APPENDIX A

THE EQUIVALENT DC BRUSH MOTOR MODEL OF THE INTERIOR PMBLDC MOTOR

As mentioned in chapter two, according to the coupling inductor model of the PMBLDC motor, one has the following electrical system model:

$$\begin{bmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_{an} \\ e_{bn} \\ e_{cn} \end{bmatrix} \tag{A.1}$$

where

v_{an}, v_{bn}, v_{cn} : stator phase voltages

i_a, i_b, i_c : stator phase currents

R_s : stator winding resistance

L_{aa}, L_{bb}, L_{cc} : self inductances of phase windings

L_{ab}, L_{ac}, L_{ba}, L_{bc}, L_{ca}, L_{cb} : mutual inductances between phase windings

e_{an}, e_{bn}, e_{cn} : three phase back emfs

If the permanent magnets are embedded in the rotor and with radial magnetization, the self inductance of the phase windings with first order approximation will be

$$L_{aa}(\theta_r) = L_{so} - L_{sm}\cos(2\theta_r) \tag{A.2}$$

$$L_{bb}(\theta_r) = L_{so} - L_{sm}\cos(2\theta_r + \frac{2}{3}\pi) \tag{A.3}$$

$$L_{cc}(\theta_r) = L_{so} - L_{sm}\cos(2\theta_r - \frac{2}{3}\pi) \tag{A.4}$$

and the mutual inductances between phase windings will be

$$L_{ab}(\theta_r) = L_{ba}(\theta_r) = L_{mo} - L_{mm}\cos(2\theta_r - \frac{2}{3}\pi) \tag{A.5}$$

$$L_{bc}(\theta_r) = L_{cb}(\theta_r) = L_{mo} - L_{mm}\cos(2\theta_r) \tag{A.6}$$

$$L_{ca}(\theta_r) = L_{ac}(\theta_r) = L_{mo} - L_{mm}\cos(2\theta_r + \frac{2}{3}\pi) \tag{A.7}$$

where θ_r is the electrical angle between the d-axis of the rotor and the a-phase winding axis. L_{so} and L_{mo} are the constant terms of the self inductance and mutual inductance of the phase windings respectively, and L_{sm} and L_{mm} are the amplitude of the double frequency terms of the self inductance and mutual inductance of the phase windings respectively. Also from equations (A.2)-(A.7) one can see that the self and mutual inductances are varied according to the θ_r angle while the motor is rotating. When $\theta_r = 0°$ ($\theta_r = \frac{2}{3}\pi$ or $\theta_r = \frac{4}{3}\pi$), the value of the self inductance L_{aa} (L_{bb} or L_{cc}) is minimum. On the contrary, when $\theta_r = 90°$ ($\theta_r = 90° + \frac{2}{3}\pi$ or $\theta_r = 90° + \frac{4}{3}\pi$), the value of the self inductance L_{aa} (L_{bb} or L_{cc}) is maximum. When $\theta_r = \frac{4}{3}\pi$ ($\theta_r = 0^\circ$ or $\theta_r = \frac{2}{3}\pi$), the value of the mutual inductance L_{ab} (L_{bc} or L_{ca}) is minimum. On the contrary, when $\theta_r = 90^\circ + \frac{4}{3}\pi$ ($\theta_r = 90^\circ$ or $\theta_r = 90^\circ + \frac{2}{3}\pi$), the value of the mutual inductance L_{ab} (L_{bc} or L_{ca}) is maximum. Substitute (A.2)-(A.7) into (2.21) one can get

$$
\begin{aligned}
v_{dc} &= \left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[v_{an}(t)\ v_{bn}(t)\ v_{cn}(t)\right]^T \\
&= R_s\left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[i_a(t)\ i_b(t)\ i_c(t)\right]^T \\
&\quad + \left[Sa(t)\ Sb(t)\ Sc(t)\right]\frac{d}{dt}\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} \\
&\quad + \left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[e_{an}(t)\ e_{bn}(t)\ e_{cn}(t)\right]^T \qquad \text{(A.8)} \\
&= R_s\left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[i_a(t)\ i_b(t)\ i_c(t)\right]^T \\
&\quad + \left[Sa(t)\ Sb(t)\ Sc(t)\right]\begin{bmatrix} L_{aa}' & L_{ab}' & L_{ac}' \\ L_{ba}' & L_{bb}' & L_{bc}' \\ L_{ca}' & L_{cb}' & L_{cc}' \end{bmatrix}\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} \\
&\quad + \left[Sa(t)\ Sb(t)\ Sc(t)\right]\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}\begin{bmatrix} i_a'(t) \\ i_b'(t) \\ i_c'(t) \end{bmatrix} \\
&\quad + \left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[e_{an}(t)\ e_{bn}(t)\ e_{cn}(t)\right]^T
\end{aligned}
$$

where

$$\left[Sa(t)\ Sb(t)\ Sc(t)\right]\begin{bmatrix} L'_{aa} & L'_{ab} & L'_{ac} \\ L'_{ba} & L'_{bb} & L'_{bc} \\ L'_{ca} & L'_{cb} & L'_{cc} \end{bmatrix}\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$$

$$= \left[Sa(t)\ Sb(t)\ Sc(t)\right]\begin{bmatrix} L'_{aa} & L'_{ab} & L'_{ac} \\ L'_{ba} & L'_{bb} & L'_{bc} \\ L'_{ca} & L'_{cb} & L'_{cc} \end{bmatrix}\begin{bmatrix} Sa(t) \\ Sb(t) \\ Sc(t) \end{bmatrix}\cdot i_{eq}(t)$$

$$\begin{aligned} &= (Sa^2 L'_{aa} + SaSbL'_{ab} + SaScL'_{ac} + SbSaL'_{ba} + Sb^2 L'_{bb} \\ &\quad + SbScL'_{bc} + ScSaL'_{ca} + ScSbL'_{cb} + Sc^2 L'_{cc})\cdot i_{eq}(t) \end{aligned} \tag{A.9}$$

$$= 2\omega_e (L_{sm} + 2L_{mm})\sin(2\theta_r + \frac{\pi}{3})\cdot i_{eq}(t)$$

$$\left[Sa(t)\ Sb(t)\ Sc(t)\right]\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}\begin{bmatrix} i'_a(t) \\ i'_b(t) \\ i'_c(t) \end{bmatrix}$$

$$= \left[Sa(t)\ Sb(t)\ Sc(t)\right]\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}\begin{bmatrix} Sa(t) \\ Sb(t) \\ Sc(t) \end{bmatrix}\cdot \frac{di_{eq}(t)}{dt}$$

$$\begin{aligned} &= (Sa^2 L_{aa} + SaSbL_{ab} + SaScL_{ac} + SbSaL_{ba} + Sb^2 L_{bb} \\ &\quad + SbScL_{bc} + ScSaL_{ca} + ScSbL_{cb} + Sc^2 L_{cc})\cdot \frac{di_{eq}(t)}{dt} \end{aligned} \tag{A.10}$$

$$= [2(L_{so} - L_{mo}) + (L_{sm} + 2L_{mm})\cos(2\theta_r - \frac{2}{3}\pi)]\cdot \frac{di_{eq}(t)}{dt}$$

Substituting (A.9) and (A.10) into (A.8) yields

$$\begin{aligned} v_{dc} &= 2[R_s + \omega_e (L_{sm} + 2L_{mm})\sin(2\theta_r + \frac{\pi}{3})]\cdot i_{eq} \\ &\quad + 2[(L_{so} - L_{mo}) + \frac{1}{2}(L_{sm} + 2L_{mm})\cos(2\theta_r - \frac{2\pi}{3})]\cdot \frac{di_{eq}}{dt} + e_{eq} \\ &= 2[R_s + R_{dq}(\theta_r)]\cdot i_{eq} + 2[(L_{so} - L_{mo} + L_{dq}(\theta_r)]\cdot \frac{di_{eq}}{dt} + e_{eq} \end{aligned} \tag{A.11}$$

where

$$R_{dq}(\theta_r) = \omega_e(L_{sm} + 2L_{mm})\sin(2\theta_r + \frac{\pi}{3}) \tag{A.12}$$

$$L_{dq}(\theta_r) = \frac{1}{2}(L_{sm} + 2L_{mm})\cos(2\theta_r - \frac{2\pi}{3}) \tag{A.13}$$

It follows from (A.11) that a simple equivalent circuit of the equivalent brush dc motor for the IPMBLDC motor can be drawn as shown in Fig. A.1. From the above result, it is seen that the three-phase PMBLDC motor can now be considered as an equivalent dc brush motor with equivalent $2[R_s + R_{dq}(\theta_r)]$ armature resistance and $2[(L_{so} - L_{mo} + L_{dq}(\theta_r)]$ armature inductance. $R_{dq}(\theta_r)$ and $L_{dq}(\theta_r)$ are the ac terms which are originated from the unequal d-axis and q-axis inductances of the interior permanent-magnet (IPM) BLDC motor. When the surface-mount PMBLDC motor is adopted, the value of the ac terms $R_{dq}(\theta_r)$ and $L_{dq}(\theta_r)$ are zero. In this case, the equivalent circuit of the equivalent brush dc motor for the SMPMBLDC motor can be drawn as a simpler model as shown in Fig. A.2. Moreover, when the voltage drops of the ac terms shown in Fig. A.1 is small compared with the system voltage, the equivalent dc brush model of Fig. A.2 for the IPMBLDC motor may as well be adopted with good approximation.

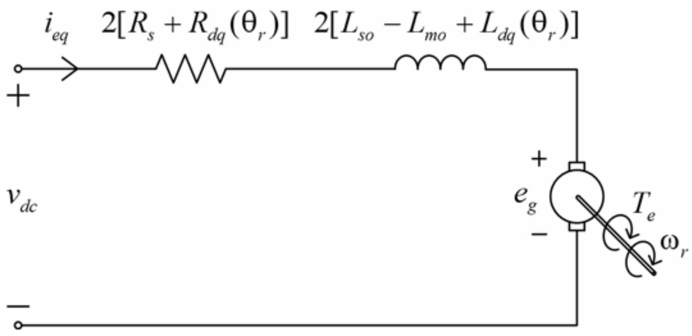

Fig. A.1 The proposed equivalent dc brush motor model for the IPMBLDC motors.

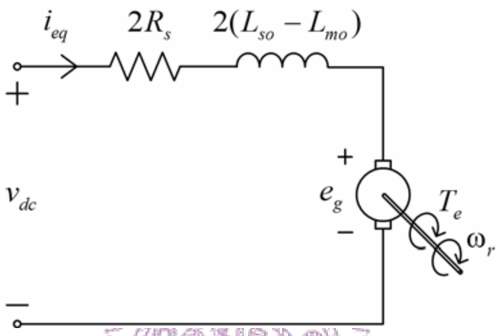

Fig. A.2 The proposed equivalent dc brush motor model for the SMPMBLDC motors.