{TN2} = 1V$$

$$\kappa_{n1} = 10 \mu A/V^2$$

$$\kappa_{n2} = 50 \mu A/V^2$$

求 V_o, I_{D1} if

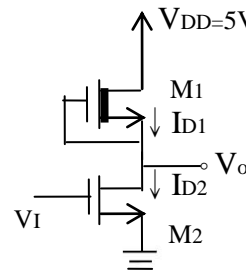
(a) $V_I = 5V$

(b) $V_I = 1.5V$

(c) $V_I = 0$

Sol:

Ex11:



$$V_{TN1} = -2V$$

$$V_{TN2} = 1V$$

$$\kappa_{n1} = 10 \mu A/V^2$$

$$\kappa_{n2} = 50 \mu A/V^2$$

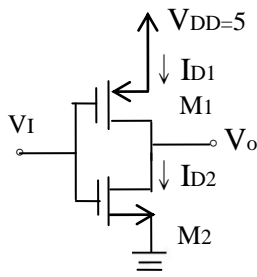
求 V_o & I_D if

(a) $V_I = 5V$

(b) $V_I = 0$

Sol:

Ex12:



$$\kappa_n = \kappa_p = 1 \text{ mA/V}^2$$

$$V_{TN} = 1\text{V} \quad V_{TP} = -1\text{V}$$

求 V_o 及 I_{D1} if

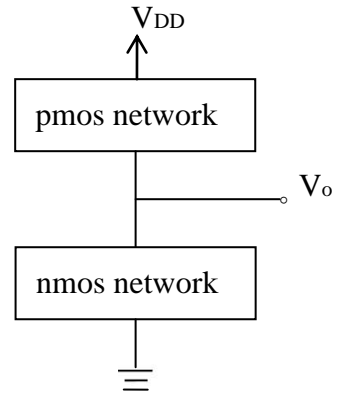
(a) $V_I = 0$

(b) $V_I = 5\text{V}$

Sol:

Complementary MOS (CMOS)

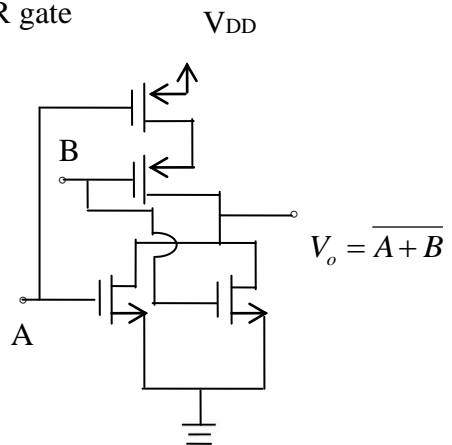
CMOS logic gate design



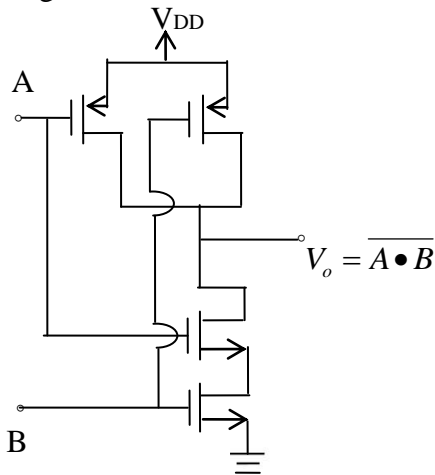
CMOS inverter

- $V_I = \text{high, nmos ON, pmos OFF, } V_o = \text{low}$
- $V_I = \text{low, nmos OFF, pmos ON, } V_o = \text{high}$

NOR gate



NAND gate



Example:

$$F = \overline{A \cdot B + C \cdot D}$$

Sol:

1. (**n**-network): **Invert** F to derive n -network
 $(\overline{F} = A \cdot B + C \cdot D)$

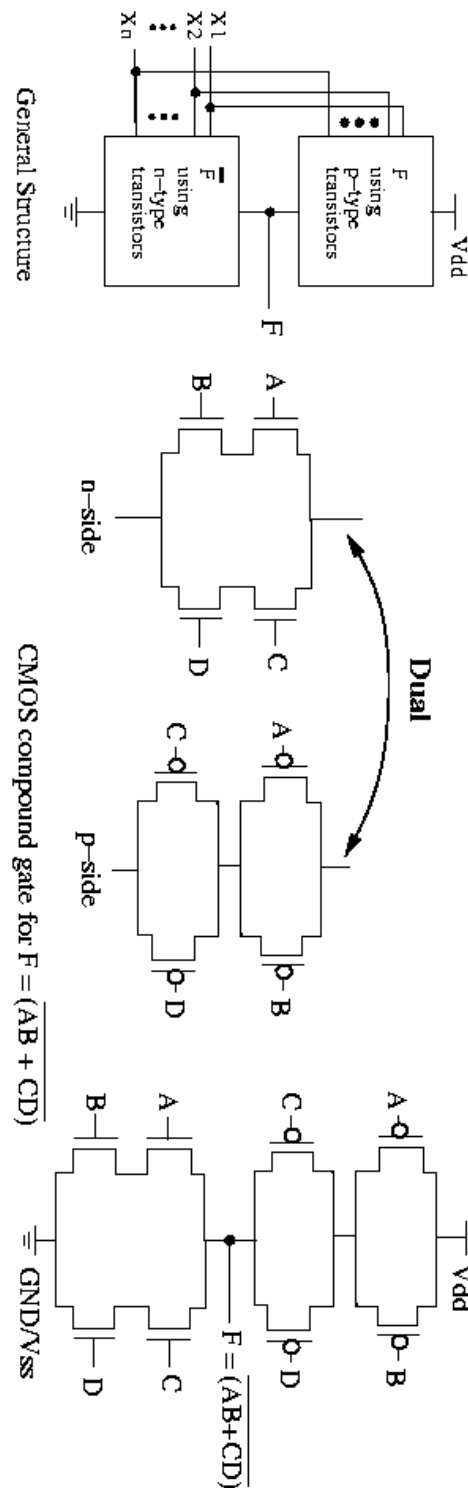
2. (**n**-network): Make connections of transistors:

- AND \Leftrightarrow Series connection
- OR \Leftrightarrow Parallel connection

3. (**p**-network): Expand F to derive p -network
 $(F = \overline{AB + CD} = \overline{AB} \cdot \overline{CD} = (\overline{A + B}) \cdot (\overline{C + D}))$
 each input is inverted

4. (**p**-network): Make connections of transistors (same as step 2)

5. Connect the n -network to GND (typically, 0V) and the p -network to VDD (5V, 3.3V, or 2.5V, etc)



Ex:

Implement $F = A \cdot B + C \cdot (D + E)$ by CMOS logic.

Sol: