Day 1: Global View of Simulaiton

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Simulation

Definition of Simulation

mimic, pretend to be, act like,...模擬, 仿真

Motivation of using Simulation

- if a problem cannot be solved analytically
- if the numerical approach is difficult
- we should try simulation approach

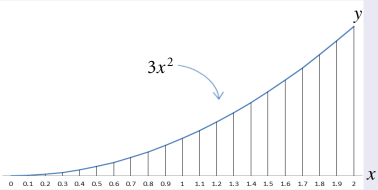
- Distinguish three approaches: analytical, numerical, and simulation

Ex 1. Integration, $\theta = \int_0^2 3x^2 dx$

Analytical (微積分)

- $\theta = \int_0^2 3x^2 dx = x^3|_0^2 = 8$

Numerical (梯形法)



Simulation (Monte-Carlo, 蒙地卡羅)

$\theta = \mathrm{E}(G(x)), G(x) = 6x^2, x \sim U(0, 2)$

sum=0

do 10 $i = 1, n$

$u \sim u(0, 1)$

$x = 2u$

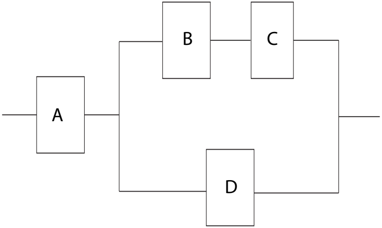
$g = 6x^2$

10 sum = sum + g

$\hat{\theta}$ = sum / n

- Discussion: advantages and disadvantages of 3 approaches

Ex 2. $\theta = $ P(current flows)



- Find: $\theta = P($ Current flows $)$
- Current flows (電流通過) if the switch is "closed"
- Given: $\mathrm{P}(A) = 0.8$, $\mathrm{P}(B) = 0.9$, $\mathrm{P}(C) = 0.7$, and $\mathrm{P}(D) = 0.6$

Ex 2. $\theta = \mathrm{P}$(current flows) (Conti)

- Analytical Approach: (Probability)

$$\begin{aligned}\theta &= \mathrm{P}((A \cap D) \cup (A \cap B \cap C)) \\ &= \mathrm{P}(A)\mathrm{P}(D) + \mathrm{P}(A)\mathrm{P}(B)\mathrm{P}(C) - \mathrm{P}(A)\mathrm{P}(B)\mathrm{P}(C)\mathrm{P}(D) \\ &\quad \text{if } A, B, C \text{ and } D \text{ are independent}\end{aligned}$$

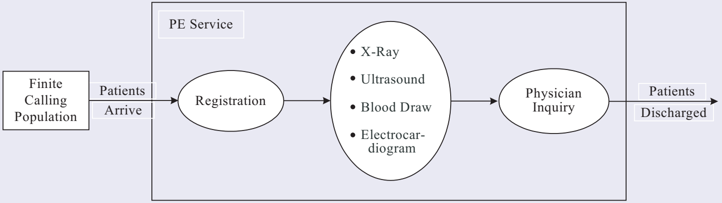
- Simulation Approach:

```
kount = 0
iseed = 123456789
do 20 i = 1, n
      A = B = C = D = .false.
      if( rand(iseed) .lt. 0.8) A = .true.
      if( rand(iseed) .lt. 0.9) B = .true.
      if( rand(iseed) .lt. 0.7) C = .true.
      if( rand(iseed) .lt. 0.6) D = .true.
      if( (A .and. D) .or. (A. and. B. and. C)) kount = kount +1
20 continue
```

$\hat{\theta}$ = kount / n

Ex 3. Physical Exam. (PE) Service (midterm Exam.)

PE Framework



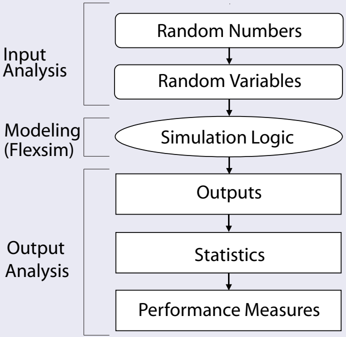
- How do we increase the efficiency of PE system?
- Can we obtain the optimal Doctor's inquiry starting time?
- Can the above problem be solved analytically? or numerically?

Tasks on Simulation

Tasks

- Basic Tasks
  - Input Analysis
  - Modelling
  - Output Analysis
- Advanced
  - Simulation Design (Variance Reduction)
  - Metamodeling
  - Simulation Optimization

Figure of Basic Tasks



Ex: Flipping a Fair Coin

Flipping a fair coin, estimate θ: the probability that showing "Head"

Analytical

- Notations:
  - Sample space S
  - event that showing "Head" E
  - θ =P(E)
- Compute: P(E) =1/2
- Formula: P(E) = (no. in E)/ (no. in S)

Simulation via Excel

- Random Numbers: rand()
- Outcome: 1 或 0: =IF(B8<0.5,1,0)
- Estimator $\hat{\Theta}$, estimate $\hat{\theta}$: =AVERAGE(C8:C1007)
- Replication: Press F9 $n = 10$ times
- Standard error: se($\hat{\theta}$)

- Discussion

Ex. Monty Hall Problem

有三個門讓你選, 三個門中只有一個門後有大獎, 其他兩個門後沒有獎。你選定後, 主持人打開其中一個你沒選中也沒有獎的門。現在再讓你作一次選擇, 你可以選擇換門或不換門。如果你選擇換門, 那得獎的機率是多少?

Analytical Appro. I

- $\mathsf{S} = \{(i, j), i = 1, 2, 3; j = 1, 2, 3\}$
- E =「參與者最後中獎」的事件
- $= \{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$, 一共有 6 個樣本點
- 計算: $\mathsf{P(E)} = 6/9 = 2/3$

Analytical Appro. II

- $\mathsf{S} = \{1, 2, 3\}$
- F =「第一次就選中有獎的門」的事件
- $\mathsf{P}(E) = \mathsf{P}(E \mid F)\mathsf{P}(F) + \mathsf{P}(E \mid F^C)\mathsf{P}(F^C) = (0)\,(1/3) + (1)\,(2/3) = 2/3$

- Discussion: 參考 "桑慧敏, 機率與推論統計學原理. p. 55, 68."

Monty Hall (MH) Problem: Simulation

simulation approach via MS Excel.

- Door-Prize: "=IF(A13<(1/3),1,IF(A13<(2/3),2,3))"
- Initial door Chosen: "1"
- Door MH chooses: "=IF(B11=A11,IF(RAND()<0.5,2,3),IF(A13=2,3,2))"
- Policy: no change: "=IF(B11=A11,1,0)"
- Policy: change door: "=IF(IF(C11=3,2,3)=A11,1,0)"
- $\hat{\Theta}$: estimated P(win, no change)=AVERAGE(D13:D1012)
- se ($\hat{\Theta}$):
- Discussion: (1) Two analytical approaches, (2) Analytical vs. Simulation

Homework

1. Review Probability distributions and use Excel to generate some random variables such as Uniform, Exponential, Normal, Weibull
   - U(0,1): rand()
   - Normal (μ, σ) x = norm.inv (rand(),μ , σ)
   - Exponential (λ): $u = 1 - \exp(-x/\beta)$
   - Weibull (α, β): $x = -\beta \ln(1-u)$
2. Apply analytical and simulation approaches to solve the following problems
   - (a) Obtain $\theta = \int_0^5 5x^3 dx$
   - (b) 現有三枚錢幣, 一枚為公正的一正一反, 一枚為兩面皆正, 一枚為兩面皆反, 若隨意擲出一枚, 出現為正面, 則另一面也是正面的機率為何?
   - (c) Discuss two analytical approaches and simulation approach for Monty Hall Problem.