1. Briefly answer the following questions:

   (a) (5%) <u>restriction conditions</u> for using the Bernoulli equation: $p + \frac{1}{2}\rho V^2 + \rho gz = \text{const.}$

   (b) (4%) <u>physical meaning</u> of $\nabla \cdot \mathbf{V}$ and $\nabla \times \mathbf{V}$, where $\mathbf{V}$ is the velocity field of a fluid flow

   (c) (6%) under what conditions do both the stream function ψ and the velocity potential ϕ exist for a flow field? when does one exist but not the other?

2. (15%) An incompressible, inviscid liquid flow (density, ρ) of depth, h_1, and velocity, V, flows under the action of gravity through a sluice gate:
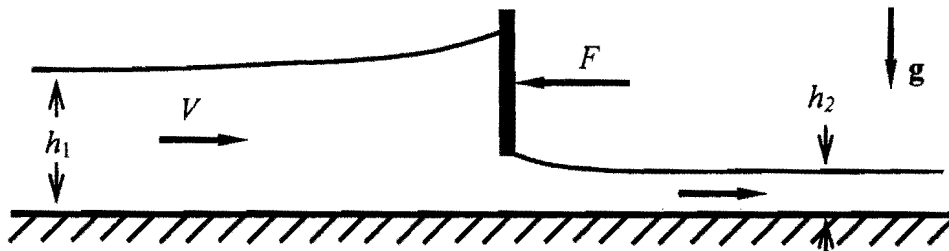
   The depth downstream of the sluice gate is denoted by h_2. Determine the force per unit width (normal to sketch), F, necessary to hold the plate in place in terms of g, ρ, h_1 and h_2.

3. (20%) The velocity profile in a turbulent boundary layer of incompressible fluid on a flat plate (U = constant) is to be approximated by the form:

 $$u/U = (y/\delta)^{\frac{1}{7}}$$

   (Disregard the fact that this does not exactly satisfy one of the constraints usually imposed on laminar boundary layer profiles namely that du/dy should tend to zero as y tends to δ). If the wall shear stress, τ_w, for this turbulent profile is assumed to be given by the empirical formula

   $$\tau_w = 0.023\rho U^2 (v/\delta U)^{\frac{1}{4}}$$

   where ρ and v are the fluid density and kinematic viscosity. Solve the Karman momentum integral equation to obtain an expression for the thickness of the boundary layer, δ, as a function of distance, x, along the plate. Assume that the layer first becomes turbulent at $x = x_0$ where the thickness is δ_0.

4. (40%) Oil is flowing upward in the channel shown in the figure below. For the section between A and B, a manometer is attached to the bottom of the channel. The distance between A and B is L, and the channel depth is h. Density of the oil is ρ, and the viscosity is μ. The manometer fluid is mercury, and the density is ρ_m. The flow is steady, laminar and fully developed. The channel width in the z direction, b, is large compared to h.
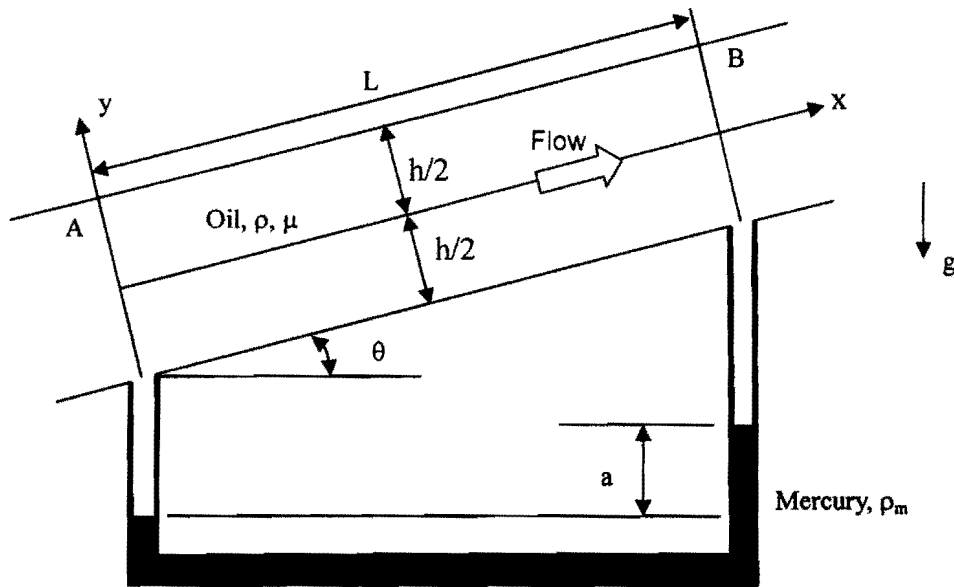
(a) What is the pressure difference between A and B?

(b) Derive expressions for the velocity profile, u, and pressure distribution, P, in the channel in terms of the given parameters. List all assumptions and show your derivation step by step.

(c) Find the expression for the friction losses, h_f, between A and B in terms of ρ, g, L θ, ρ_m, a.

5. (10 %) A flat plate of infinite width and breadth oscillates sinusoidally in its own plane beneath a viscous fluid as shown in the figure below. The fluid is at rest far above the plate. Making as many simplifying assumptions as you can, set up the governing differential equation and boundary conditions for finding the velocity field u in the fluid. DO NOT SOLVE.

Sinusoidal oscillation