SINUSOIDAL MODELING

EE6641 Analysis and Synthesis of Audio Signals

Yi-Wen Liu

Nov 3, 2015

Last time: Spectral Estimation

- Resolution
  - Scenario: multiple peaks in the spectrum
  - Choice of window type and window length
  - Rule of thumb: $\Delta\omega = O(1/\Delta\tau)$
- Accuracy
  - Scenario: signal in noise
  - Quadratic interpolation of FFT log-magnitude
  - Fisher Information and Cramer-Rao lower bounds
  - Coherent frequency estimation: $\Delta\omega = O(\Delta\tau^{-3/2})$

Today's agenda

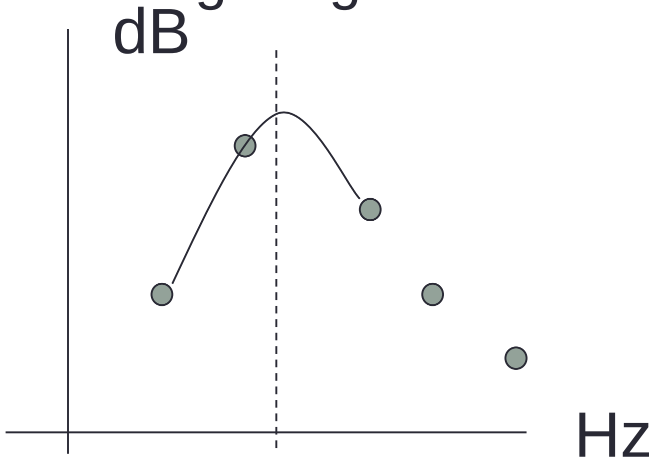
- Sinusoidal modeling
  - Analysis
    - Formula for QI-FFT
    - Choice of window
    - Peak tracking
  - Synthesis
  - Applications

- Spectral decomposition
  - (S + N) vs. (S + N + T)
  - Spectral subtraction

QI-FFT: a fast frequency estimator

- **Windowing**: time-domain multiplication with a shaping function
- **Zero-padding**: append zeros in time ⇔ interpolate in frequency
- **FFT**: apply fast Fourier transform
- **Peak detection**: find every peak
- **Quadratic interpolation**: fit a parabola in log-magnitude

⇒ The location of the parabola is the frequency estimate.



- Reference: M. Abe and J. Smith, "Design criteria for simple sinusoidal parameter estimation based on quadratic interpolation of FFT magnitude peaks", (AES 2004)

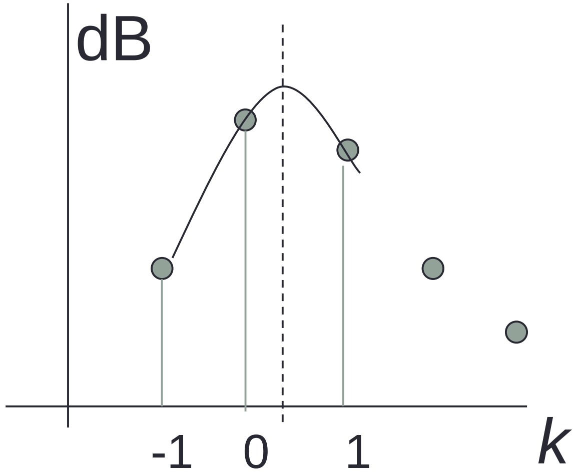
A formula for QI-FFT

$$L(x) = Ax^2 + BX + C$$

$$L(-1) = L^-$$

$$L(0) = L^0$$

$$L(1) = L^+$$



$$A = \frac{L^+ + L^- - 2L^0}{2}$$

$$B = (L^+ - L^-)/2$$

$$C = L^0$$

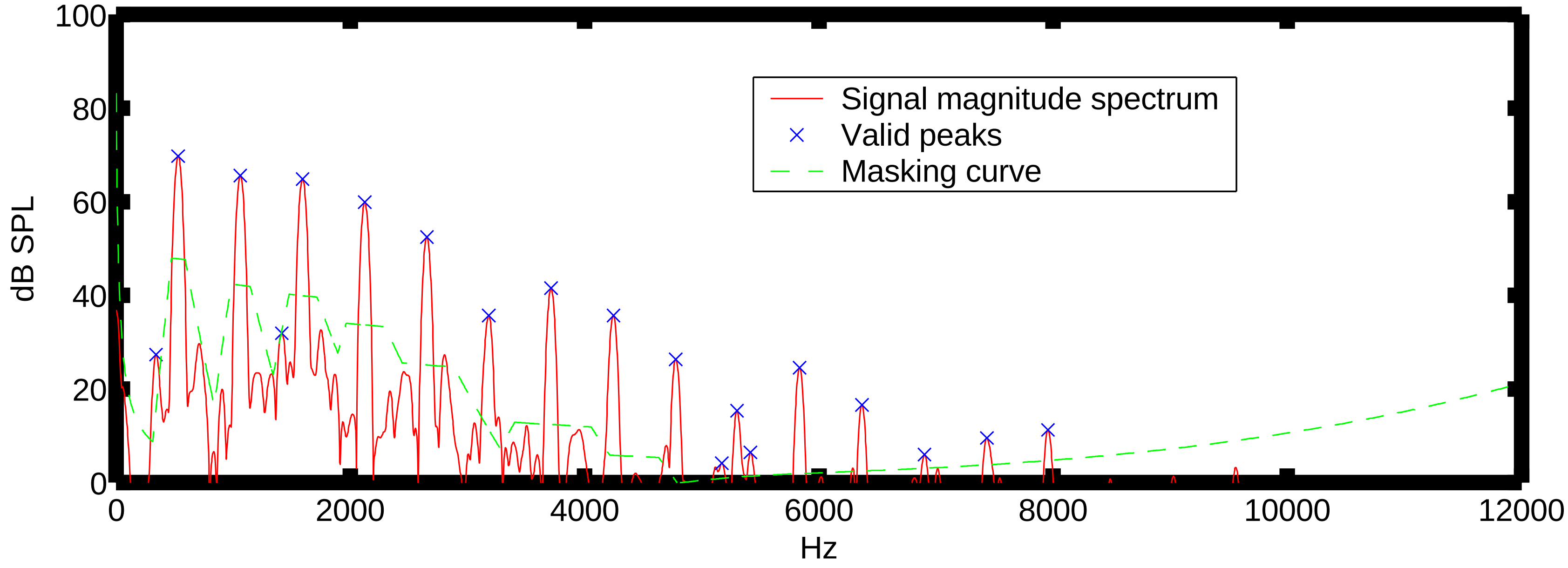
$$L(x) = A(x-q)^2 + \hat{L}$$

$$q = -\frac{B}{2A}$$

$$\hat{L} = L^0 - B^2/4A$$

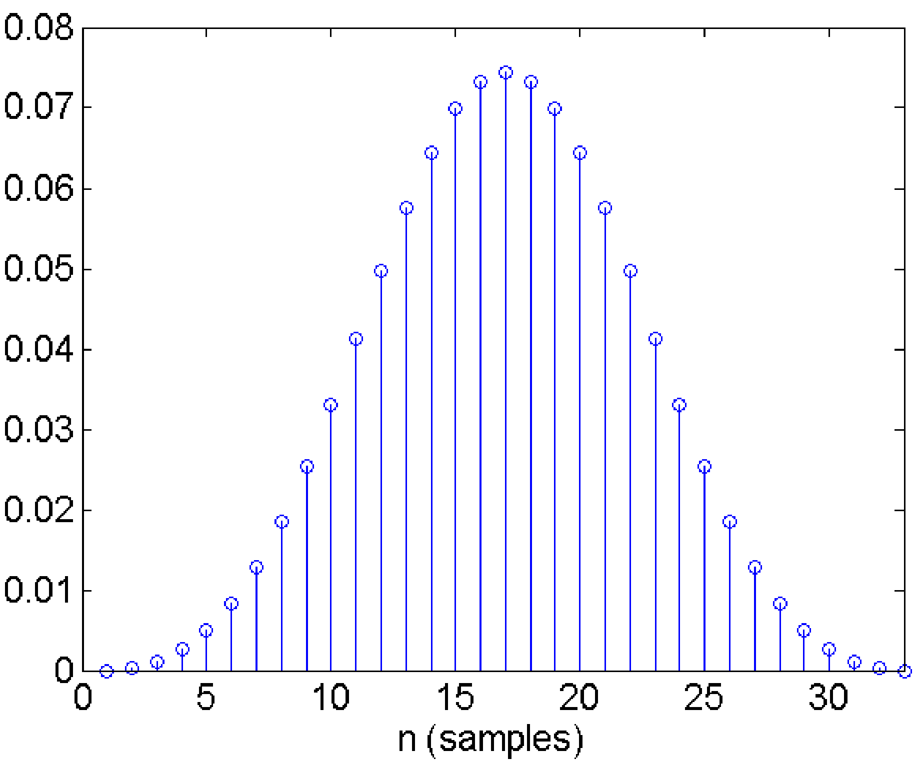
Example: finding peaks in a spectrum

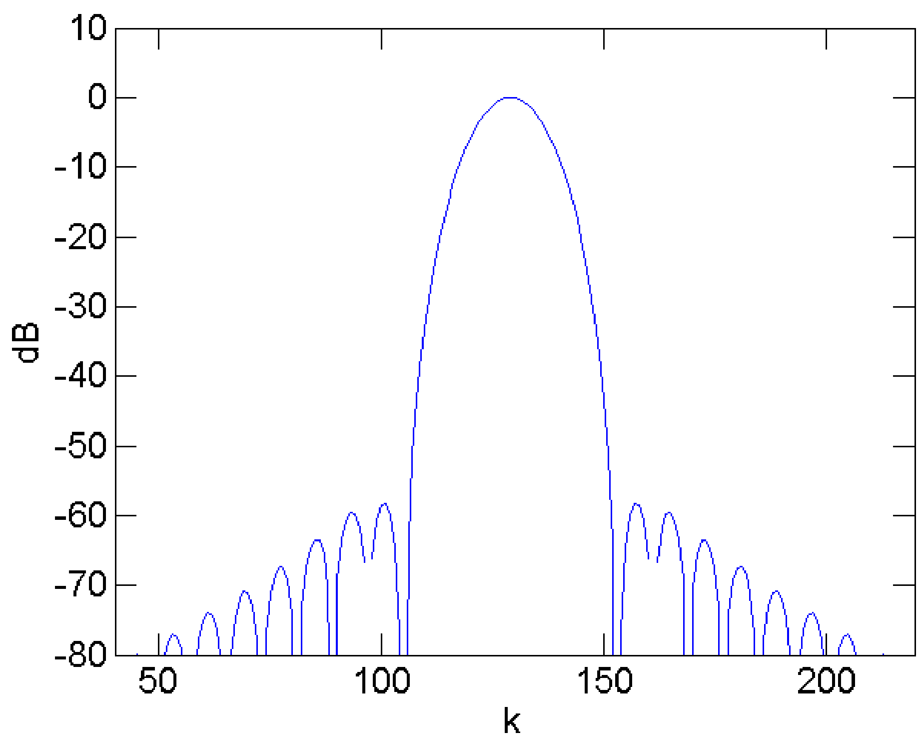
- Peaks can move
- Peaks can grow or attenuate
- Need to ignore side lobes
  - But how?



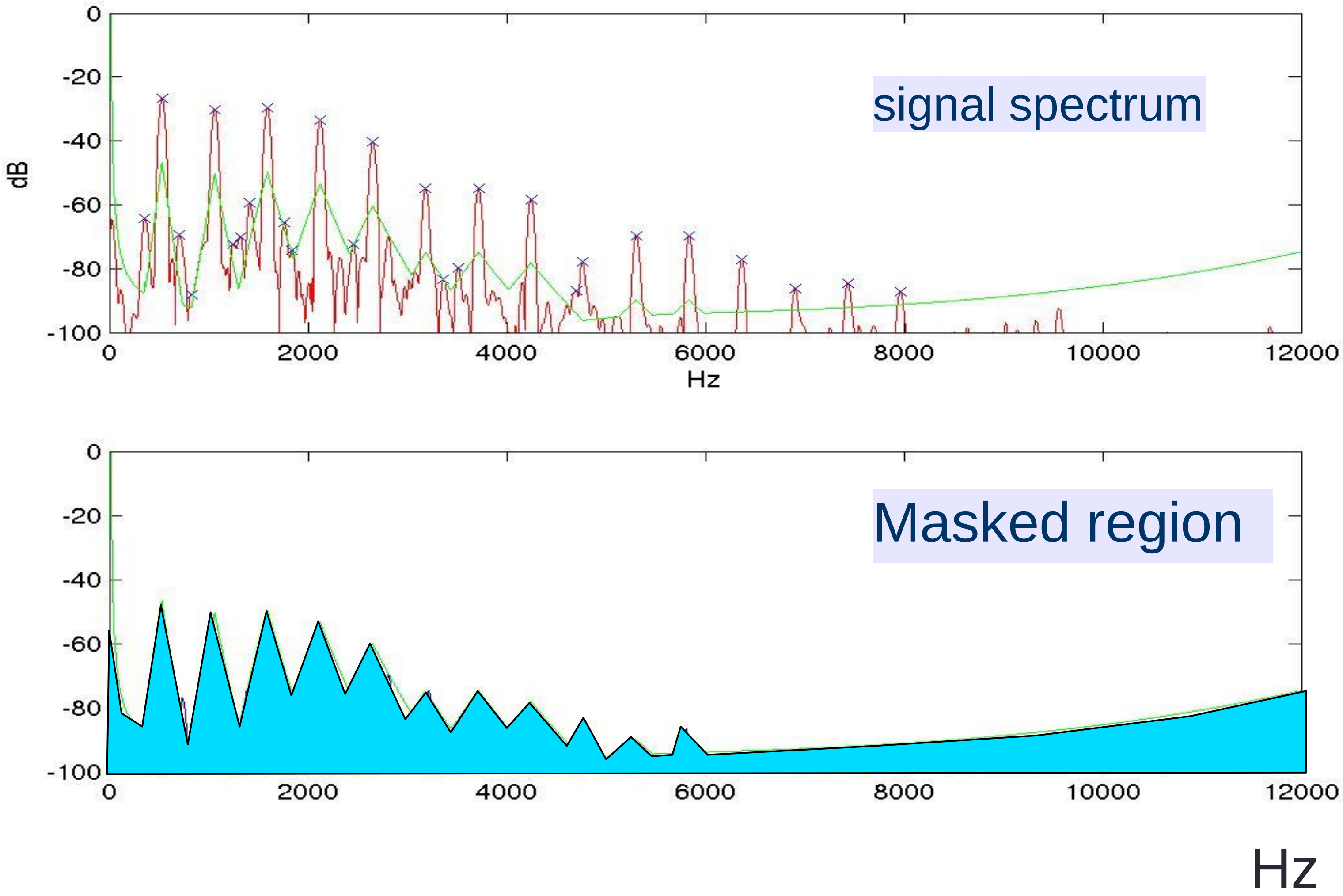
Blackman window
has > 50 dB side-lobe suppression

- All side lobes beneath the *masking threshold*.





Example:
Hann-windowed transform of trumpet sounds

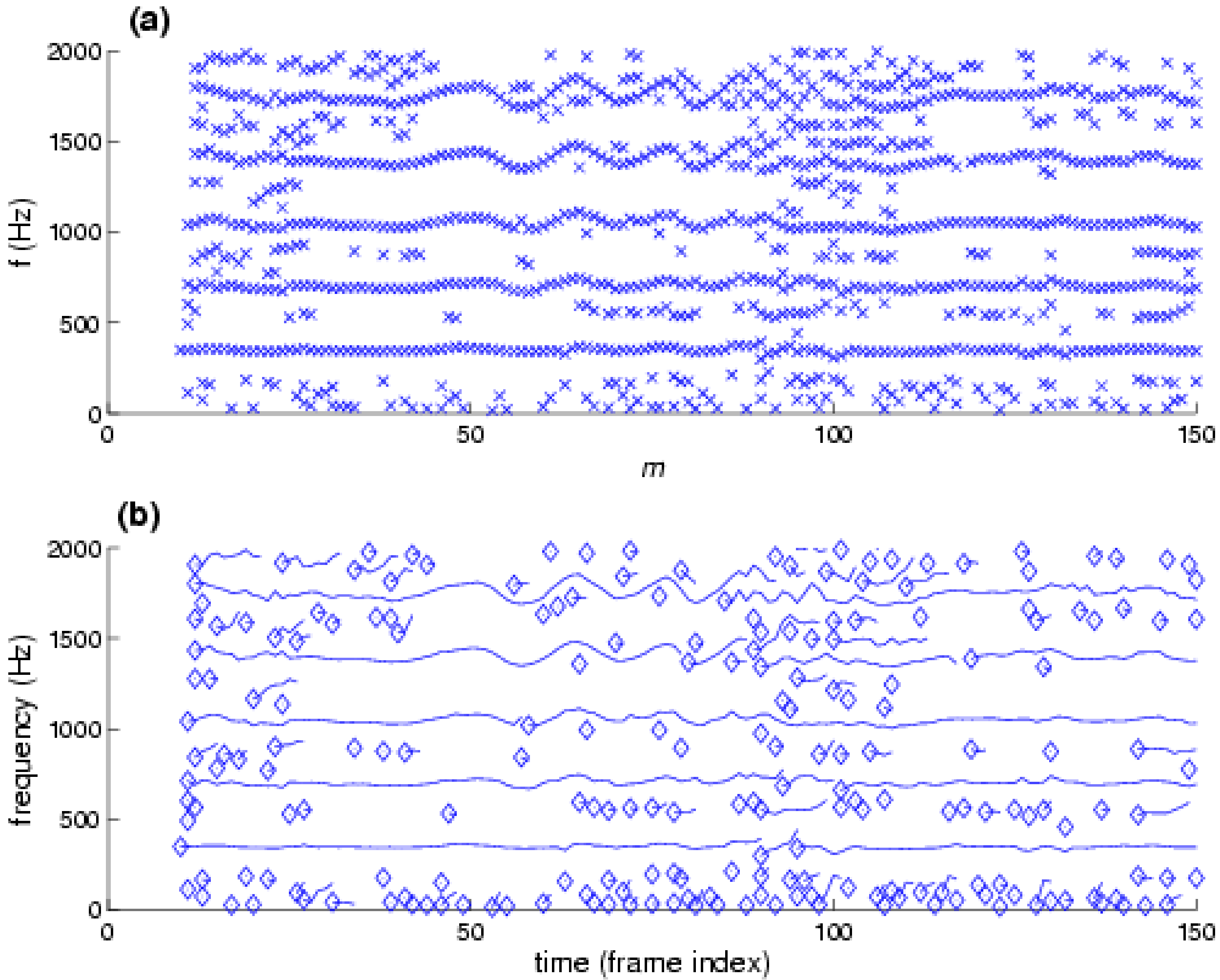


Psychoacoustic masking

- Masking refers to the fact that softer sounds cannot be heard due to the presence of stronger sounds.
  - Forward masking
  - Simultaneous
  - Backward masking

- We will cover psychoacoustics later in this semester.

Sinusoidal Modeling: from discrete peaks to trajectories

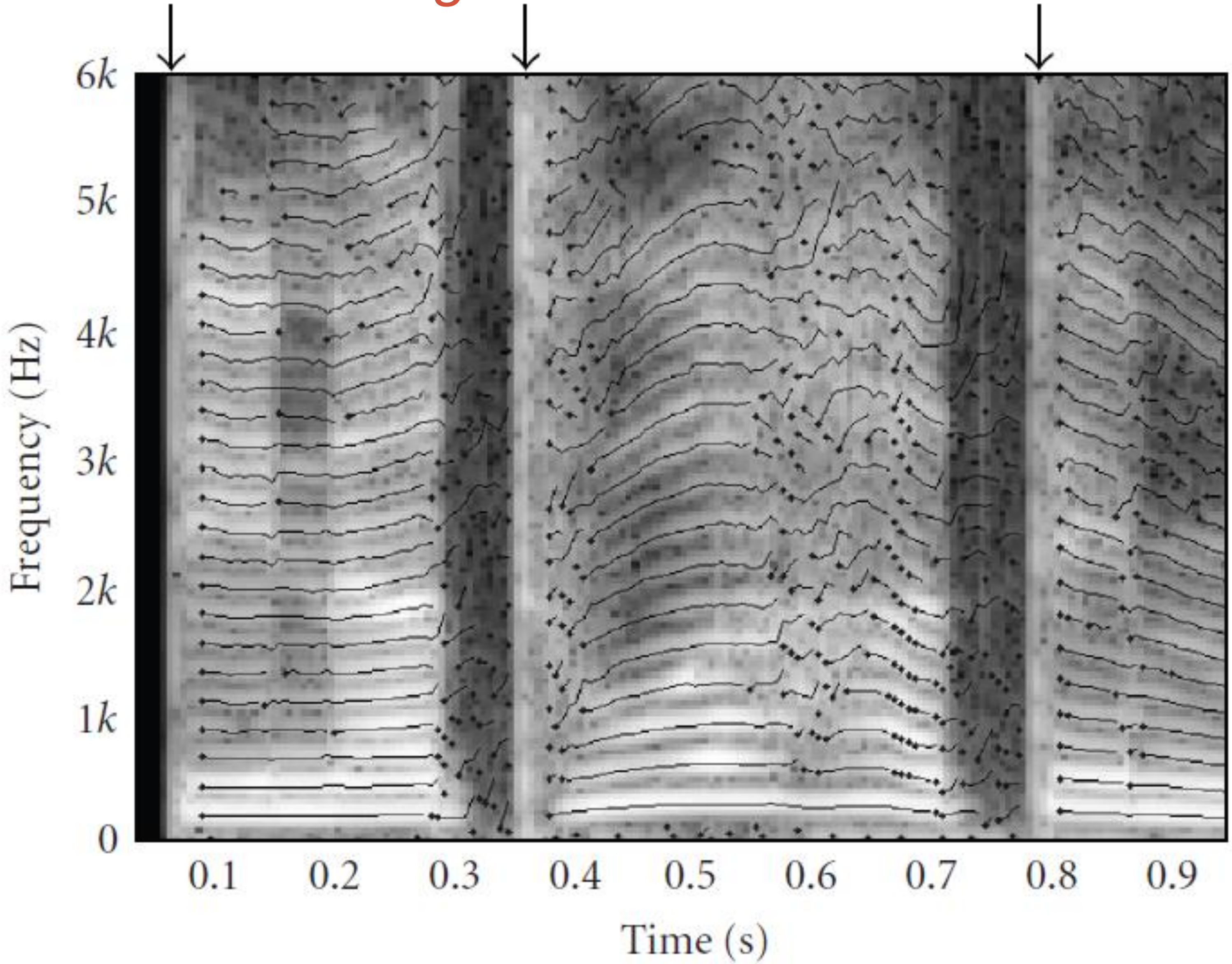


Trajectory formation

- Basic peak tracking involves:
  - Finding shortest link
  - Resolving splits

- Advanced technique includes
  - Forbidding large jumps
  - Transient detection
  - Highest peak chooses first

Frequency-trajectories of a speech signal, arrows indicate *transient regions*



Part II: Synthesis

- Sinusoidal modeling
  - Analysis
  - **Synthesis**
    - Linear-interpolation
    - Phase continuation
    - A window-based synthesis method

- Spectral decomposition
  - (S + N) vs. (S + N + T)
  - Spectral subtraction
  - Noise modeling (Nov. 16, 23)

Sinusoidal synthesis (ii): the linear interpolation method

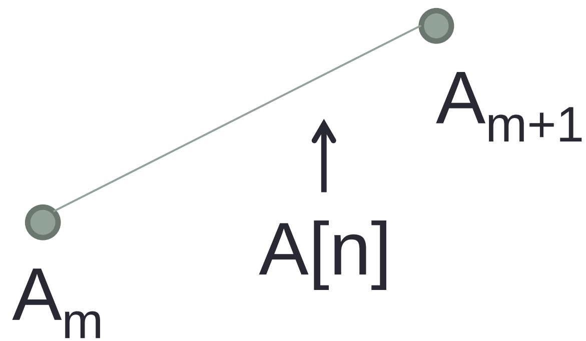
$\{A_m\}_{m=0}^{M-1}$ ....an amplitude envelope

$\{f_m\}_{m=0}^{M-1}$ ....a frequency envelope

$$A[n] = \beta A_m + (1-\beta)A_{m+1}$$

$$f[n] = \beta f_m + (1-\beta)f_{m+1}$$

$$\beta = \frac{(m+1)h-n}{h}$$



Remark: amplitude can be log or linear scale.

The linear-interpolation method (cont'd)

- Between time *mh* and (*m*+1)*h*, do this:

$$A[n] = \beta A_m + (1-\beta)A_{m+1}$$
$$f[n] = \beta f_m + (1-\beta)f_{m+1}$$

- update the phase sample-by-sample:

$$\varphi_n = \varphi_{n-1} + 2\pi f[n]/f_s$$

- For each trajectory *j*, do:

$$s_j[n] = A[n]\cos(\varphi_n)$$

- Finally, sum over all *j*

From one trajectory to a signal:
a window-based method

$\{A_m\}_{m=0}^{M-1}$ ....an amplitude envelope

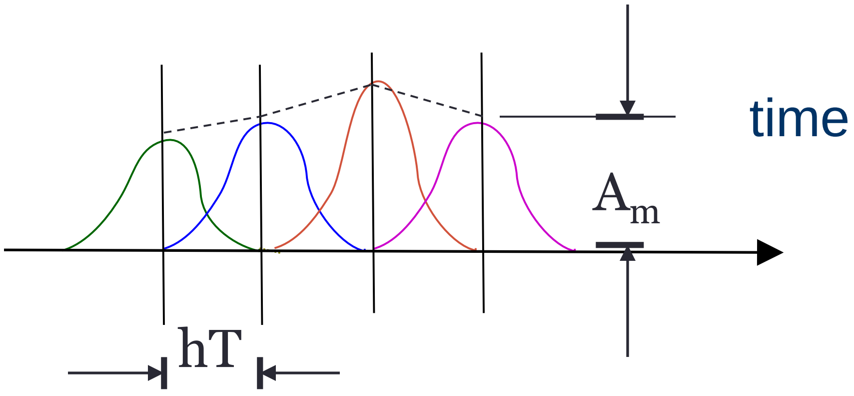
$\{f_m\}_{m=0}^{M-1}$ ....a frequency envelope

Synthesis:

$$s[n] = \sum_{m=0}^{M-1} A_m w[n - mh] \cos\left(2\pi f_m nT + \phi_m\right)$$

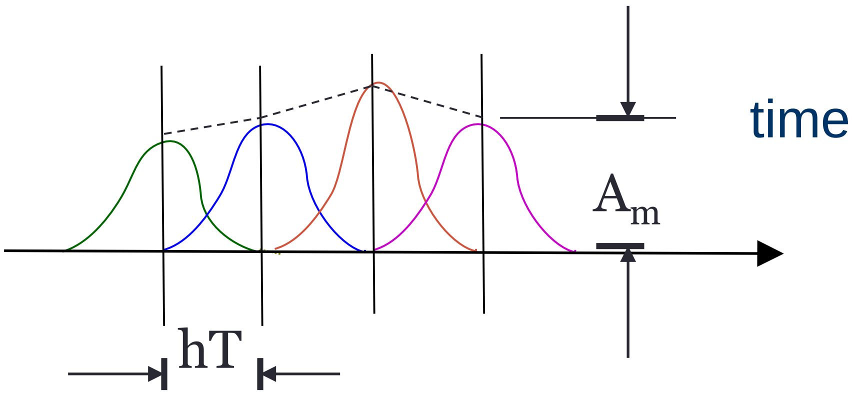
Continuous phase updates:

$$\phi_m = \phi_{m-1} + 2\pi \left(\frac{f_{m-1} + f_m}{2}\right) hT$$



summation over all trajectories

$$s[n] = \sum_{\text{trajectories}} \sum_{m=0}^{M-1} A_m w[n - mh] \cos\left(2\pi f_m nT + \varphi_m\right)$$



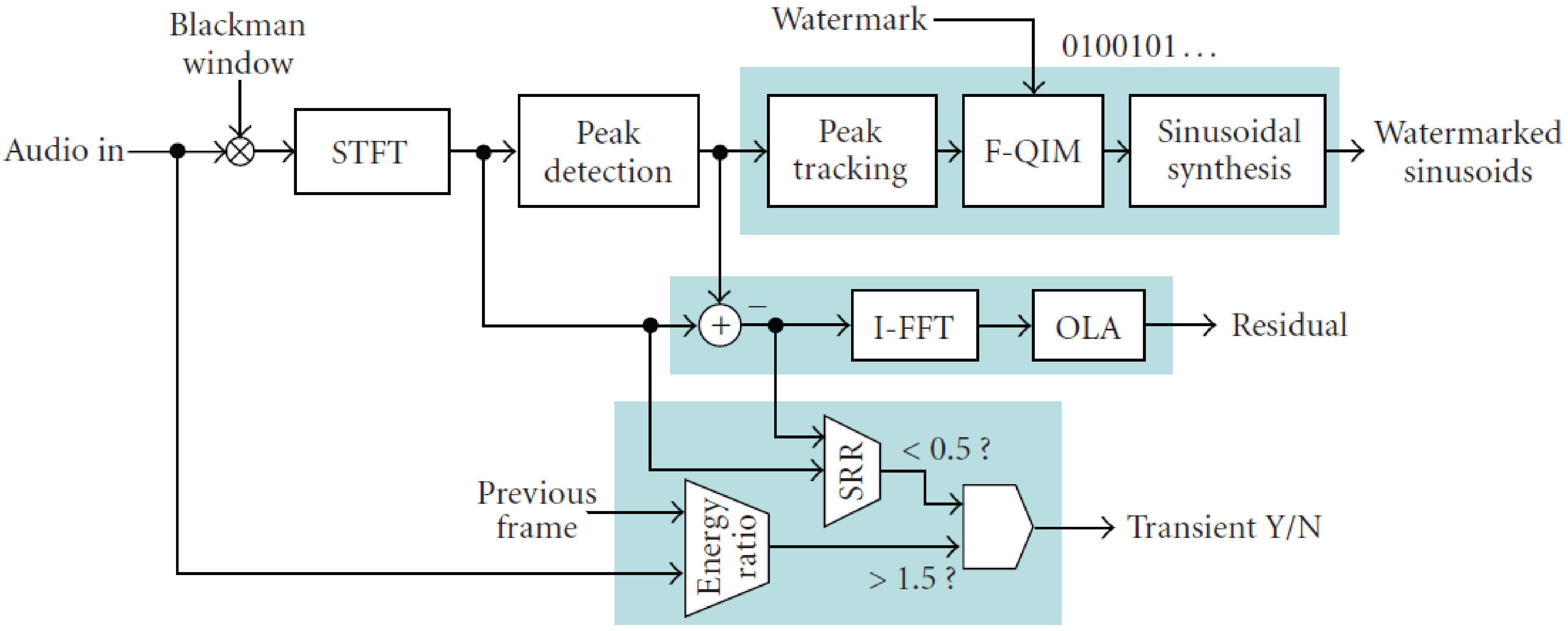
Applications of sinusoidal modeling

- Noise removal/ speech enhancement
  - Potentials in hearing aids/ cochlear implants
- Musical effects:
  - Pitch shifting/ time warping
- Parametric audio coding:
  - MPEG-4 structured audio
  - B. Vercoe et al. (1998). “Structured audio: Creation, transmission, and rendering of parametric sound representations,” *Proc. IEEE,* Vol. 86, No. 5, 922-40.

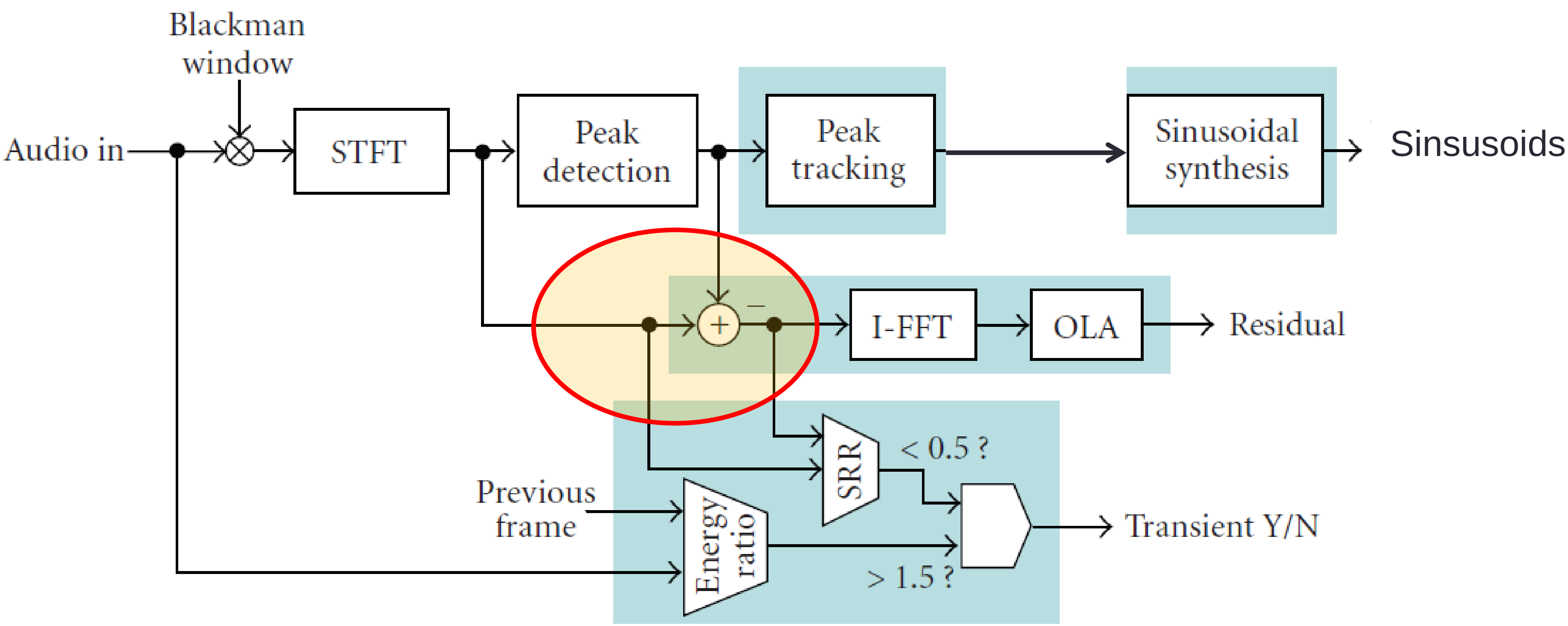
Part III: Spectral decomposition

- Sinusoidal modeling
  - Analysis
  - **Synthesis**
    - Linear-interpolation
    - Phase continuation
    - A window-based synthesis method
- Signal decomposition
  - Spectral subtraction
  - (S + N) vs. (S + N + T)
  - Noise modeling (**Possible final project idea**)

Example: **a watermarking system based on signal decomposition** (Liu & Smith, 2007)

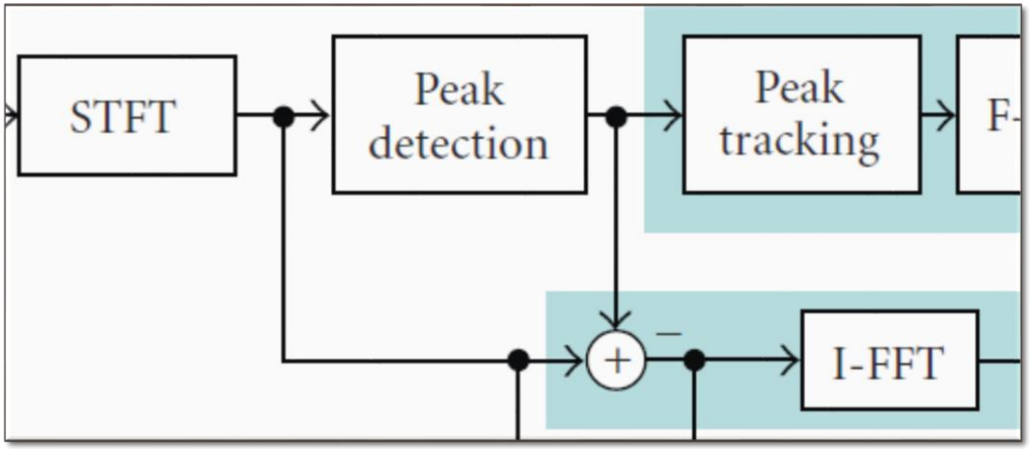


Sines + Noise + Transient (S+N+T) decomposition

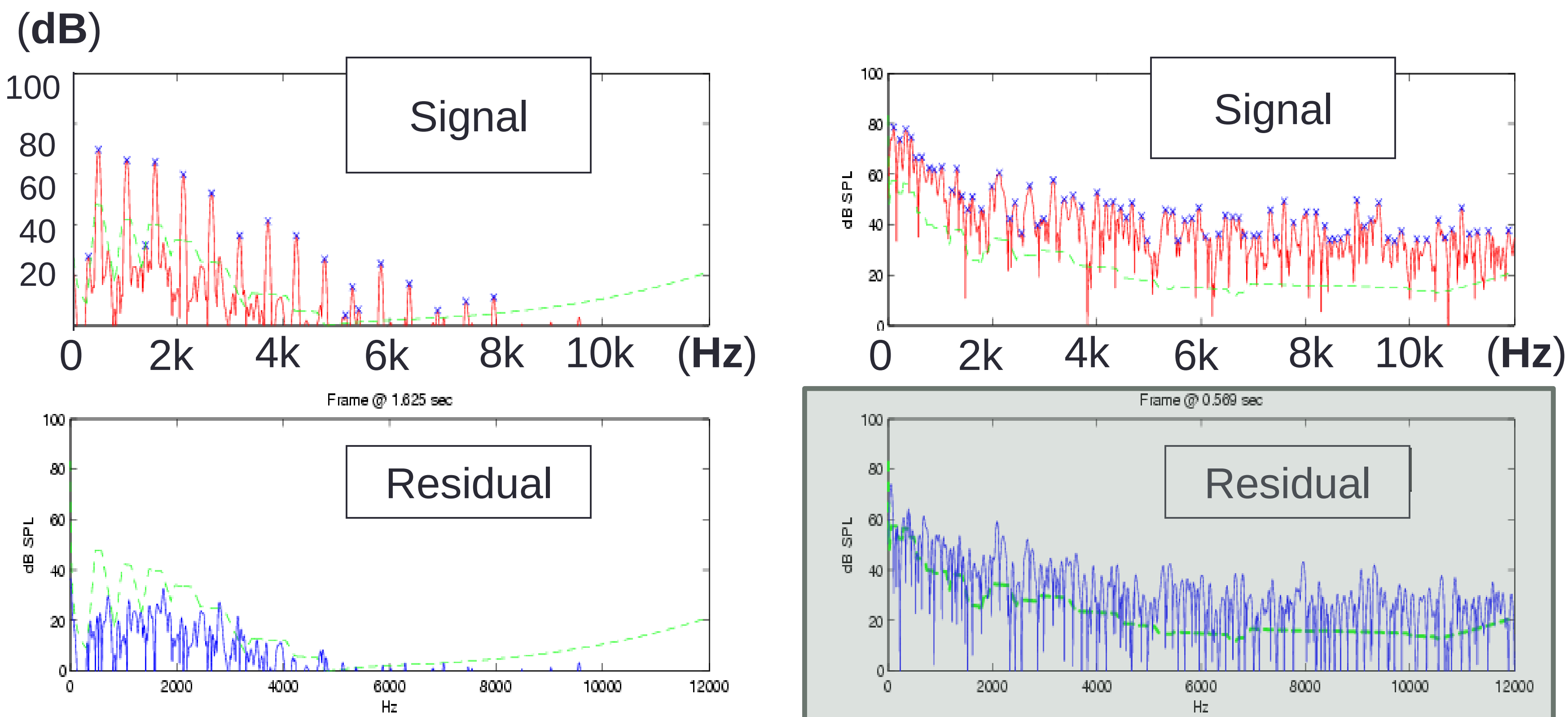


Spectral subtraction

- For each peak in the spectrum, do
  1. fit the mainlobe of Blackman window
  2. subtract the mainlobe
  3. Inverse FFT to synthesis the residual (“noise”)



Sines + "noise" decomposition by spectral subtraction

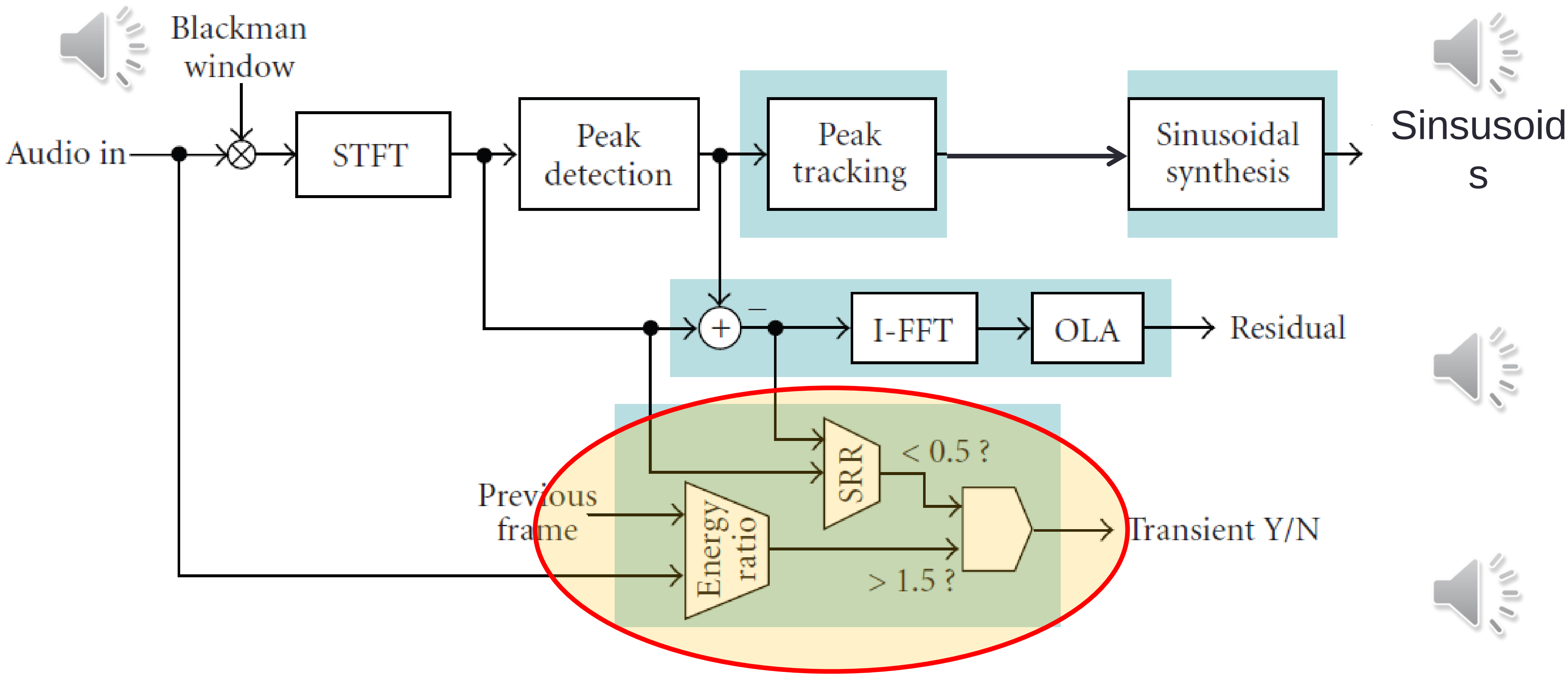


(a) Trumpet

(b) Dance music

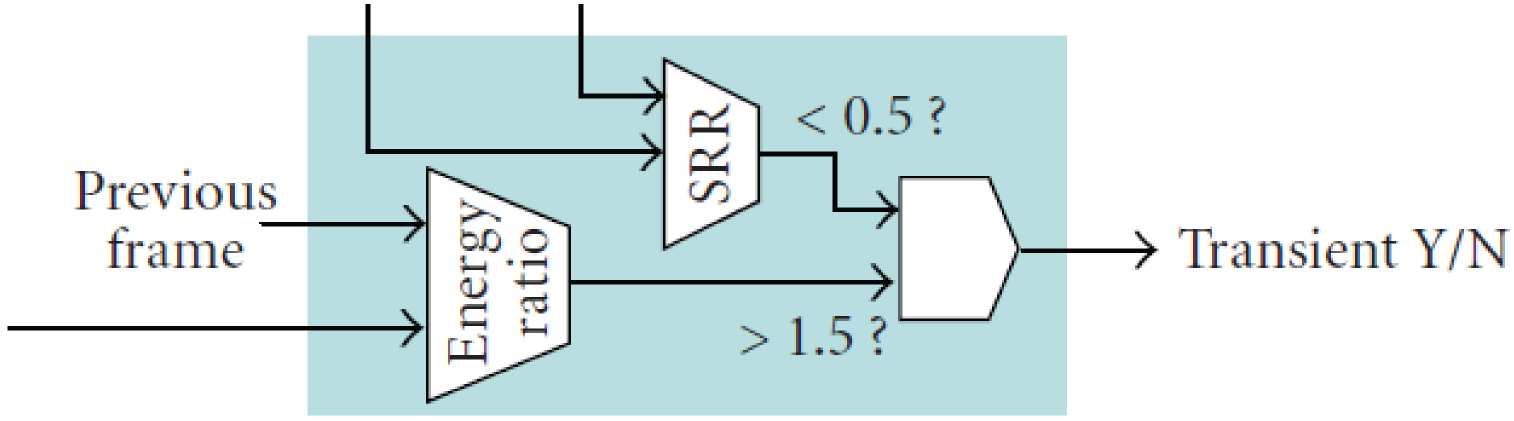
Parametric noise modeling?

Transient detection



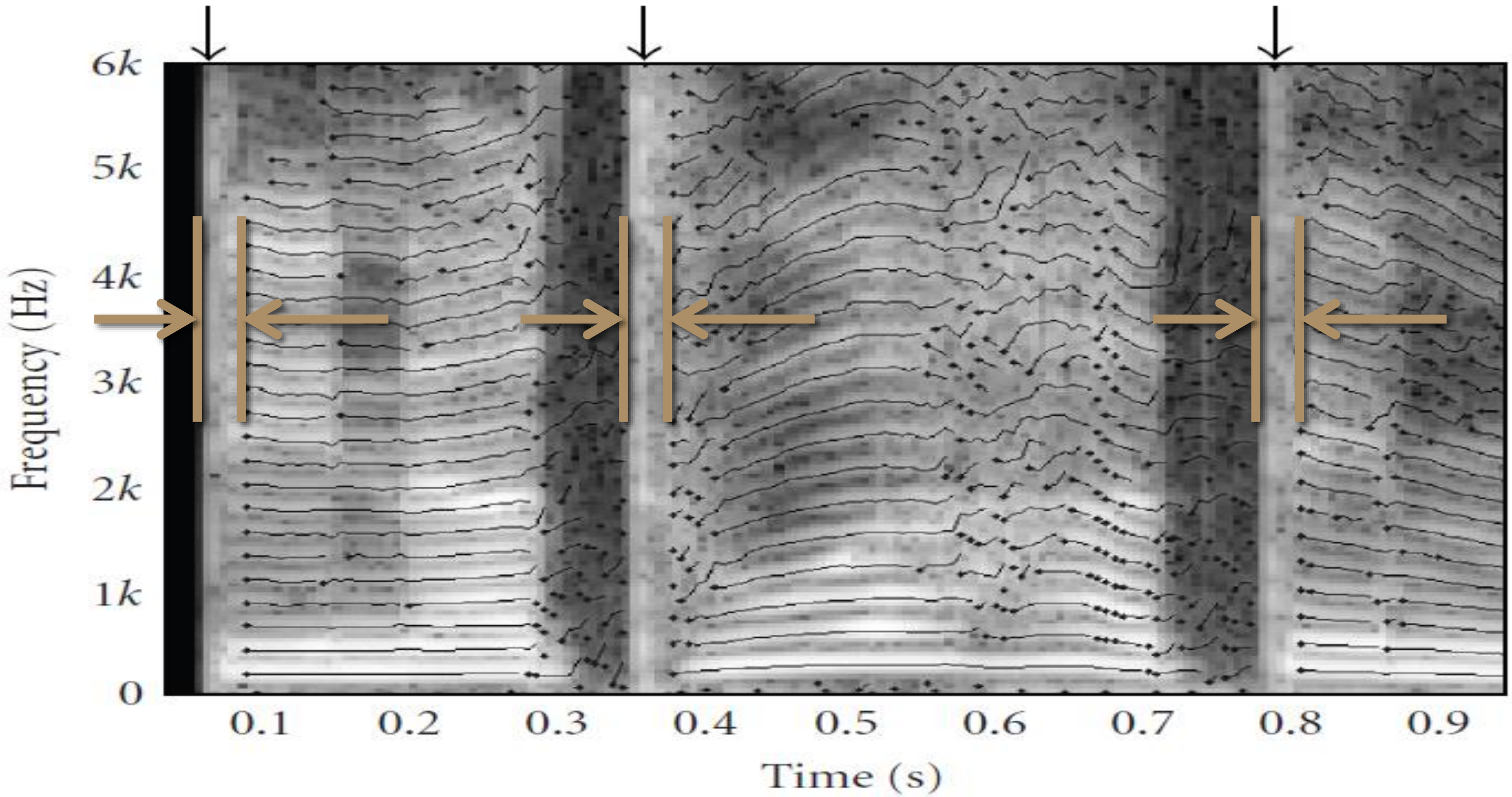
Transient detection: there's no gold standard

- The following is just a heuristic approach
  - Energy comparison
  - Sine-to-residual ratio (SRR)



Entering and exiting the transients

- Once a transient is detected, sinusoidal model is **halted** for a period of time.



Possible project ideas

- Trajectory formation
  - **Causal implementation for real-time applications**
- Applications of spectral subtraction
  - Noise-removal / Speech enhancement
- Audio recognition
  - Sound source segregation
  - Audio fingerprinting/ watermarking
  - And so on…