Input Modeling: Part I

W. M. Song 桑慧敏
Tsing Hua Univ. 清華大學

2015.11.11

Recall: R.V. X, 證件 and 屬性

Identification (ID, 證件)

- cdf: $F_X(x) = \mathrm{P}(X \le x)$
- pmf or pdf:

$$f_X(x) = \begin{cases} F_X(x) - F_X(x^-) = \mathrm{P}(X = x), & \text{若 } X \text{ 為離散型隨機變數} \\ \dfrac{dF_X(x)}{dx}, & \text{若 } X \text{ 為連續型隨機變數} \end{cases}$$

- mgf: $\mathrm{M}_X(t) = \mathrm{E}(e^{tX})$

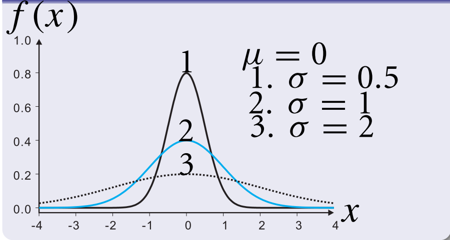
Characteristics (屬性)

- Moments (動差): Expected value, variance, skewness, and kurtosis,...
- quartiles (四分位數), Percentiles (百分位數)
- Tail events probability (尾端機率) : Markov and Chebyshev's Inequalities

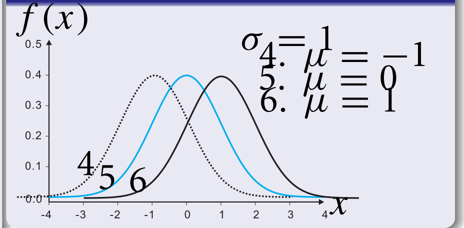
- In general: Given 證件, we can derive 屬性.
  But given 屬性, we may not be able to derive the 證件

Plots of pdf for Normal(μ, σ)

Same Mean

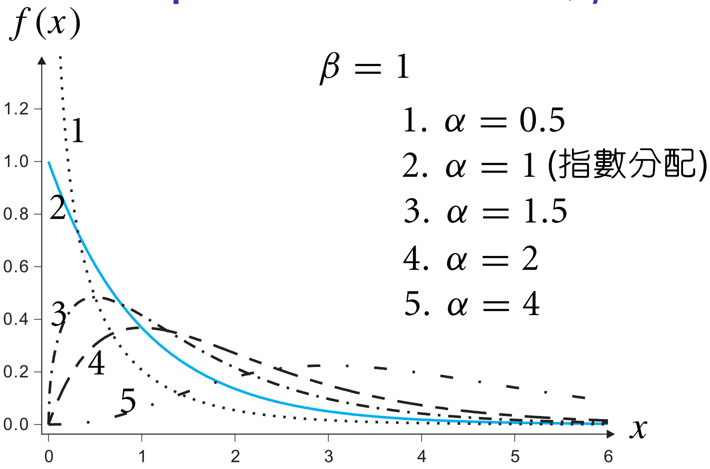


Same Variance



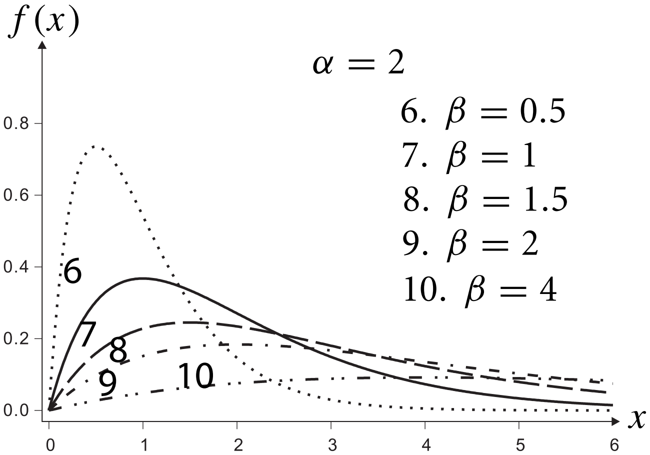
- There is no shape parameter; No of Shape parameters: $k = 0$.

Plots of pdf for Gamma(α, β)



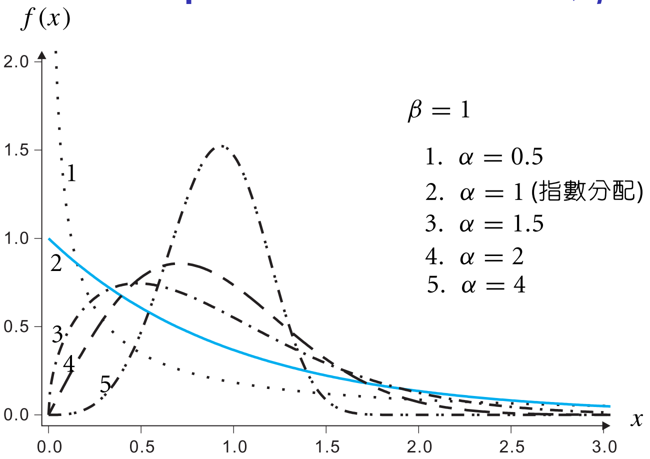
- Shape parameters α; No of Shape parameters $k = 1$.

Plots of pdf for Gamma(α, β)



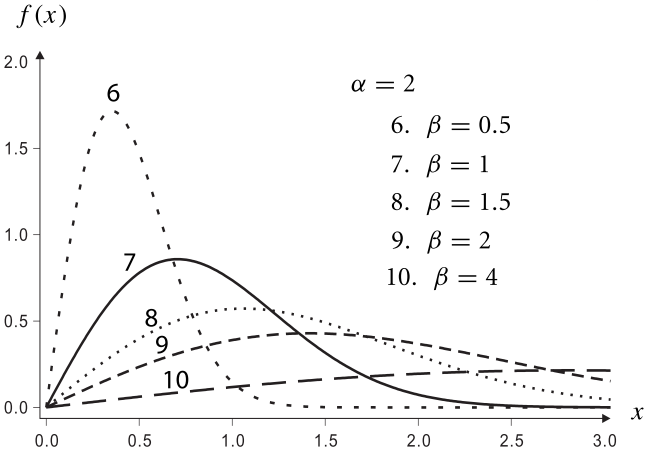
- Shape parameters α; No of Shape parameters $k = 1$.

PLots of pdf for Weibull(α, β)



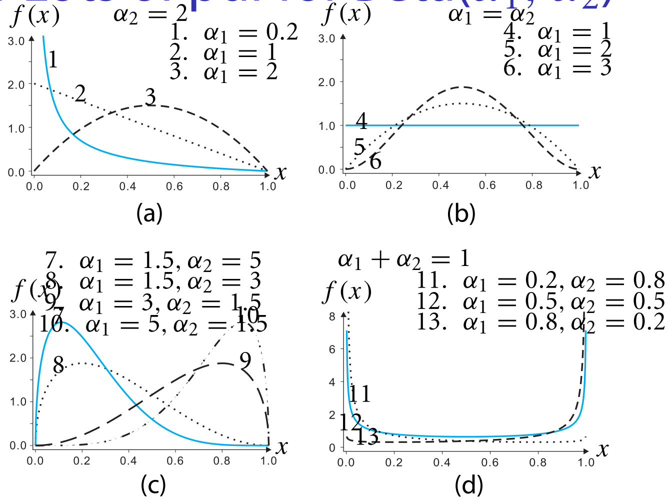
- Shape parameters α; No of Shape parameters $k = 1$.

PLots of pdf for Weibull(α, β)



- Shape parameters α; No of Shape parameters $k = 1$.

PLots of pdf for Beta(α_1, α_2)



- Shape parameters α_1, α_2; No of Shape parameters $k = 2$.

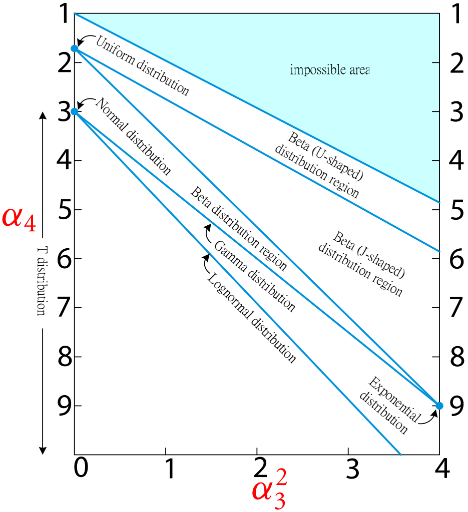
Skewness and Kurtosis: Pdf

偏度: $\alpha_3 = \mathrm{E}(X - \mu_X)^3/(\sigma_X^3)$

峰度: $\alpha_4 = \mathrm{E}(X - \mu_X)^4/(\sigma_X^4)$

- Q1: Given $\alpha_3 = 0, \alpha_4 = 3$, can you tell the name of the R.V.?
- A1: Normal distributon
- Q2: Given $\alpha_3 = 2, \alpha_4 = 9$, can you tell the name of the R.V.?
- A2: Exponential distribution
- Q3: In general, given α_3 and α_4, can we tell the R.V.?
- A3: Yes. There is a relationship among the random variables and the standardized 3rd and 4th moments. (See next page)
- Reference: Statistical Models in Engineering by G.J. Hahn and S. S. Shapiro, John Wiley: New York, 1967.

PDF with the 3rd and 4th moments



偏度:

$\alpha_3 = \mathrm{E}(X-\mu_X)^3/(\sigma_X^3)$

峰度:

$\alpha_4 = \mathrm{E}(X-\mu_X)^4/(\sigma_X^4)$

k: no. of shape para.

$k = 0$: a point

$k = 1$: a line

$k = 2$: an area

Probability Plots for Normal

- Ordered Statistics: $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ are n ordered statistics
- Empirically, $\hat{F}_n(x_{(i)})$ can be estimated by $\frac{i-0.5}{n}$

$x_{(i)}$ and $\Phi^{-1}(\frac{i-0.5}{n})$ have linear relation

- $\underline{x} = x_1, x_2, \cdots, x_n$
- $\underline{x}_{(i)} = x_{(1)}, x_{(2)}, \cdots, x_{(n)}$
- $F_X(x_{(i)}) = P(X \le x_{(i)}) = P(Z \le \frac{x_{(i)}-\mu}{\sigma}) = \Phi(\frac{x_{(i)}-\mu}{\sigma}) \simeq \frac{i-0.5}{n}$
- $\frac{x_{(i)}-\mu}{\sigma} \simeq \Phi^{-1}(\frac{i-0.5}{n}) \Longrightarrow \Phi^{-1}(\frac{i-0.5}{n}) \simeq -(\mu/\sigma) + (1/\sigma)x_{(i)}$
- That is, ($x_{(i)}$,$\Phi^{-1}(\frac{i-0.5}{n})$) should be a "line", if $x_{(i)} \sim$ Normal(μ, σ).