

# Common Random Numbers-Set Random Streams

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# Control Random Numbers

## Distributions in FLexsim

- Exponential(location, scale, [stream](#))
  - Normal(Mean Std Dev, [stream](#))
  - Beta(Min, Max, Shape 1, Shape 2, [stream](#))
  - General(xxx,xxx,xxx,...,[stream](#))
- The last value in the parenthesis of any distribution is [stream](#), which is called random stream, used to control random numbers.

# Random Numbers (亂數)

## Definitions

$U_1, U_2, \dots, U_n$  are random numbers if

- They follow uniform (0,1)
  - They are independent
- 
- random numbers are independent uniform (0,1) random variables.
  - All random variables can be generated based on random numbers. (see next page)

# Inverse CDF $F_X^{-1}(u)$

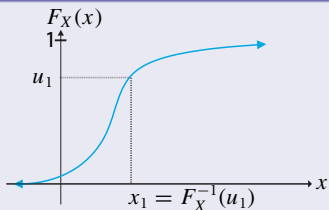
## Definition

- $F_X(x) \equiv P\{X \leq x\}, \forall x \in R$
- $F_X^{-1}(u) \equiv \inf\{x \in R : F_X(x) \geq u\}, u \in [0, 1]$

## Results

- $F_X(x) \sim U(0, 1)$  for conti.  $X$
- $U \sim U[0, 1] \Rightarrow F_X^{-1}(u) \sim F_X(\cdot)$

## cdf Plot



## Generate $x_1, x_2, \dots, x_n$

- Step 1. Generate  $u_1$  from  $U(0,1)$
- Step 2. Obtain  $x_1 = F_X^{-1}(u_1)$
- Repeat the above 2 steps  $n$  times;  
we have data:  $x_1, x_2, \dots, x_n$

# Generate $X$ based on RN

Let  $U_1, U_2, \dots, U_n$  be random numbers

## Random variables

- Ex 1.  $X \sim \text{Exponential}(\beta)$ ,  $E(X) = \beta$
- Ex 2.  $X \sim \text{Weibull}(\lambda, \beta)$
- Ex 3.  $X \sim \text{Normal}(\mu, \sigma)$

## Inverse CDF

- Ex 1.  $X_i = -\beta \ln(1 - U_i)$
- Ex 2.  $X_i = -\beta [\ln(1 - U_i)]^{1/\alpha}$
- Ex 3.  $X_i = \frac{(U_i^{0.135} - (1 - U_i)^{0.135})}{0.1975}$

- Use Fit-of-Goodness to test whether the above Inverse CDF algorithms work.

# Set Random Stream = 0

## 1 Queue.



Source1 Properties

Source: Source1

Arrival Style: Inter-Arrival Time

FlowItem Class: Box

Inter-Arrival Time: exponential(0, 2.1, 0)

Global Table - ArrivalTime

Row	ArrivalTime
Row 1	1.11
Row 2	2.94
Row 3	2.98
Row 4	3.13
Row 5	4.62
Row 6	5.53
Row 7	5.83
Row 8	6.07
Row 9	6.67
Row 10	9.64
Row 11	20.23
Row 12	30.97
Row 13	32.78
Row 14	15.38
Row 15	14.62

## 2 Queues.



Source2 Properties

Source: Source2

Arrival Style: Inter-Arrival Time

FlowItem Class: Box

Inter-Arrival Time: exponential(0, 2.1, 0)

Global Table - ArrivalTime

Row	ArrivalTime
Row 1	1.11
Row 2	2.96
Row 3	4.34
Row 4	4.81
Row 5	5.09
Row 6	8.06
Row 7	8.65
Row 8	10.46
Row 9	11.85
Row 10	11.09
Row 11	11.89
Row 12	14.36
Row 13	15.30
Row 14	19.30

- Click "Repeat Random Streams"
- "Set random stream = 0" → Arrival Times differ

# Set Random Stream = 1

## System 1

The screenshot shows the 'Source1 Properties' dialog box with the 'Repeat Random Streams' checkbox checked. The 'Inter-Arrival Time' is set to 'exponential(0, 2, 1)'. Below it, the 'Global Table - ArrivalTime' window displays a table of arrival times for 15 rows.

Row	ArrivalTime
Row 1	0.81
Row 2	2.74
Row 3	3.78
Row 4	3.98
Row 5	4.35
Row 6	4.36
Row 7	5.01
Row 8	6.35
Row 9	7.83
Row 10	10.41
Row 11	12.09
Row 12	16.87
Row 13	20.01
Row 14	24.07
Row 15	28.05

## System 2

The screenshot shows the 'Source2 Properties' dialog box with the 'Repeat Random Streams' checkbox checked. The 'Inter-Arrival Time' is set to 'exponential(0, 2.1, 0)'. Below it, the 'Global Table - ArrivalTime' window displays a table of arrival times for 14 rows.

Row	ArrivalTime
Row 1	0.81
Row 2	2.74
Row 3	3.78
Row 4	3.98
Row 5	4.35
Row 6	4.36
Row 7	5.01
Row 8	6.35
Row 9	7.83
Row 10	10.41
Row 11	12.09
Row 12	16.87
Row 13	20.01
Row 14	24.07

- Click "Repeat Random Streams"
- Set "random stream  $\neq 0$ "  $\rightarrow$  Arrival Time identical

# Common Random Numbers (CRN)

- Goal: To use CRN in two systems in Flexsim

## Way 1. Default distri.

- Two systems can not be in the same Flexsim file
- Click "Repeat Random Streams"
- Set the same **stream** value. Ex. exponential (location, scale, **stream**), stream value cannot be 0
- It is not easy to trace the RN

## Way 2. Inverse CDF

- Use **inverse cdf** to generate data.
- Store the RN ( $U_i, i = 1, 2, \dots$ ) in Global Table
- Ex. Suppose we want to generate Exponential (mean =  $1/\lambda$ )
- $X_i = -\ln(1 - U_i)/\lambda$ , where  $U_i$  are RN
- It is easy to trace the RN

- Q: How can we use CRN to control two systems in the same flexsim file?
- A: Way 1 is not valid. We need to use Way 2