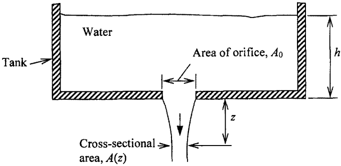
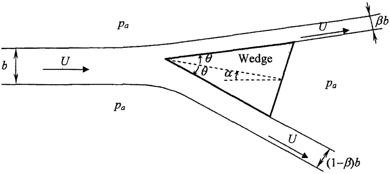
※ 考生請注意：本試題 ☑可 □不可 使用計算機

1 (10%) A water tank has an orifice in the bottom of the tank.



The height, h, of water in the tank is kept constant by a supply of water which is not shown. A jet of water emerges from the orifice; the cross-sectional area of the jet, $A(z)$, is a function of the vertical distance, z. Neglecting friction (viscous effects) and surface tension find an expression for $A(z)$ in terms of A_0, h and z, where A_0 is the cross-sectional area at z=0. Assume that the area of the tank free surface is very large compared with A_0.

2. (20%) A wedge with a vertex angle 2θ is inserted into a jet of water (density, ρ) of width, b, and velocity, U, as shown in the sketch below.



(背面仍有題目.請繼續作答)

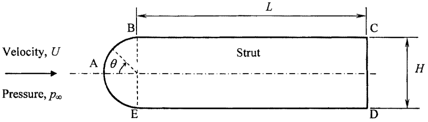
The angle of attack, α, of the wedge is also defined in the sketch. The result is that the single incident jet is divided into two jets both of which leave the back edges of the wedge with the velocity, U The widths of the two departing jets are βb and $(1-\beta)b$ as indicated in the figure. It is assumed that the flow is planar and that the pressure in surrounding air is everywhere atmospheric.

(1) Find the lift and drag on the wedge per unit length normal to the sketch as functions of ρ, U, b, β, α and θ.

(2) If the angle of attack, α, is varied while ρ, U, b, β and θ remain fixed, find the angle of attack at which the lift is zero.

(3) If, on the other hand, the wedge is moved in a direction perpendicular to the incident jet while ρ, U, b, α and θ remain fixed then β will change. There is one such position at which the lift is zero; what is the value of β at this position in terms of α and θ? If the wedge were free to move in such a way would this position represent a position of stable or unstable equilibrium?

3. (20%) The sketch below shows the cross-sectional geometry of a supporting strut. The actual strut is quite streamlined, in other words L/H is large. It is intended for use in a uniform stream of incompressible fluid (density, ρ) of velocity, U, parallel to the side of length, L. The flow can be considered to be planar



Further assume:

- that the velocity distribution over the cylindrical nose, BAE, is the same as in potential flow, that is to say the velocity outside the boundary layer is $2U\sin\theta$.

- that the flow separates at the sharp trailing edges C and D so that the pressure coefficient (defined based on the upstream dynamic pressure) acting on the base CD is

$$C_p = -\frac{1}{3} - \frac{3}{2}\frac{H}{L}$$

- that the skin friction forces on the cylindrical nose are negligible.

If the drag coefficient is defined based on the upstream dynamic pressure and the frontal projected area (H times a unit depth perpendicular to the sketch), find

(1) The contribution of the form drag to the total drag coefficient (denoted this by C_{DF}).

(2) An estimate of the contribution of the skin friction on the sides of the strut to the total drag coefficient, assuming the boundary layer remains laminar This should be in terms of the Reynolds number, $Re=UL/\nu$, where ν is the kinematic viscosity of the fluid.

(3) For what aspect ratio, L/H, will the drag be comprised of equal parts of form and skin friction drags if $Re=1000$?

4. (32 %) Oil is being pumped through rectangular channel shown in the figure. The pump characteristic curve is given as $H=200-110Q^2$ The unit of the head, H, is in meters, and in m^3/s for the volumetric flow rate, Q. The channel walls have an average roughness of 0.04 mm. The width and height of the channel are 1 m and 2 cm, respectively The density of the oil is 900 kg/m^3 and the kinematic viscosity is 0.0001 m^2/s. The velocity profile of the steady, incompressible, turbulent, fully developed flow in the very wide rectangular channel is given as $u/U_{max}=(y/h)^{1/7}$ between the bottom wall and the centerline (the bottom wall is defined as y=0, so the centerline is at y=h). The profile above the centerline is symmetric to that below

(1) Could the approximate profile $u/U_{max}=(y/h)^{1/7}$ be used to obtain the shear stress at the wall and at the centerline of this flow? Explain with a few sentences.

(2) Obtain wall shear stress. The friction factor is between 0.01 and 0.05 A few iterations may be required to get the friction factor, and a value accurate to the third decimal place will be good enough. You may also need the following relationship to obtain the friction factor

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$

(3) If $\tau_w=825$ N/m^2 and V=14.3 m/s and total shear stress distribution in this fully developed turbulent flow is linear, calculate the turbulent and viscous stresses at y=0.2 cm.

(4) If the present pump is replaced with a new geometrically similar and a diameter 1.5 times larger pump running at the same speed, what would be the new flow rate Q?

(背面仍有題目,請繼續作答)

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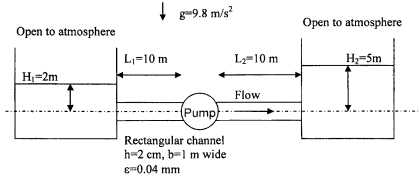

(Drawing NOT TO SCALE)

5. (18 %) Water film is flowing down a very wide slope as shown in the figure below The flow is steady, laminar, and fully developed. Use the symbol ν to represent the kinematic viscosity of water, θ for the inclined angle of the slope, h for the film thickness, and g for gravity Show all assumptions, calculations and derivations step by step.
   (1) Use Navier-Stokes equations to derive expressions for the ratio between the velocity profile, $u(y)$, and the maximum velocity, U_{max}, in terms of y and h, as well as U_{max} in terms of h, g, θ, and ν
   (2) Is there energy loss as water flows down the slope? If yes, what energy of the flow is converted to losses? Where does this energy show up?