CHAPTER II

REVIEW OF SWITCHING FLOW-GRAPH MODELING TECHNIQUE FOR DC-DC SWITCHING CONVERTERS

2.1 Introduction

The Switching Flow-Graph (SFG) technique [32]-[35] is a graphic nonlinear modeling tool for DC-DC switching converter. The SFG technique utilizes the state-space averaging theory and based on the extension of the signal flow-graph concept to develop the unified large-signal, steady-state and small-signal models. Unlike the conventional modeling technique requires laborious mathematics, when providing SFG method to model a switching system, the complex mathematic derivations are not essential.

The SFG technique introduces the concept of switching branches and switching functions into the signal-flow graph theory to model a switching system in a flow-graph form. When the switch is ON and OFF periodically, the switching system is interchanged between two different circuit topologies. According to the two circuit topologies, two flow-graphs can be obtained. The switching branches carried with their switching functions are used to merge the two flow-graphs to generate a SFG. The switching branches are the only nonlinear part of the SFG. Therefore, the modeling work is reduced to the switching branches. The large-signal, small-signal and steady-state SFG models can immediately be obtained by replacing the switching branches with their large-signal, small-signal and steady-state switching branch models respectively.

From the SFG model, one can easily observe the cause-effect relationship between the circuit variables. Moreover, based on the small-signal SFG model, the transfer functions of the switching system can easily be obtained by using Mason's

Rule; it is helpful for the controller design.

2.2 The Linear Signal Flow-Graph Technique

Before presenting the SFG modeling technique for DC-DC switching converters, it is appropriate to give a rough review about the existing signal flow-graph technique [39]. The signal flow-graph is an easy and straightforward modeling tool used for describing linear dynamic systems in a schematic form. The first step of creating the signal flow-graph model is to choose the meaningful and interesting variables of circuits and set these variables to be nodes. The second step is to connect these nodes with directional branches. The branches carry the transmittance just as the usual gain, impedance or admittance that represents the relationship between the adjacent nodes. As input signal passes through the branch, the signal is multiplied by the transmittance on the branch and sent to the connected nodes. The node variable is the sum of all signals entering into the same node. The flow-graph model of a linear circuit can, thus, be established by following this procedure.

2.3 Switching Flow-Graph Modeling Technique for DC Converters

Based on the concept of the signal flow-graph technique and make use of the switching branches and switching functions, the SFG technique can be applied to model the DC-DC switching circuits. The modeling procedures for a PWM DC-DC converter can be summarized as follows [32]:

(1) Find the sub-circuits corresponding to all the switching states.

Consider a DC-DC switching converter operated in continuous current mode (CCM) as an example; there are two switching states related to switch ON and OFF. In each switching state, one can find their corresponding sub-circuits, namely ON-circuit and OFF-circuit.

(2) Draw the flow-graphs for all the sub-circuits with the same node distribution.

By using the theory of signal flow-graph, one can create two flow-graphs, G_{ON} and G_{OFF}, for ON-circuit and OFF-circuit. The two flow-graphs share the same nodes and part of the branches.

(3) The flow-graphs are then combined to obtain the switching flow-graph.

One can find that some branches exist both in G_{ON} and G_{OFF}, but other branches exist in only one of them. The two flow-graphs can be merged to one SFG, G, by utilizing the switching branches and their switching functions. The switching functions for switch S are defined as following:

$$F_S(t) = \begin{cases} 1 & , \quad \text{when} \quad S \quad \text{is} \quad ON \\ 0 & , \quad \text{when} \quad S \quad \text{is} \quad OFF \end{cases} \tag{2.1a}$$

$$\overline{F_S}(t) = \begin{cases} 1 & , \quad \text{when} \quad S \quad \text{is} \quad OFF \\ 0 & , \quad \text{when} \quad S \quad \text{is} \quad ON \end{cases} \tag{2.1b}$$

Branches exist in G_{ON} but not in G_{OFF} are replaced by the switching branches carried with switching function $F_S(t)$. Similarly, branches exist in G_{OFF} but not in G_{ON} are replaced by the switching branches carried with switching function $\overline{F}_S(t)$.

(4) The switching branches are replaced by their steady state switching branch model (or small-signal switching branch model, large-signal switching branch model) to obtain the corresponding steady state model (or small-signal model, large-signal model).

It can be seen that G is linear except the part of switching branch. Therefore, the modeling work can be reduced to switching branches. Assume the filter corner frequency is much smaller than the switching frequency and the input signal and output signal of the switching branch are represented as $x(t)$ and $y(t)$ respectively.

A. The large-signal model

Derived from the state-space averaging theory, the relations between $x(t)$ and $y(t)$ for switching branches are described in the Equations (2.2) and (2.3); where T_S means the switching period, T_{ON}/T_{OFF} means the ON/OFF time duration of switch, and $d(t)$ and $d'(t)$ are the duty ratios of switch.

$$y(t)=\frac{1}{T_S}\int_{T_{ON}} x(t)dt \approx x(t)d(t) \tag{2.2a}$$

$$y(t)=\frac{1}{T_S}\int_{T_{OFF}} x(t)dt \approx x(t)d'(t) \tag{2.2b}$$

$$d(t)+d'(t)=1 \tag{2.3}$$

From equations (2.2), the large-signal models of the switching branches can be obtained by multiplying the input signal of the switching branches together with their duty ratios. Therefore, replacing the switching branches with the large-signal models of switching branches can develop the large signal model of the switching converter.

B. The steady-state model approach

In steady state, the input signal $x(t)$, output signal $y(t)$ and the duty ratios $d(t)$ and $d'(t)$ are approached to be constant, X, Y, D and D' respectively. According to equation (2.2), the steady-state relations for the switching branches are described in the equations (2.4). Similarly, the steady-state model of the switching branches can be obtained by multiplying the input signal of the switching branches with their duty ratios, as described in equations (2.4). Assuming $s \to 0$ and substituting the steady-state model into the SFG for the switching branches, the steady-state model of the switching converter is obtained.

$$Y = XD \qquad \text{for } F_S \text{ branch} \tag{2.4a}$$

$$Y = XD' \qquad \text{for } \overline{F}_S \text{ branch} \tag{2.4b}$$

C. Small-signal model

Introducing the small perturbations $\hat{x}$, $\hat{y}$, and $\hat{d}$ near the working point,

X , Y , and D . One can obtain the following equations:

$$x(t) = X + \hat{x}(t) \tag{2.5}$$
$$y(t) = Y + \hat{y}(t) \tag{2.6}$$
$$d(t) = D + \hat{d}(t) \tag{2.7}$$
$$d'(t) = D - \hat{d}(t) \tag{2.8}$$
$$d(t) + d'(t) = 1 \tag{2.9}$$

Substitute equations (2.5) - (2.9) into equation (2.2), the relations of the small-signal perturbation can be described as equation (2.10).

for F_S branches :

$$\begin{aligned} Y + \hat{y}(t) &= (X + \hat{x}(t))(D + \hat{d}(t)) \\ &= XD + X\hat{d}(t) + D\hat{x}(t) + \hat{x}(t)\hat{d}(t) \end{aligned} \tag{2.10a}$$

for $\overline{F_S}$ branches :

$$\begin{aligned} Y + \hat{y}(t) &= (X + \hat{x}(t))(D' - \hat{d}(t)) \\ &= XD - X\hat{d}(t) + D\hat{x}(t) - \hat{x}(t)\hat{d}(t) \end{aligned} \tag{2.10b}$$

By neglecting the second order perturbations, the equations (2.10) can be modified as:

$$\hat{y}(t) = X\hat{d}(t) + D\hat{x}(t) \qquad \text{for } F_S(t) \text{ branches} \tag{2.11a}$$
$$\hat{y}(t) = -X\hat{d}(t) + D\hat{x}(t) \qquad \text{for } \overline{F_S}(t) \text{ branches} \tag{2.11b}$$

The same, substituting the small-signal model into the SFG for the switching branches can easily generate the small-signal model of the switching converter.

(5) The algebraic rules of the flow-graph can be used to simplify the resulting models.

By means of the flow-graph algebraic rule, the small-signal dynamic model can be further simplified. Based on the simplified small-signal model, the analytic form of the transfer functions for the switching converters can easily be derived by using Mason's Rule.

2.4 Examples

Consider a boost converter operated in CCM as an example [40-41], as shown in Fig. 2.1(a), where v_s is the input voltage, L is the inductor, C is the capacitor, R is the resistor, R_L is the parasitic resistance of the inductor, and S and D are the controllable switch and diode respectively. Define v_L, v_O, i_L and i_O as the inductor voltage, output voltage, inductor current and output current respectively. As the boost converter operated in CCM, there are only two switching states corresponding to switch ON and OFF. When the switch is ON, the diode is reversed biased and the voltage source supplies power to the inductor. When the switch is OFF, the output stage receives the power from the inductor and input source. The two sub-circuits, ON-circuit and OFF-circuit, are shown in Figs. 2.1(b) and (c).

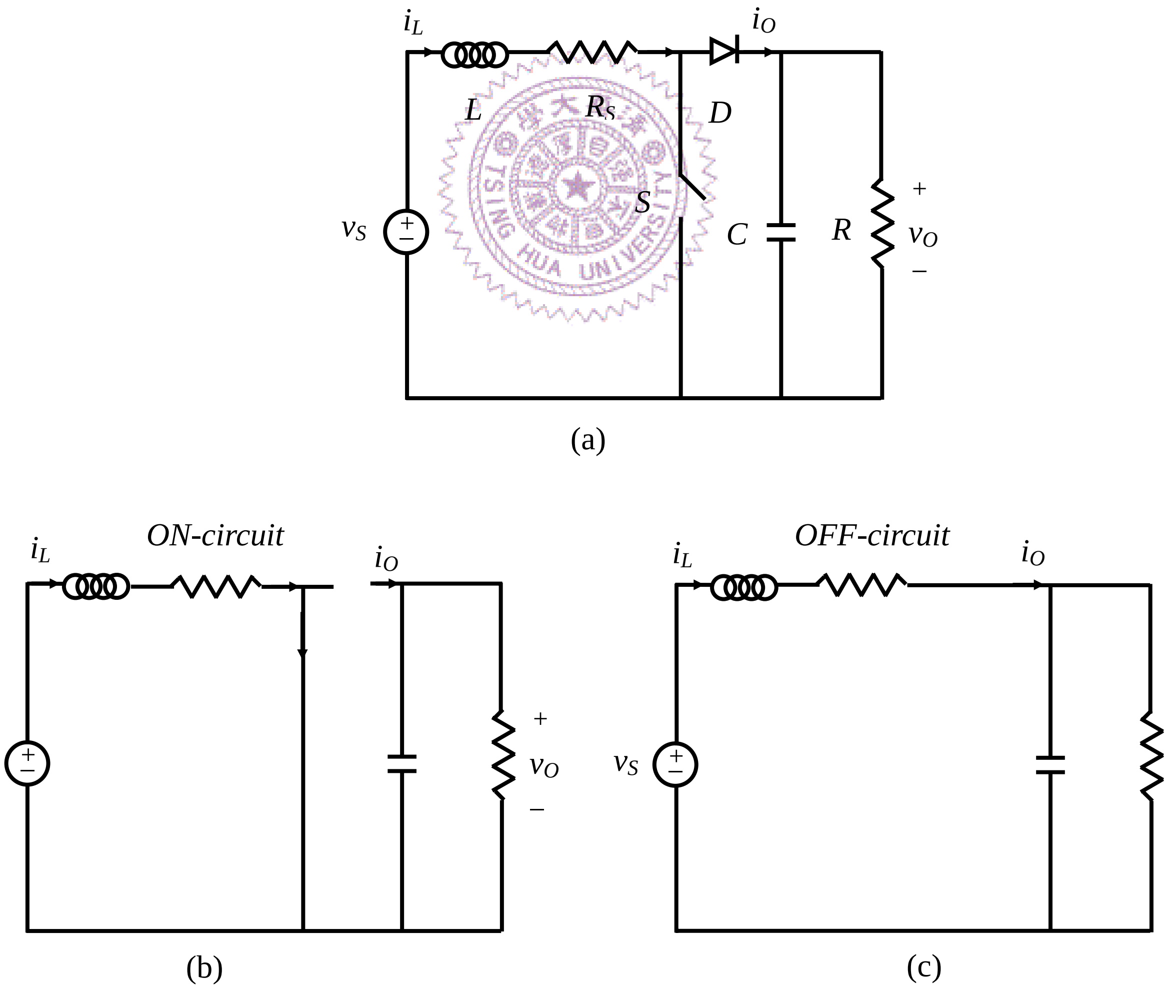

Fig. 2.1 Boost converter: (a) the circuit diagram (b) switch ON, (c) switch OFF.

According to the procedure of constructing the signal flow-graph, the two sub-circuits can be described by their flow-graphs, G_{ON} and G_{OFF}, as shown in Figs. 2.2 (b) and 2.2(c). The switching function of S is defined the same as equations (2.1a) and (2.1b). Then, by adding the corresponding switching branches carried with the switching functions, the two flow-graphs can be merged to one switching flow-graph, G, as shown in Fig. 2.2(c).

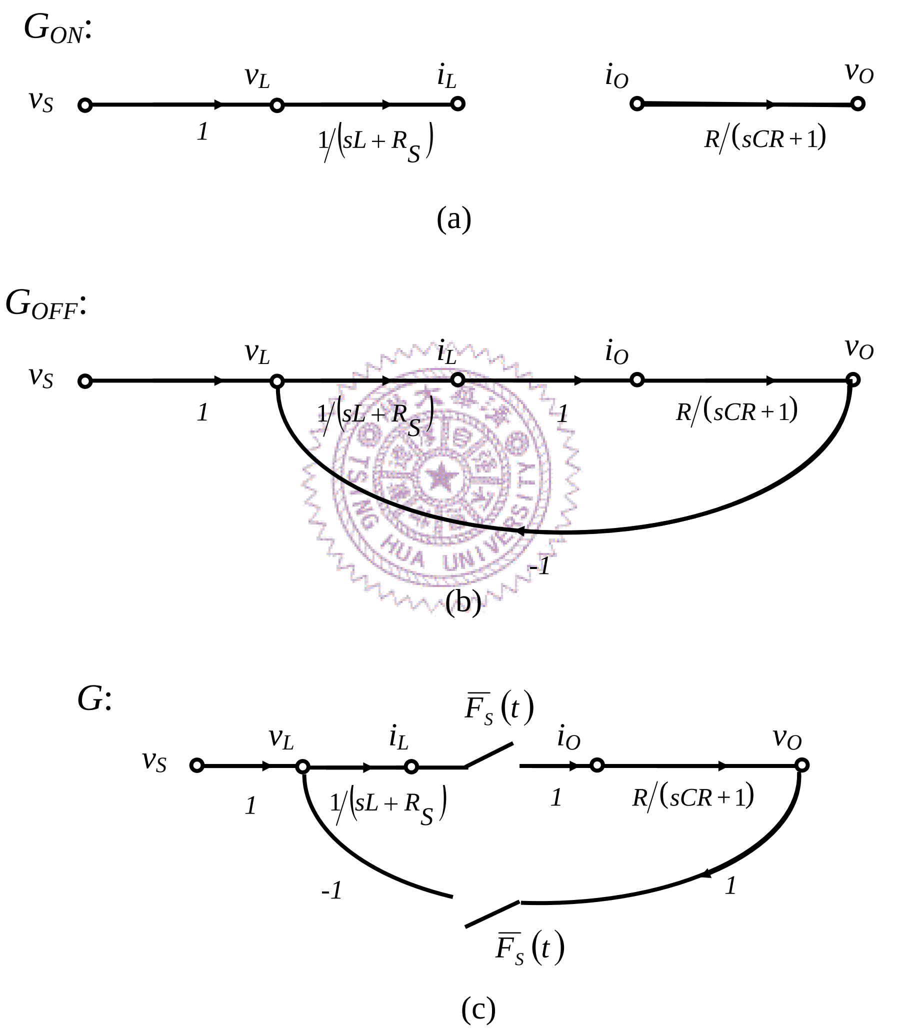

Fig. 2.2 (a) The flow-graph of ON-circuit, (b) the flow-graph of OFF-circuit, (c) the switching flow-graph of the boost converters.

As illustrated in previous section, the large-signal model, steady-state model and small-signal model can be represented as Figs. 2.3(a), 2.3(b) and 2.3(c) respectively. In Fig. 2.3(b), V_S, V_L, V_O, I_L, I_O and D represent the steady-state signals of v_S, v_L, v_O, i_L, i_O and d. The small perturbations of v_S, v_L, v_O, i_L, i_O and d are symbolized as $\hat{v}_S$, $\hat{v}_L$, $\hat{v}_O$, $\hat{i}_L$, $\hat{i}_O$ and $\hat{d}$ in Fig. 2.3(c).

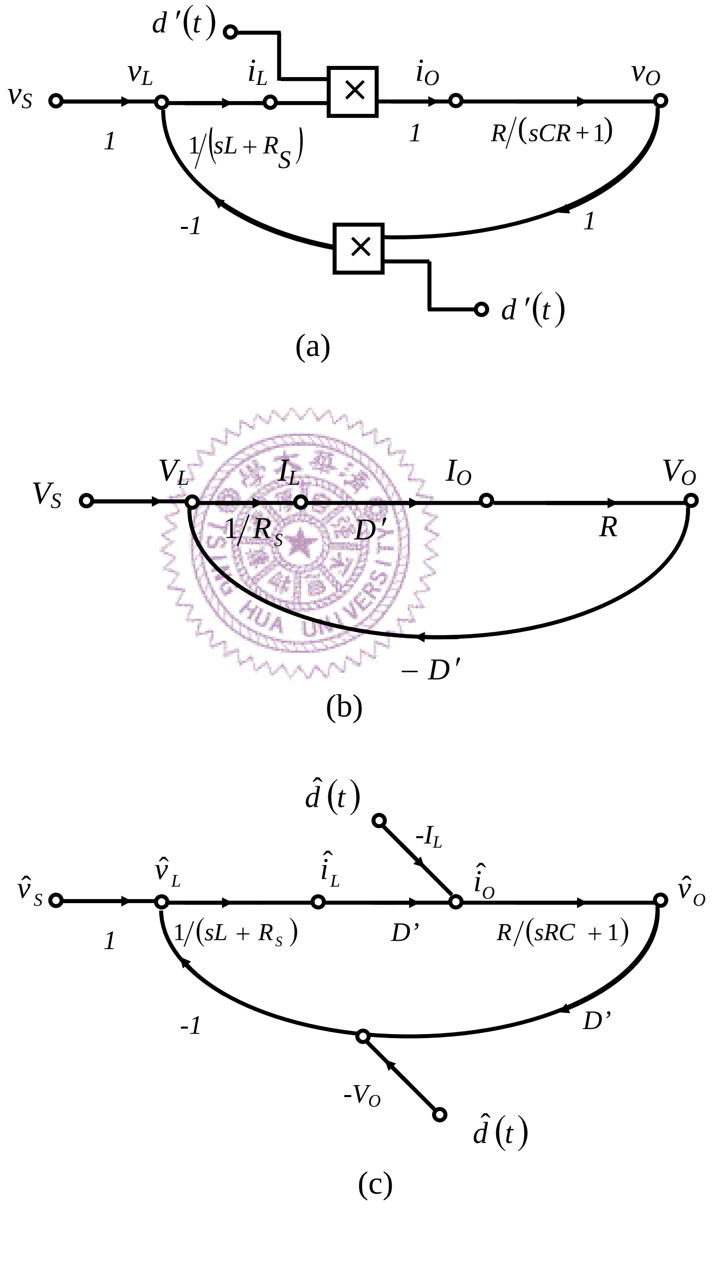

Fig. 2.3 (a) The large-signal model, (b) the steady-state model, (c) the small-signal model of the boost converter operated in CCM.

From Fig. 2.3(c), one can easily derive the input and output impedance by using Mason's Rule. The input impedance z_{in} and output impedance z_{out} are shown as follows:

$$z_{in} = \frac{\hat{v}_S}{\hat{i}_L} = \frac{CRLs^2 + (CRR_S + L)s + (R_S + RD'^2)}{sCR + 1} \tag{2.12}$$

$$z_{out} = \frac{\hat{v}_O}{\hat{i}_O} = \frac{sLR + R_S R}{CRLs^2 + (CRR_S + L)s + (R_S + RD'^2)} \tag{2.13}$$

The transfer functions from arbitrary state variable to another arbitrary state variable can be obtained by using the same method, such as input to output gain and control to output gain, etc.

This example illustrates that it is easier, faster by using the SFG technique to model a DC-DC converter. However, it is seen that for a three-phase switching converter with six controllable switches and six parallel diodes, the step of finding sub-circuits for the multiple switching states would involve a lot of efforts [36]. The merit of easy implementation of the existing switching flow-graph for DC-DC converters will be lost. Therefore, in the following sections the concept of virtual switch and virtual switching function will be proposed to overcome this dilemma.