系所組別：機械工程學系甲組
考試科目：流體力學 考試日期：0223，節次：1

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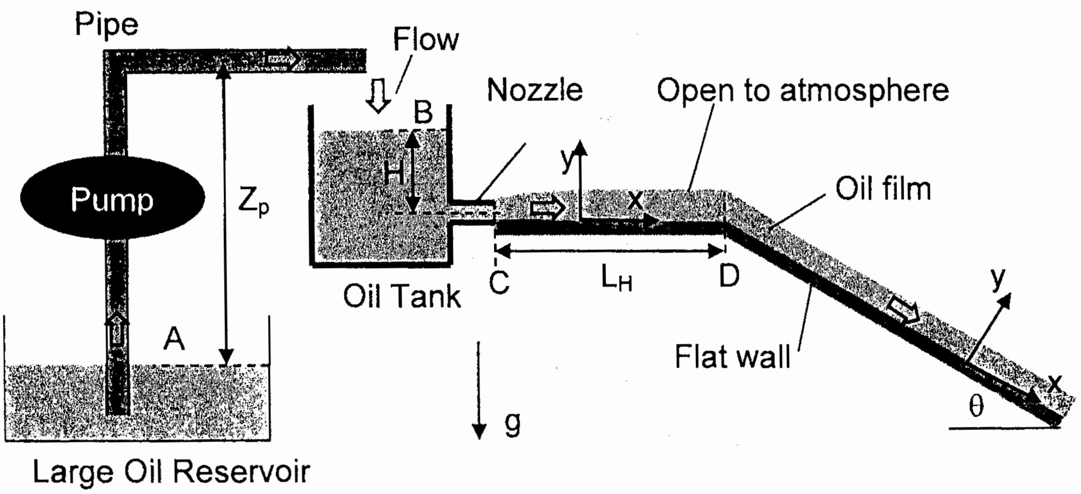
1. (50 %) Steady oil flow exists in the system shown in the figure below. The pump delivers enough oil from a large reservoir through the inverted L-shaped pipe to keep the oil level in the oil tank, H, constant. Total length of the pipe is L_P, and the diameter is d_P. The height of the horizontal pipe section from the free surface of the oil reservoir is Z_P. Oil flows out of the oil tank through a nozzle onto a horizontal flat plate. Assume frictionless flow in the tank up to the nozzle exit. The velocity of the oil flow at point C is therefore constant and equals to U across the nozzle height, c. Boundary layer begins to develop in the oil film as soon as the oil film leaves the nozzle, and the boundary layer thickness is defined as δ. The oil film thickness at point D equals to a, and the distance between point C and D is L_H. The flow goes down an inclined plate at an angle of θ with the horizontal direction beyond point D, and becomes fully-developed with a constant thickness of h after a short adjustment distance. Widths of the plates are large, and both equal to b. There are side walls so that the oil does not spill from the sides. Density and kinematic viscosity of the oil are ρ and ν, respectively. Specific gravity, g, is pointing vertically downwards. The directions of x and y coordinates are shown in the figure. Show all assumptions, derivations and calculations for the problems below.



(1) Derive the expression for U as a function of g and H.

(2) The velocity profile in the boundary layer of the oil film on the horizontal section is given as $u/U=2(y/\delta)-(y/\delta)^2$. Derive the relationship among a, δ, and c.

(3) Find the expression for the drag force on the surface from point C to D by applying control volume analysis.

(4) If the velocity profile in the fully-developed flow region on the inclined section is $u/U_{max}=2(y/h)-(y/h)^2$, what is the drag force per unit length in this section? Justify each step in the solution and eliminate U_{max} in the final answer.

(5) Find an expression for the pump power necessary to keep the oil in the oil tank at a constant height of H when the flow in the pipe is laminar. Neglect minor losses.

(6) If the working fluid is replaced with water (kinematic viscosity smaller than oil), how will the boundary layer thickness at point D be affected given H remains the same, and why?

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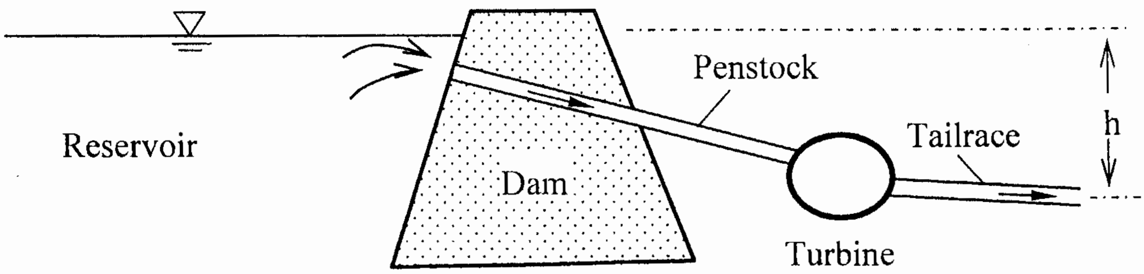
2. (10%) The following is the stream function for a particular steady, planar, incompressible and inviscid flow:

$$\psi = A(x^2y - \frac{y^3}{3})$$

where A is a know constant.

(1) Find expressions for the velocity components u and v in this flow.
(2) Find an expression for the vorticity.
(3) We can define a velocity potential for this flow. Why? Find an expression for the velocity potential by assuming that the value of the velocity potential at the origin is zero.
(4) Make a rough sketch of the streamlines of this flow.
(5) Find an expression for the pressure in this flow assuming that the pressure, p_0, at the origin is known. Denote the fluid density by ρ and neglect all body forces. What shape are the lines of constant pressure (isobars)?

3. (20%) The tailrace (discharge pipe) of a hydro-electric turbine installation is at an elevation, h, below the water level in the reservoir:



The frictional loss in the penstock (the pipe leading to the turbine) and the tailrace are represented by the loss head, $kU^2/2g$, where U is the mean velocity in those pipes (which have the same cross-sectional area, A) and g is the gravitational acceleration. The flow discharges to atmospheric pressure at the exit from the tailrace. The water density is denoted by ρ.

(1) What is the drop in total head across the turbine?
(2) What is the power developed by the turbine assuming that it is 90% efficient?
(3) What is the optimum velocity, U, which will produce the maximum power output from the turbine assuming that h, k, A, ρ and g are constant?

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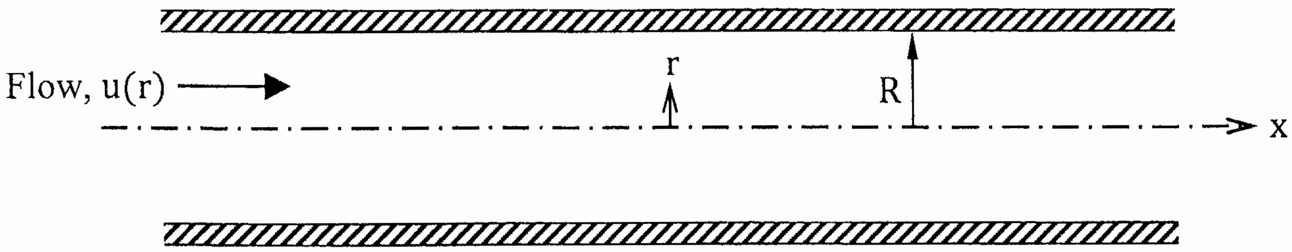
4. (20%) Consider the fully-developed pipe flow of an incompressible, non-Newtonian fluid:



The fluid is such that the normal stress in the x-direction is equal to $-p$, where p is the pressure and the shear stress, σ, is related to the velocity gradient by

$$\sigma = C\left(-\frac{du}{dr}\right)^2$$

where C is a known constant. Find the friction factor, f, for this pipe flow in terms of C, r (the fluid density) and R (the radius of the pipe).

[Note: Remember the definition

$$f = \frac{4R}{\rho\bar{u}^2}\left(-\frac{dp}{dx}\right)$$

where $\bar{u}$ is the average velocity of the pipe flow.]