CHAPTER 2

DERIVATION OF EQUIVALENT DC BRUSH MOTOR MODEL OF THE PMBLDCM FOR SYSTEM INTEGRATION

2.1 Introduction

The traditional dc brush motors have been widely used for quite a long time due to their mature production technique, low cost and easy control features [2]. However, due to the major drawback of the necessity of maintenance of the mechanic commutator and brush assembly, the PMBLDC motor have almost replaced the traditional brush dc motors. The present issue of the PMBLDC motors is mainly focused on the reduction of production cost as well as further improvement in the dynamic performance. Considering the special feature of easy control of dc brush motors, it is the main purpose of this chapter to derive an equivalent dc brush motor model for the three phase PMBLDC motors for more efficient system integration of the drive system in latter chapters to achieve lower cost and better performance.

2.2 Review of the Conventional Mathematic Models

In order to simplify the three-phase control of the PMBLDC motors to a simple scalar control of the traditional brush dc motors, first, consider the mathematical model of a traditional dc brush motor drive as shown in Fig. 2.1 as follows. [20] [45]

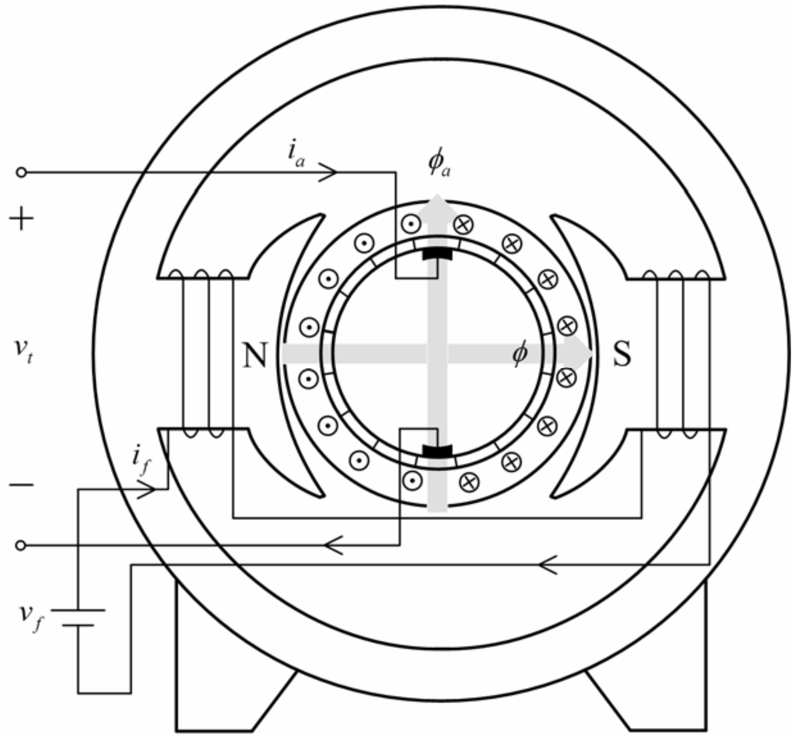

Fig. 2.1 Schematic diagram of an elementary two-poles dc machine.

Electrical system model:

$$v_t = R_a i_a + L_a \frac{di_a}{dt} + e_g \tag{2.1}$$

$$v_f = R_f i_f + L_f \frac{di_f}{dt} \tag{2.2}$$

Mechanical model:

$$T_e = J \frac{d\omega_r}{dt} + B\omega_r + T_l \tag{2.3}$$

where

$$e_g = K_a \phi \omega_r \tag{2.4}$$

$$T_e = K_a \phi i_a \tag{2.5}$$

and the corresponding notations are described as follows:

v_t	: the terminal input voltage
i_a	: the armature current
e_g	: the back emf
R_a	: the armature resistance
L_a	: the armature inductance
v_f	: the terminal voltage of the field circuit
i_f	: the field excitation current
R_f	: the field winding resistance
ϕ	: magnetic flux per pole of the field winding
ω_r	: rotor mechanical angular velocity
ω_e	: electrical angular velocity
K_a	: the back emf constant
T_l	: load torque
J	: inertia of the motor
B	: damping coefficient of the motor

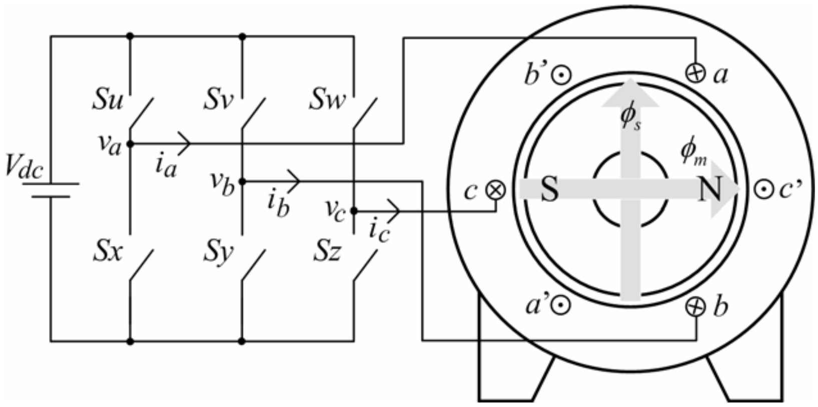

Fig. 2.2 Schematic diagram of a typical PMBLDC motor with a full-bridge inverter.

From (2.5) one can see that the developed electromagnetic torque T_e is proportional to the armature current i_a if the field excitation current, i_f, is fixed. Naturally, the resulting scalar control of the drive system becomes rather easy.

Next, consider a PMBLDC motor which has symmetric three-phase stator windings and trapezoidal air gap flux distribution and with a full bridge inverter as shown in Fig. 2.2. The basic operation principle is quite well known [46-47] and only some typical waveforms of armature emfs, armature phase currents as well as the gating signals of the full bridge inverter of Fig. 2.2 are shown in Fig. 2.3 for reference. From Fig. 2.3, one can see that, ideally, the waveforms of the emfs are with trapezoidal shape. Also, for any time instant, there are only two armature windings which conduct a constant dc current. The gating signals, namely, *Su, Sv, Sw, Sx, Sy, Sz* are controlled such that the PMBLDC motor armature currents can achieve the desired waveform and magnitude. From the coupling

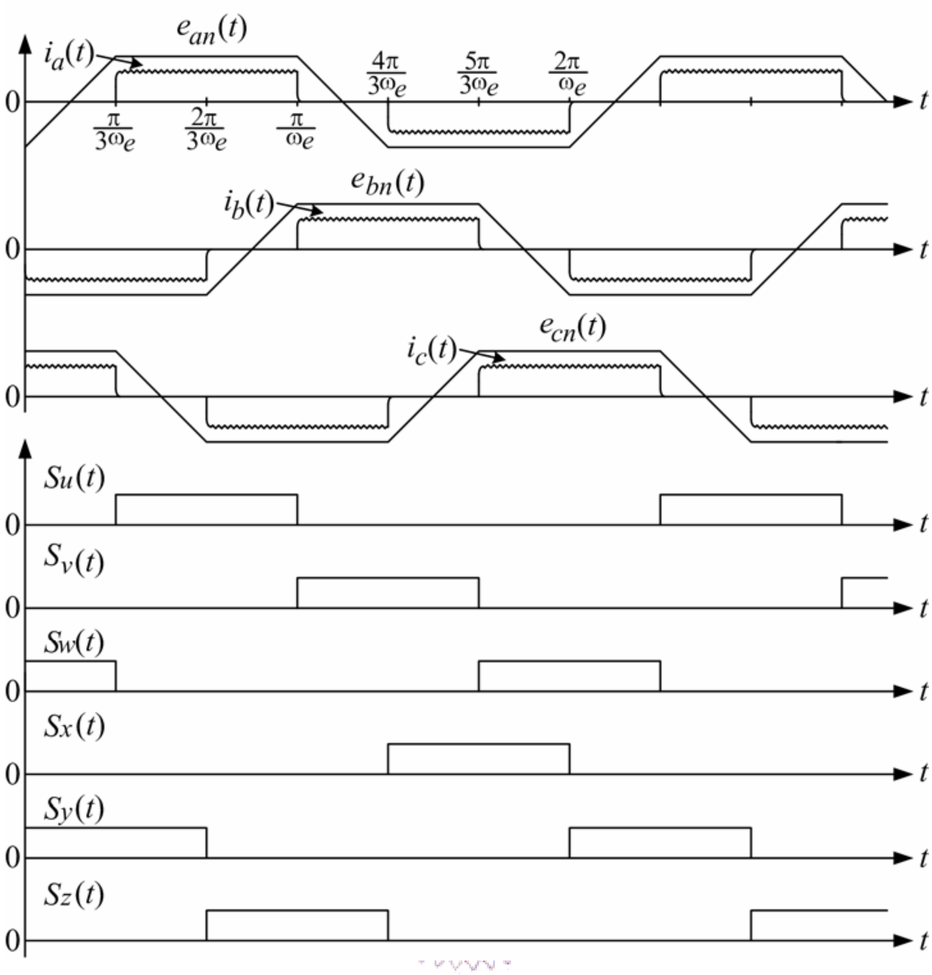

Fig. 2.3 Typical waveforms of the emfs, phase currents and the corresponding gating signals for the PMBLDC motor drives.

inductor model of the PMBLDC motor [46-47], one has the following electrical system model:

$$\begin{bmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_{an} \\ e_{bn} \\ e_{cn} \end{bmatrix} \tag{2.6}$$

where

v_{an}, v_{bn}, v_{cn} : stator phase voltages

i_a, i_b, i_c : stator phase currents

R_s : stator winding resistance

L_{aa}, L_{bb}, L_{cc} : self inductances of phase windings

L_{ab}, L_{ac}, L_{ba}, L_{bc}, L_{ca}, L_{cb} : mutual inductances between phase windings

e_{an}, e_{bn}, e_{cn} : three phase back emfs

Since the permanent magnets are embedded in the rotor and with proper magnetization direction, the induced emfs, namely e_{an}, e_{bn} and e_{cn} in the stator windings of a PMBLDC motor as the rotor is rotating are in the trapezoidal waveform as shown in Fig. 2.3. Thus, the ideal emfs in Fig. 2.3 for one period can be expressed as follows:

$$e_{an}(t)=\begin{cases} \dfrac{6\omega_e^2}{\pi}\lambda_m\left(t-\dfrac{\pi}{6\omega_e}\right) & \text{if} \quad 0\le t\le \dfrac{\pi}{3\omega_e} \\ \omega_e\lambda_m & \text{if} \quad \dfrac{\pi}{3\omega_e}\le t\le \dfrac{\pi}{\omega_e} \\ -\dfrac{6\omega_e^2}{\pi}\lambda_m\left(t-\dfrac{7\pi}{6\omega_e}\right) & \text{if} \quad \dfrac{\pi}{\omega_e}\le t\le \dfrac{4\pi}{3\omega_e} \\ -\omega_e\lambda_m & \text{if} \quad \dfrac{4\pi}{3\omega_e}\le t\le \dfrac{2\pi}{\omega_e} \end{cases} \tag{2.7}$$

$$e_{bn}(t)=e_{an}\left(t-\frac{2\pi}{3\omega_e}\right) \tag{2.8}$$

$$e_{cn}(t)=e_{an}\left(t+\frac{2\pi}{3\omega_e}\right) \tag{2.9}$$

where λ_m is the flux linkage amplitude of the stator windings. Also, the corresponding electromagnetic torque can be obtained from the conservation of power as follow:

$$T_e = (e_n i_a + e_{bn} i_b + e_{cn} i_c) / \omega_r \tag{2.10}$$

As to the mechanical system, one has the same governing equation as (2.3).

2.3 Derivation of the Proposed Equivalent DC Brush Motor Model

From equation (2.4) and (2.5) one can obtain:

$$T_e = \frac{e_g i_a}{\omega_r} \tag{2.11}$$

Also, from Fig. 2.3 one can observe that for each 60° electrical angle, there are only two stator windings which conduct the same dc current with constant amplitude. In other words, within this time interval, both back emf and phase current are constant and exactly like that of a traditional brush dc motor. Based on this observation, one can define the following commutation functions, namely $Sa(t)$, $Sb(t)$ and $Sc(t)$ as shown in Fig. 2.4:

$$\begin{aligned} Sa(t) \equiv \sum_{n=0}^{\infty} [&u(t - \frac{2n\pi}{\omega_e} - \frac{\pi}{3\omega_e}) - u(t - \frac{2n\pi}{\omega_e} - \frac{\pi}{\omega_e}) \\ &- u(t - \frac{2n\pi}{\omega_e} - \frac{4\pi}{3\omega_e}) + u(t - \frac{2n\pi}{\omega_e} - \frac{2\pi}{\omega_e})] \end{aligned} \tag{2.12}$$

$$Sb(t) = Sa(t - \frac{2\pi}{3\omega_e}) \tag{2.13}$$

$$Sc(t) = Sa(t - \frac{4\pi}{3\omega_e}) \tag{2.14}$$

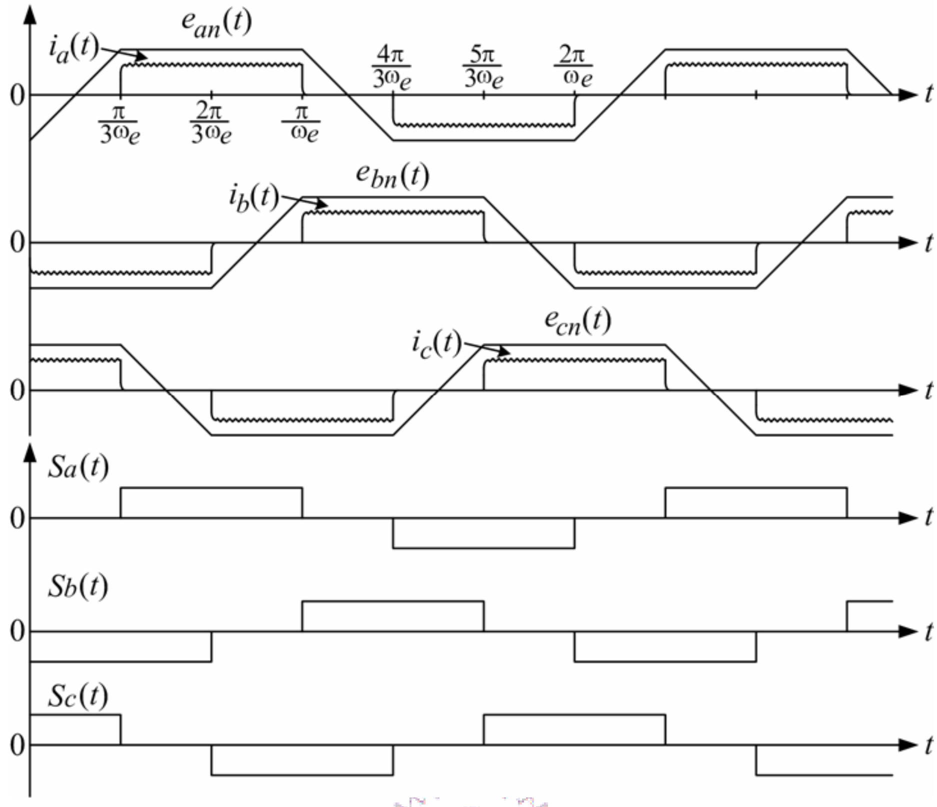

Fig. 2.4 Typical waveforms of the emfs, phase currents and the proposed commutation functions for the PMBLDC motor drives.

where $u(t)$ is the unit step function. It turns out from Fig. 2.4 that the equivalent line to line back emf, namely, $e_{eq}(t)$ for the PMBLDC motor becomes

$$e_{eq}(t) = \left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[e_{an}(t)\ e_{bn}(t)\ e_{cn}(t)\right]^T \tag{2.15}$$

According to the above definition, the magnitude becomes two times that of the peak emf of each phase. Similarly, from Fig. 2.4 one can define the following equivalent line current

$$i_{eq}(t) = \frac{1}{2}\left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[i_a(t)\ i_b(t)\ i_c(t)\right]^T \tag{2.16}$$

It is interesting to see that from (2.15) and (2.16) that

$$e_{eq}(t)i_{eq}(t)=\left[Sa(t)\ Sb(t)\ Sc(t)\right]\begin{bmatrix}e_{an}(t)\\ e_{bn}(t)\\ \mathrm{e}_{cn}(t)\end{bmatrix}\frac{1}{2}\left[Sa(t)\ Sb(t)\ Sc(t)\right]\begin{bmatrix}i_a(t)\\ i_b(t)\\ i_c(t)\end{bmatrix}$$

$$=\frac{1}{2}\left[e_{an}(t)\ e_{bn}(t)\ \mathrm{e}_{cn}(t)\right]\begin{bmatrix}Sa(t)^2 & Sa(t)Sb(t) & Sa(t)Sc(t)\\ Sb(t)Sc(t) & Sb(t)^2 & Sb(t)Sc(t)\\ Sc(t)Sa(t) & Sc(t)Sb(t) & Sc(t)^2\end{bmatrix}\begin{bmatrix}i_a(t)\\ i_b(t)\\ i_c(t)\end{bmatrix} \tag{2.17}$$

$$=\frac{1}{2}\left[e_{an}(t)\ e_{bn}(t)\ \mathrm{e}_{cn}(t)\right]\begin{bmatrix}2i_a(t)\\ 2i_b(t)\\ 2i_c(t)\end{bmatrix}$$

$$=e_{an}(t)i_a(t)+e_{bn}(t)i_b(t)+e_{cn}(t)i_c(t)$$

Therefore, from (2.10) one can obtain the electromagnetic torque in terms of the equivalent dc line current and equivalent dc line emf as follow:

$$T_e=(e_ai_a+e_bi_b+e_ci_c)/\omega_r=e_{eq}i_{eq}/\omega_r \tag{2.18}$$

Also, from definition (2.15) one can see that e_{eq}, in fact, is equal to two times of the peak valve of the emf, namely $2\omega_r\lambda_m$. It follows from (2.18) that

$$T_e=2\lambda_m i_{eq} \tag{2.19}$$

Now consider the electromagnetic torque equation in (2.19) and the same motion equation in (2.3), the equivalent armature current to the rotor angular speed transfer function becomes:

$$\frac{Laplace\ [\omega_r(t)]}{Laplace\ [i_{eq}(t)]} = \frac{2\lambda_m}{Js+B} \triangleq G_P(s) \tag{2.20}$$

which will be used in the later chapter.

In summary, by synthesizing the three phase voltages and currents through the proposed commutation functions, one can obtain an equivalent brush dc motor with the following equivalent back emf e_{eq} and equivalent armature current i_{eq}:

$$e_{eq}(t) = \left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[e_{an}(t)\ e_{bn}(t)\ e_{cn}(t)\right]^T$$

$$i_{eq}(t) = \frac{1}{2}\left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[i_a(t)\ i_b(t)\ i_c(t)\right]^T$$

Similarly, application of the commutation functions to the three phase voltages of the PMBLDC motor, namely (2.6), yields

$$\begin{aligned}
&\left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[v_{an}(t)\ v_{bn}(t)\ v_{cn}(t)\right]^T \\
&\quad = R_s\left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[i_a(t)\ i_b(t)\ i_c(t)\right]^T \\
&\qquad + \left[Sa(t)\ Sb(t)\ Sc(t)\right]\cdot\frac{d}{dt}\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc}\end{bmatrix}\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t)\end{bmatrix} \\
&\qquad + \left[Sa(t)\ Sb(t)\ Sc(t)\right]\left[e_{an}(t)\ e_{bn}(t)\ e_{cn}(t)\right]^T \\
&\quad = 2[R_s + \omega_e(L_{sm}+2L_{mm})\sin(2\theta_r+\frac{\pi}{3})]\cdot i_{eq} \\
&\qquad + 2[(L_{so}-L_{mo})+\frac{1}{2}(L_{sm}+2L_{mm})\cos(2\theta_r-\frac{2\pi}{3})]\cdot\frac{di_{eq}}{dt} + e_{eq} \\
&\quad = 2[R_s + R_{dq}(\theta_r)]\cdot i_{eq} + 2[(L_{so}-L_{mo}+L_{dq}(\theta_r)]\cdot\frac{di_{eq}}{dt} + e_{eq}
\end{aligned} \tag{2.21}$$

where

$$R_{dq}(\theta_r) = \omega_e(L_{sm} + 2L_{mm})\sin(2\theta_r + \frac{\pi}{3}) \tag{2.22}$$

$$L_{dq}(\theta_r) = \frac{1}{2}(L_{sm} + 2L_{mm})\cos(2\theta_r - \frac{2\pi}{3}) \tag{2.23}$$

The more detail derivation process about (2.21) is given in the Appendix A. It follows from (2.21) that a simple equivalent circuit of the equivalent brush dc motor for the PMBLDC motor can be drawn as shown in Fig. 2.5. From the above result, it is seen that the three-phase PMBLDC motor can now be considered as an equivalent dc brush motor with equivalent $2[R_s + R_{dq}(\theta_r)]$ armature resistance and $2[(L_{so} - L_{mo} + L_{dq}(\theta_r)]$ armature inductance. $R_{dq}(\theta_r)$ and $L_{dq}(\theta_r)$ are the ac terms which are originated from the unequal d-axis and q-axis inductances of the interior permanent-magnet (IPM) BLDC motor. Compared with the system voltage, the voltage drop of $R_{dq}(\theta_r)$ and $L_{dq}(\theta_r)$ are small, such that the ac terms can approximately be ignored. In addition, if the magnets of the

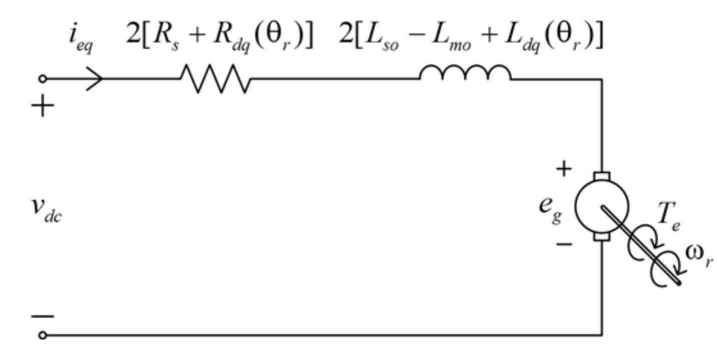

Fig. 2.5 The proposed equivalent dc brush motor model for the PMBLDC motors.

PMBLDC motor are mounted on the surface of the rotor core, the disturbance terms $R_{dq}(\theta_r)$ and $L_{dq}(\theta_r)$ will thus be vanished.

Since the electromagnetic torque of the PMBLDC motor derived in (2.19) is only proportional to the equivalent armature current of the motor, hence, instead of using the complicated control of three phase currents $i_a(t)$, $i_b(t)$ and $i_c(t)$, only one control variable, i.e., the equivalent armature current $i_{eq}(t)$ need to be controlled to drive the PMBLDC motor. This will simplify greatly the PMBLDC motor control. Finally, it is worth mentioning that the proposed commutation functions basically serve as an equivalent electronic commutator. In fact, the proposed commutation functions can be obtained indirectly from the three Hall sensor signals as can be seen in the next chapter. On the other hand, given the input dc voltage and $i_{eq}(t)$ in case necessary, one can get

$$\left[i_a(t)\ i_b(t)\ i_c(t)\right] = \left[Sa(t)\ Sb(t)\ Sc(t)\right] \cdot i_{eq}(t)$$

$$\left[v_{an}(t)\ v_{bn}(t)\ v_{cn}(t)\right] = \frac{1}{2}\left[Sa(t)\ Sb(t)\ Sc(t)\right] \cdot v_{dc}$$