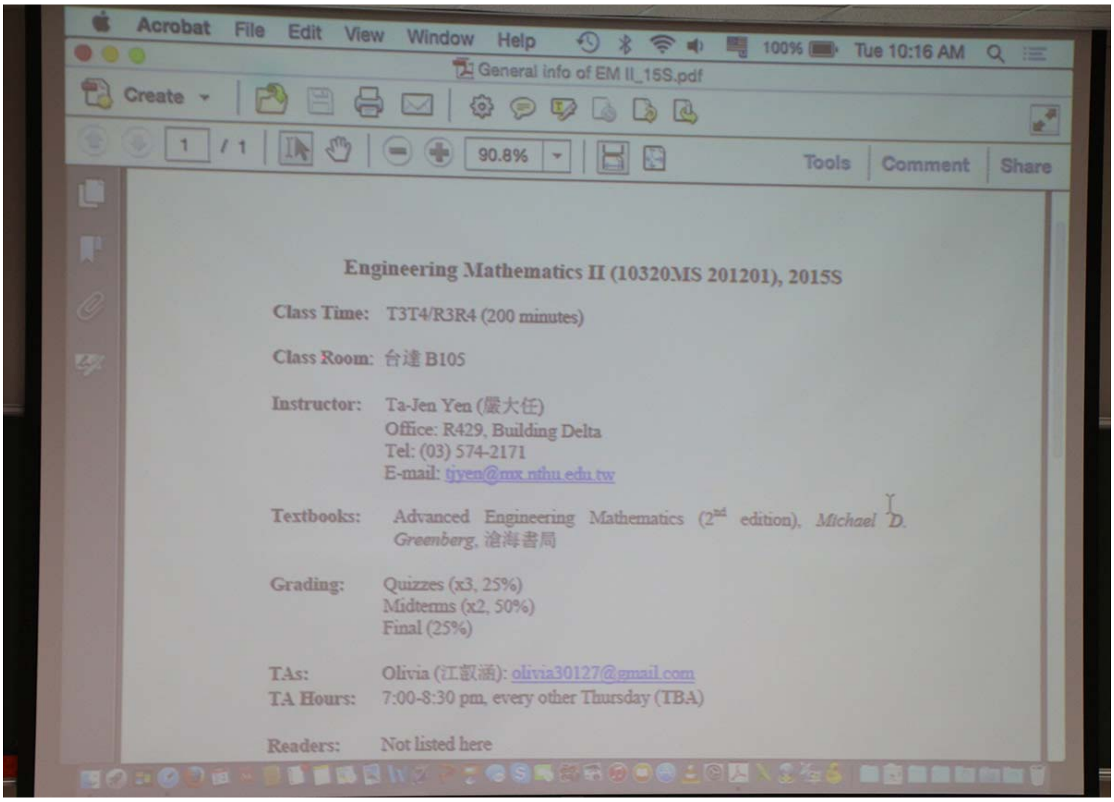
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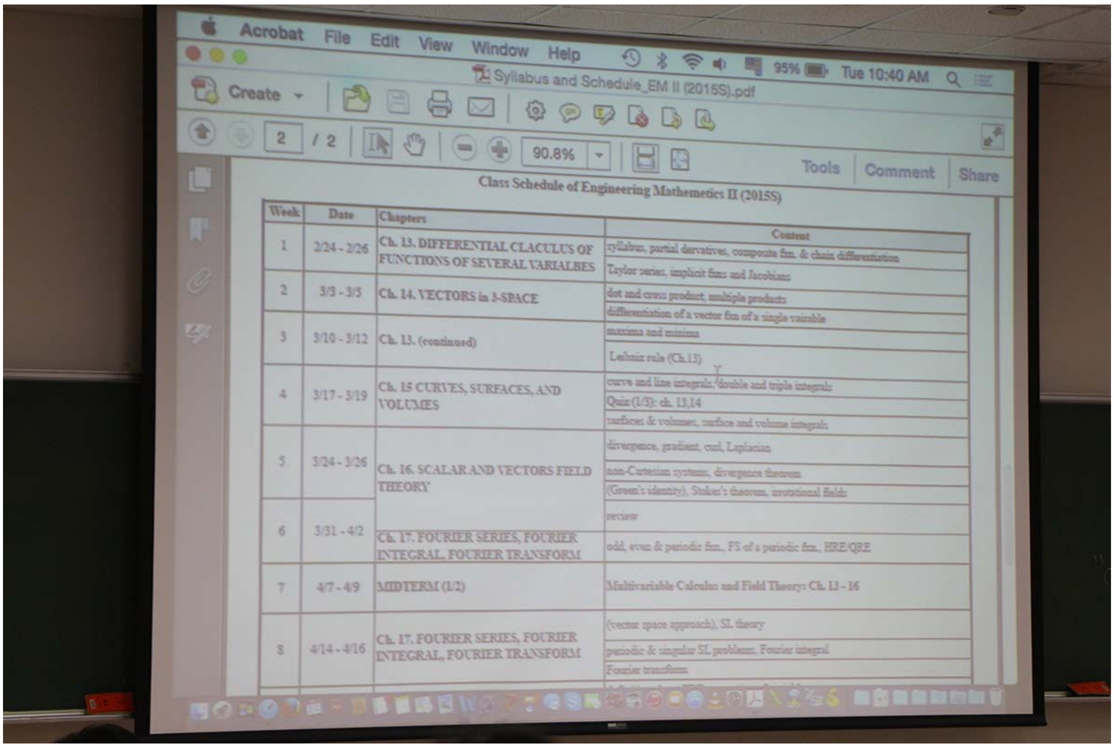
Engineering Mathematics II (10320MS 201201), 2015S
Class Time: T3T4/R3R4 (200 minutes)
Class Room: 台達 B105
Instructor: Ta-Jen Yen (嚴大任)
Office: R429, Building Delta
Tel: (03) 574-2171
E-mail: tjyen@mx.nthu.edu.tw
Textbooks: Advanced Engineering Mathematics (2nd edition), Michael D. Greenberg, 滄海書局
Grading: Quizzes (x3, 25%)
Midterms (x2, 50%)
Final (25%)
TAs: Olivia (江叡涵): olivia30127@gmail.com
TA Hours: 7:00-8:30 pm, every other Thursday (TBA)
Readers: Not listed here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Class Schedule of Engineering Mathemetics II (2015S)
Week Date Chapters Content
1 2/24 - 2/26 Ch. 13. DIFFERENTIAL CLACULUS OF FUNCTIONS OF SEVERAL VARIALBES
syllabus, partial dervatives, composite fcn. & chain differentiation
Taylor series, implicit fcns and Jacobians
2 3/3 - 3/5 Ch. 14. VECTORS in 3-SPACE
dot and cross product, multiple products
differentiation of a vector fcn of a single vairable
3 3/10 - 3/12 Ch. 13. (continued)
maxima and minima
Leibniz rule (Ch.13)
4 3/17 - 3/19 Ch. 15 CURVES, SURFACES, AND VOLUMES
curve and line integrals, double and triple integrals
Quiz (1/3): ch. 13,14
surfaces & volumes, surface and volume integrals
5 3/24 - 3/26 Ch. 16. SCALAR AND VECTORS FIELD THEORY
divergence, gradient, curl, Laplacian
non-Cartesian systems, divergence theorem
(Green's identity), Stokes's theorem, irrotational fields
6 3/31 - 4/2
review
Ch. 17. FOURIER SERIES, FOURIER INTEGRAL, FOURIER TRANSFORM
odd, even & periodic fcn., FS of a periodic fcn., HRE/QRE
7 4/7 - 4/9 MIDTERM (1/2)
Multivariable Calculus and Field Theory: Ch. 13 - 16
8 4/14 - 4/16 Ch. 17. FOURIER SERIES, FOURIER INTEGRAL, FOURIER TRANSFORM
(vector space approach), SL theory
periodic & singular SL problems, Fourier integral
Fourier transform

13.2. Preliminaries. (x9)

13.3. Partial derivatives ; 偏導數.

1. Definition.

$$f(x,y) \to f_x\Big|_{(x_0,y_0)} \; ; \; f_y\Big|_{(x_0,y_0)} \; ?$$

$$\equiv \frac{\partial f}{\partial x} \qquad \equiv \frac{\partial f}{\partial y} \; (x \text{ fixed}$$

general info.

13.2 Preliminaries

13.3 Partial derivatives

13.4 composite fxns. & chain rule.

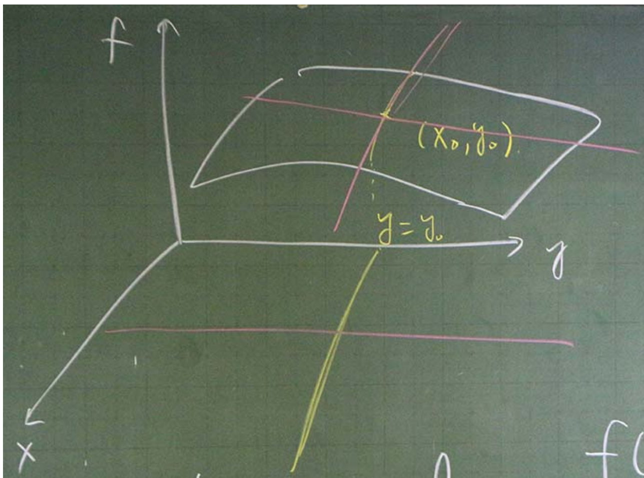
13.2. Preliminaries. (x9)

13.3. Partial derivatives ;

1. Definition.

$$f(x,y) \to f_x\Big|_{(x_0,y_0)}$$

$$\equiv \frac{\partial f}{\partial x}$$



$$f_x\Big|_{(x_0,y_0)} \equiv \lim_{\Delta x\to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}\Big|_{(x_0,y_0)}$$

$$f_{xy} = \partial\left(\frac{\partial f}{\partial x}\right)/\partial y = \frac{\partial^2 f}{\partial y\,\partial x}$$ #

<Ex.1> Let $f(x,y) = x\sin(xy^2)$, find f_x, f_y

f_{xy}, f_{yx}.

Sol

$$f_x = \sin(xy^2) + x\cos(xy^2)\cdot y^2$$

$$= \sin(xy^2) + \underline{xy^2\cos(xy^2)}$$

$$f_y = x\cos(xy^2)\cdot(2xy) = \underline{2x^2y\cos(xy^2)}$$

$$\sin(xy^2)$$

$$f_{xy} = \cos(xy^2)(2xy) + \underline{2xy}\,\underline{\cos(xy^2)}$$

$$- xy^2\sin(xy^2)(2xy) = -2x^2y^3 + 4xy\cos(xy^2)$$

$$f_{yx} = 4xy\cos(xy^2) - 2x^2y\sin(xy^2)\cdot y^2$$

$$= 4xy\cos(xy^2) - 2x^2y^3\sin(xy^2)$$

$$f_{xy} = f_{yx}$$

#

<Ex.2>

$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

find f_{xy} & f_{yx} ? $\rightarrow$ $f_{xy} \neq f_{yx}$

<Theorem> Equality of derivatives.

$f(x,y) \rightarrow$ if f_x, f_y, f_{xy}, f_{yx} are all "continuous" in some neighborhood of (x_0, y_0)

, then $f_{xy} = f_{yx}$

13.4 Composite fxns. & chain rule

1. single-variable fxns

$$f(x) = f(x(t)) \equiv F(t)$$

composite fxn.

"prime"

$$\frac{dF(t)}{dt} = F'(t) = \underbrace{\frac{df(x)}{dx} \cdot \frac{dx}{dt}}_{\text{chain rule!}}$$

$$f(x,y) = f(x(t), y(t)) \equiv F(t), \quad \text{find } F'(t)$$

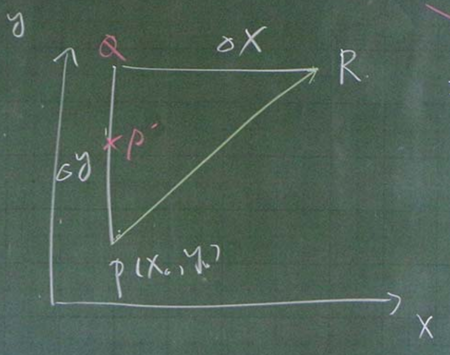
Pf

PS: $\begin{cases} x(t+\Delta t) = x(t) + \Delta x \\ y \sim \end{cases}$

$$\Delta F = F(t+\Delta t) - F(t)$$

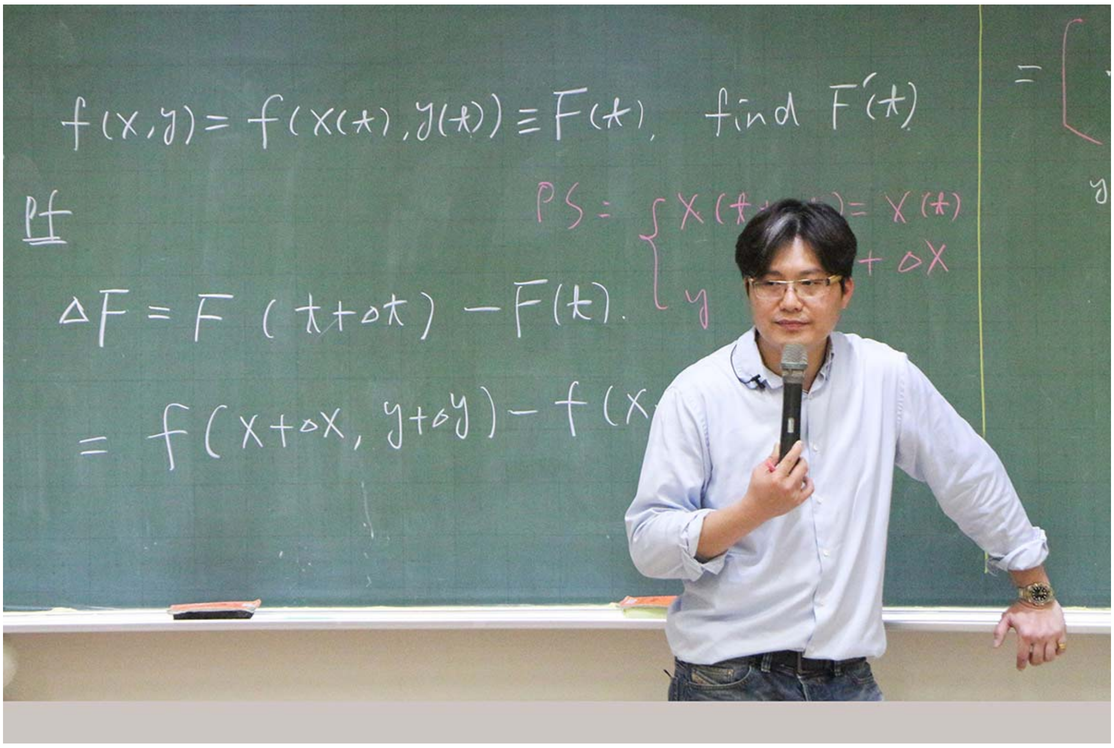
$$= f(x+\Delta x, y+\Delta y) - f(x,y)$$

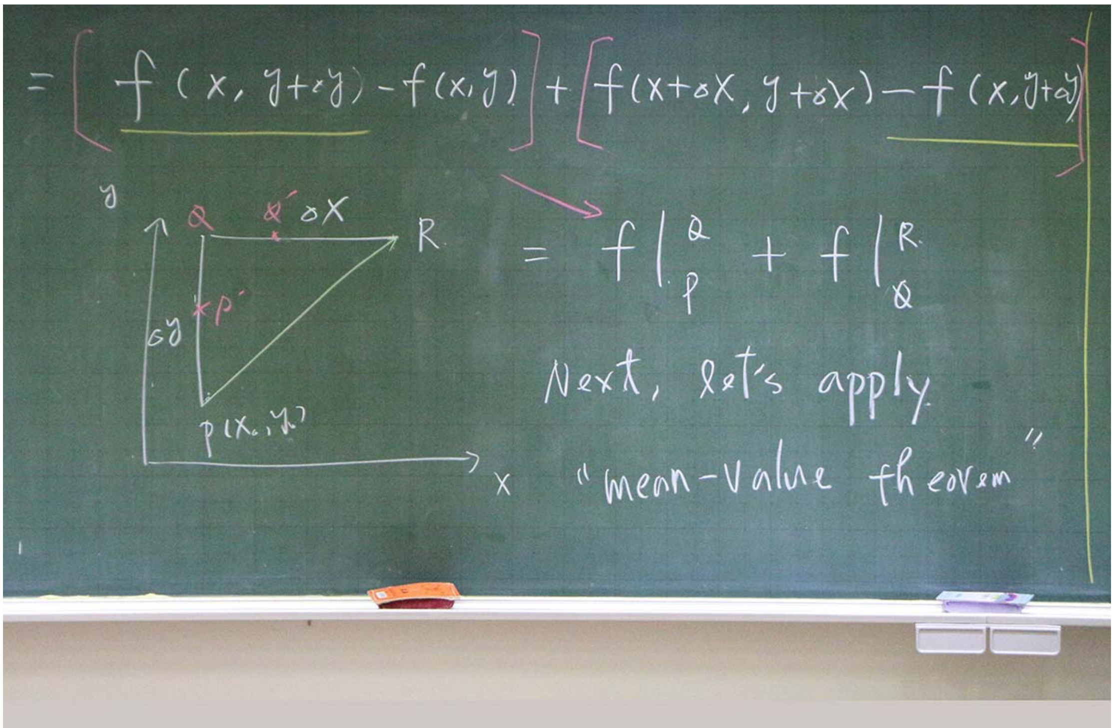
$$= [f(x, y+\Delta y) - f(x,y)] + [f(x+\Delta x, y+\Delta x) - f(x, y+\Delta y)]$$



$$= f\Big|_P^Q + f\Big|_Q^R$$

Next, let's apply "mean-value theorem"


f(x,y) = f(x(t), y(t)) ≡ F(t), find F'(t)
Pf
ΔF = F(t+Δt) − F(t)
= f(x+Δx, y+Δy) − f(x


= [f(x, y+Δy) − f(x,y)] + [f(x+ΔX, y+ΔX) − f(x,y+Δy)]
y
Q
Q'
ΔX
R
Δy
P'
P(x,y)
x
= f|P Q + f|Q R
Next, let's apply
"mean-value theorem"

$$\Delta F = \left.\frac{\partial f}{\partial y}\right|_{\underline{\underline{P'}}} \Delta y + \left.\frac{\partial f}{\partial x}\right|_{\underline{\underline{Q'}}} \Delta x$$

As $\Delta t \to 0$, $\Delta x \to 0$ and $\Delta y \to 0$

Then, $P' \to P$, $Q' \to P$

$$\lim_{\Delta t \to 0} \frac{\Delta F}{\Delta t} = \lim_{\Delta t \to 0} \left(f_y \frac{\Delta y}{\Delta t} + f_x \frac{\Delta x}{\Delta t} \right)$$

$$\Delta F = \left.\frac{\partial f}{\partial y}\right|_{\underline{\underline{P'}}} \Delta y + \left.\frac{\partial f}{\partial x}\right|_{\underline{\underline{Q'}}} \Delta x$$

As $\Delta t \to 0$, $\Delta x \to 0$ and $\Delta y \to 0$

Then, $P' \to P$, $Q' \to P$

$$\lim_{\Delta t \to 0} \frac{\Delta F}{\Delta t} = \lim_{\Delta t \to 0} \left(f_y \frac{\Delta y}{\Delta t} + f_x \frac{\Delta x}{\Delta t} \right)$$

$= f_x x' + f_y y'$ #

<Ex.1>

composite fxn

$r(x,y) = x^2 y - e^{2y} = R(t)$, where

$x(t) = 3t^2$, $y(t) = \sin t$. Find $R'(t)$

Sol. $R'(t) = \frac{\partial r}{\partial x}\frac{dx}{dt} + \frac{\partial r}{\partial y}\frac{dy}{dt}$

$= f_x x' + f_y y'$ #

<Ex.1>

composite fxn

$r(x,y) = x^2 y - e^{2y} = R(t)$, where

$x(t) = 3t^2$, $y(t) = \sin t$. Find $R'(t)$

Sol. $R'(t) = \frac{\partial r}{\partial x}\frac{dx}{dt} + \frac{\partial r}{\partial y}\frac{dy}{dt}$

$x(t)$

$$= 2xy\,(6t) + (x^2 - 2e^{2y})\cos t$$

$y(t)$

$$= \sim$$