CHAPTER 4B

CIRCUIT THEOREMS

CONTENTS

4.6 Superposition Theorem

x → $f(g)$ → *y*

input → → *output*

The relationship f (x) between cause x and effect y is linear if f (•) is both additive and homogeneous.

definition of additive property：

If $f(x_1)=y_1$, $f(x_2)=y_2$ then $f(x_1+x_2)=y_1+y_2$

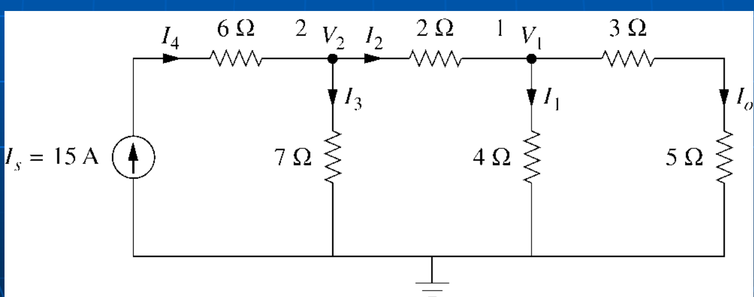
definition of homogeneous property：

If $f(x)=y$ and α is a real number then $f(\alpha x)= \alpha y$

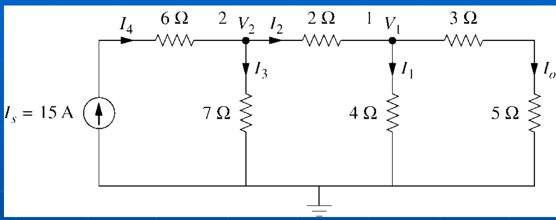
4.6 Superposition Theorem

- Example 4.6.1

  Assume I_0 = 1 A and use linearity to find the actual value of I_0 in the circuit in figure.



4.6 Superposition Theorem



If $I_0 = 1\text{A}$, then $V_1 = (3+5)I_0 = 8\text{V}$

$I_1 = \dfrac{V_1}{4} = 2\text{A}$, $I_2 = I_1 + I_0 = 3\text{A}$

$V_2 = V_1 + 2I_2 = 8 + 6 = 14\text{V}$, $I_3 = \dfrac{V_2}{7} = 2\text{A}$

$I_4 = I_3 + I_2 = 5\text{A} \Rightarrow I_S = 5\text{A}$

$I_0 = 1A \rightarrow I_S = 5\text{A}$, $I_0 = 3\text{A} \rightarrow I_S = 15\text{A}$

4.6 Superposition Theorem

For a linear circuit N consisting of n inputs , namely u_1 , u_2 , …… , u_n , then the output y can be calculated as the sum of its components：

$$y = y_1 + y_2 + \dots\dots + y_n$$

where

$$y_i = f(u_i) \ , \ i = 1,2,\dots\dots,n$$

4.6 Superposition Theorem

Proof： Consider the nodal equation of the corresponding circuit for the basic case as an example

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nn} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} I_{1s} \\ I_{2s} \\ \vdots \\ I_{ns} \end{bmatrix} \cdots\cdots\cdots (A)$$

$$[G]\, e = I_s \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (B)$$

Let $G_k = [\, G_{k1}\ G_{k2} \ldots G_{kn}\,]^T$

Then $[G] = [\, G_1\ G_2 \ldots G_n\,]$

4.6 Superposition Theorem

- **Cramer’s Rule for solving Ax=b**

Take n=3 as an example.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Let

$\det A = \triangle \neq 0$

4.6 Superposition Theorem

Then

$$x_1 = \frac{\det \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\Delta}$$

$$x_2 = \frac{\det \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\Delta}$$

$$x_3 = \frac{\det \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\Delta}$$

4.6 Superposition Theorem

Suppose that the kth nodal voltage e_k is to be found.

Then from Cramer's rule one has

$$e_k = \frac{\det\left[G_1\ G_1 \cdots I_s \cdots G_n\right]}{\det\left[G\right]}$$

$$= \sum_{j=1}^{n} \frac{\Delta_{jk}}{\Delta} I_{js}$$

$$where\ \Delta \triangleq \det\left[G\right]$$

$$\therefore\ e_k = e_{k1} + e_{k2} + \cdots \cdots + e_{kn}$$

4.6 Superposition Theorem

where

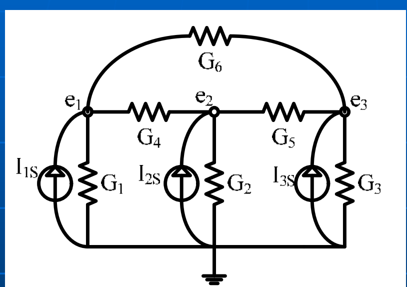
$$e_{k1} = \frac{\Delta_{1k}}{\Delta} I_{1s} \ , \ \text{due to } I_{1s} \text{ only}$$

$$\vdots \qquad \vdots$$

$$e_{kn} = \frac{\Delta_{nk}}{\Delta} I_{ns} \ , \ \text{due to } I_{ns} \text{ only}$$

4.6 Superposition Theorem

- Example 4.6.2



Find $e_2 = ?$

Nodal Equation

$$\begin{bmatrix} G_1+G_4+G_6 & -G_4 & -G_6 \\ -G_4 & G_2+G_4+G_5 & -G_5 \\ -G_6 & -G_5 & G_3+G_5+G_6 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} I_{1S} \\ I_{2S} \\ I_{3S} \end{bmatrix}$$

4.6 Superposition Theorem

By using Cramer's rule

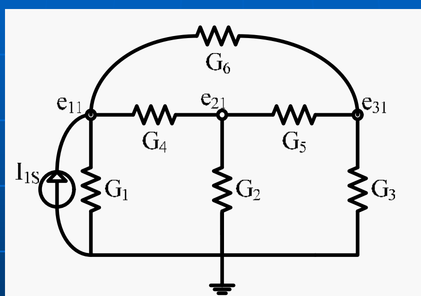
$$e_2 = \frac{\det\begin{pmatrix} G_1+G_4+G_6 & I_{1S} & -G_6 \\ -G_4 & I_{2S} & -G_5 \\ -G_6 & I_{3S} & G_3+G_5+G_6 \end{pmatrix}}{\Delta}$$

$$= \frac{\Delta_{12}}{\Delta} I_{1S} + \frac{\Delta_{22}}{\Delta} I_{2S} + \frac{\Delta_{32}}{\Delta} I_{3S}$$

$$= e_{21} + e_{22} + e_{23}$$

4.6 Superposition Theorem

Where e_{21} is due to I_{1S} only，$I_{2S} = I_{3S} = 0$



$$\begin{bmatrix} G_1+G_4+G_6 & -G_4 & -G_6 \\ -G_4 & G_2+G_4+G_5 & -G_5 \\ -G_6 & -G_5 & G_3+G_5+G_6 \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{21} \\ e_{31} \end{bmatrix} = \begin{bmatrix} I_{1S} \\ 0 \\ 0 \end{bmatrix}$$

4.6 Superposition Theorem

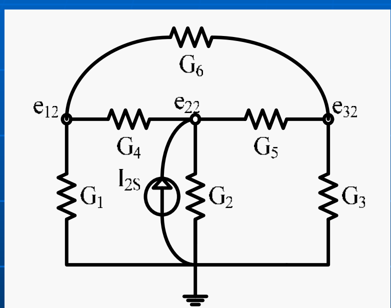
$G_1+G_4+G_6$	$-G_4$	$-G_6$	e_{11}		I_{1S}
$-G_4$	$G_2+G_4+G_5$	$-G_5$	e_{21}	=	0
$-G_6$	$-G_5$	$G_3+G_5+G_6$	e_{31}		0

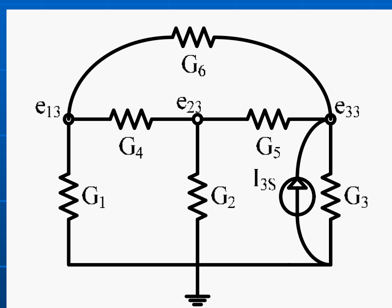
$$\therefore e_{21} = \frac{\det\begin{pmatrix} G_1+G_4+G_6 & I_{1S} & -G_6 \\ -G_4 & 0 & -G_5 \\ -G_6 & 0 & G_3+G_5+G_6 \end{pmatrix}}{\Delta}$$

$$= \frac{\Delta_{12}}{\Delta} I_{1S} \quad , \quad \textit{due to } I_{1S} \textit{ only}$$

4.6 Superposition Theorem

Similarly





Duo to I_{2S} *only*

$I_{1S} = I_{3S} = 0$

Duo to I_{3S} *only*

$I_{1S} = I_{2S} = 0$

4.7 Thevenin's Theorem

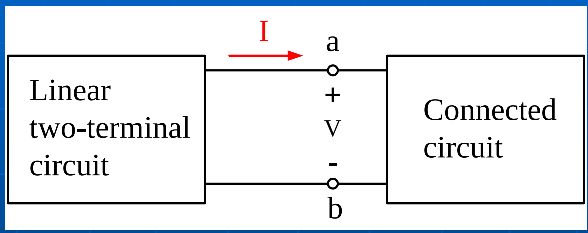
In high school, one finds the equivalent resistance of a two terminal resistive circuit without sources.

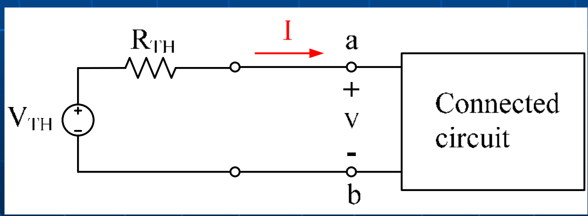
Now, we will find the equivalent circuit for two terminal resistive circuit with sources.

4.7 Thevenin's Theorem

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{TH} in series with a resistor R_{TH} where V_{TH} is the open circuit voltage at the terminals and R_{TH} is the input or equivalent resistance at the terminals when the independent sources are turned off .

4.7 Thevenin's Theorem

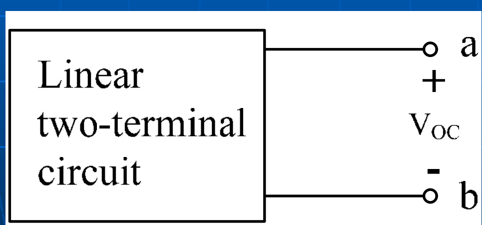




4.7 Thevenin's Theorem

Equivalent circuit: same voltage-current relation at the terminals.

$V_{TH} = V_{OC}$: Open circuit voltage at a-b

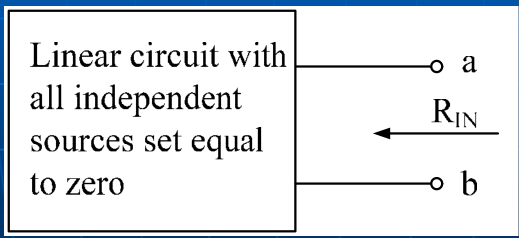


$$V_{TH} = V_{OC}$$

4.7 Thevenin's Theorem

$R_{TH} = R_{IN}$: input resistance of the dead circuit

Turn off all independent sources



$R_{TH} = R_{IN}$

4.7 Thevenin's Theorem

CASE 1

If the network has no dependent sources:

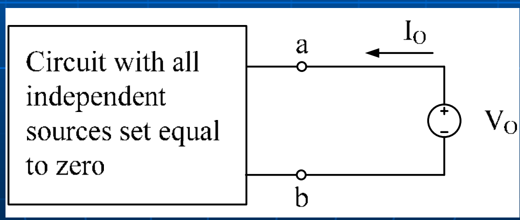
- Turn off all independent source.
- R_{TH} : input resistance of the network looking into a-b terminals

4.7 Thevenin's Theorem

CASE 2

If the network has dependent sources
-Turn off all independent sources.
-Apply a voltage source V_O at a-b

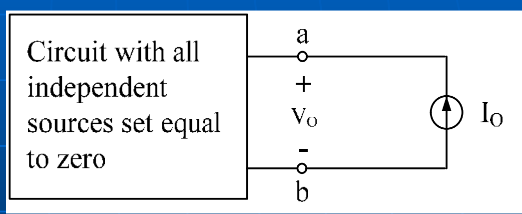
$$R_{TH} = \frac{V_O}{I_O}$$



4.7 Thevenin's Theorem

-Alternatively, apply a current source I_O at a-b

$$R_{TH} = \frac{V_O}{I_O}$$



If $R_{TH} < 0$, the circuit is supplying power.

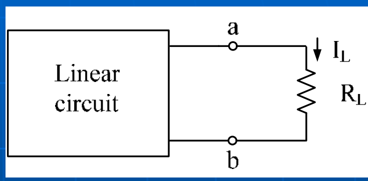
4.7 Thevenin's Theorem

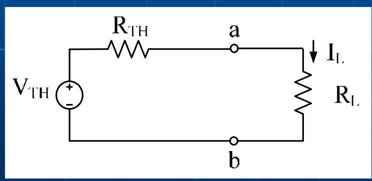
Simplified circuit

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{TH} + R_L} V_{TH}$$

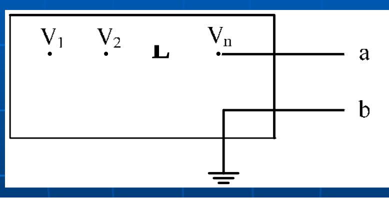
Voltage divider





4.7 Thevenin's Theorem

Proof : Consider the following linear two terminal circuit consisting of n+1 nodes and choose terminal b as datum node and terminal a as node n .



$$\begin{pmatrix} G_{11} & \cdots & G_{1n} \\ \vdots & \ddots & \vdots \\ G_{n1} & \cdots & G_{nn} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{pmatrix} = \begin{pmatrix} I_{1s} \\ I_{2s} \\ \vdots \\ I_{ns} \end{pmatrix}$$

4.7 Thevenin's Theorem

Then nodal voltage V_n when a-b terminals are open can be found by using Cramer's rule .

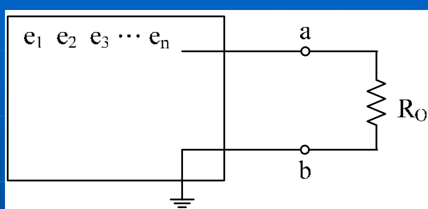
$$V_n = \frac{1}{\Delta}\sum_{k=1}^{n} \Delta_{kn} I_{ks} \quad \cdots\cdots\cdots \quad (A)$$

Δ is the determinant of [G] matrix

Δ_{ku} is the corresponding cofactor of G_{kn}

Now connect an external resistance R_o to a-b terminals . The new nodal voltages will be changed to e_1 , e_2 , … , e_n respectively .

4.7 Thevenin's Theorem



Nodal equation

$$\begin{pmatrix} G_{11} & \cdots & G_{1n}+0 \\ \vdots & & G_{2n}+0 \\ \vdots & & \vdots \\ G_{n1} & \cdots & G_{nn}+\frac{1}{R_o} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{pmatrix} = \begin{pmatrix} I_{1s} \\ I_{2s} \\ \vdots \\ I_{ns} \end{pmatrix} \quad \cdots\cdots\cdots \quad (B)$$

4.7 Thevenin's Theorem

Note that

$$\det\begin{pmatrix} G_{11} & \cdots & G_{1n}+0 \\ \vdots & & G_{2n}+0 \\ \vdots & & \vdots \\ G_{n1} & \cdots & G_{nn}+\dfrac{1}{R_o} \end{pmatrix} = \det[G] + \det\begin{pmatrix} G_{11} & \cdots & 0 \\ G_{21} & & 0 \\ \vdots & & \vdots \\ G_{n1} & \cdots & \dfrac{1}{R_o} \end{pmatrix}$$

$$= \Delta + \frac{1}{R_o}\Delta_{nn}$$

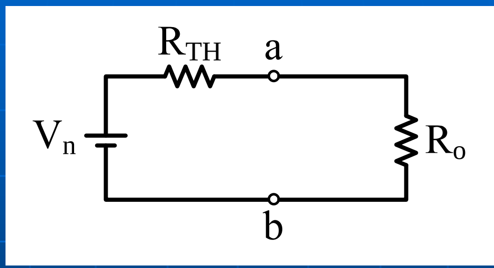
4.7 Thevenin's Theorem

Hence , e_n can be obtained as follows .

$$e_n = \frac{\det\begin{pmatrix} G_{11} & \cdots & I_{1s} \\ \vdots & \ddots & \vdots \\ G_{n1} & \cdots & I_{ns} \end{pmatrix}}{\Delta + \dfrac{1}{R_o}\Delta_{nn}} = \frac{\sum_{k=1}^{n}\Delta_{kn}I_{ks}}{\Delta + \dfrac{1}{R_o}\Delta_{nn}} = \frac{\dfrac{1}{\Delta}\sum_{k=1}^{n}\Delta_{kn}I_{ks}}{1+\dfrac{1}{R_o}\dfrac{\Delta_{nn}}{\Delta}} = \frac{R_o}{R_o + R_{TH}}V_n$$

where $R_{TH} \triangleq \dfrac{\Delta_{nn}}{\Delta}$

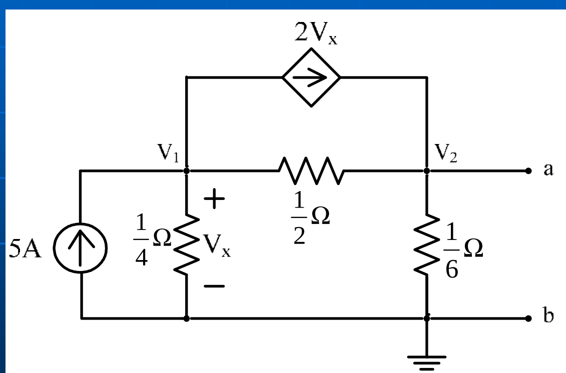
4.7 Thevenin's Theorem



In other words , the linear circuit looking into terminals a-b can be replaced by an equivalent circuit consisting of a voltage source V_{TH} in series with an equivalent resistance R_{TH} , where V_{TH} is the open circuit voltage V_n and $R_{TH} = \frac{\Delta_{nn}}{\Delta}$.

4.7 Thevenin's Theorem

Example 4.7.1



4.7 Thevenin's Theorem

Example 4.7.1 (cont.)

Find open circuit voltage V_2

$$\begin{pmatrix} 2+4 & -2 \\ -2 & 2+6 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 5-2V_x \\ 2V_x \end{pmatrix}$$

$$2V_x = 2V_1$$

$$\begin{pmatrix} 2+4+2 & -2 \\ -2-2 & 2+6 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$$\Delta = \det\begin{pmatrix} 8 & -2 \\ -4 & 8 \end{pmatrix} = 64 - 8 = 56$$

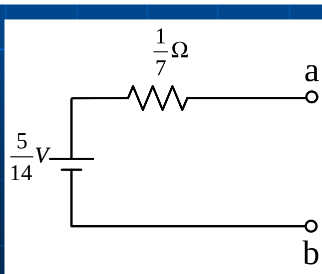
4.7 Thevenin's Theorem

Example 4.7.1 (cont.)

$$\therefore V_2 = \frac{\det\begin{pmatrix} 8 & 5 \\ -4 & 0 \end{pmatrix}}{56} = \frac{20}{56} = \frac{5}{14} V = V_{TH}$$

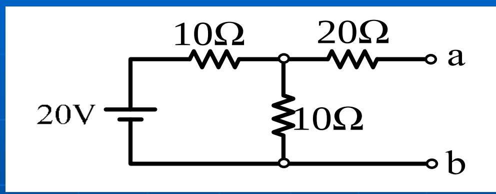
$$R_{TH} = \frac{\Delta_{22}}{\Delta} = \frac{8}{56} = \frac{1}{7}\Omega$$

∴ Ans.



4.7 Thevenin's Theorem

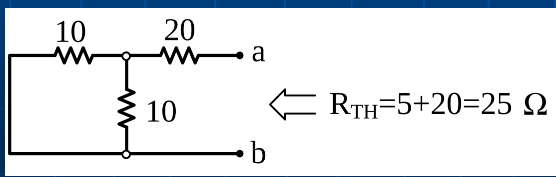
Example 4.7.2



By voltage divider principle :

open circuit voltage V_{TH}=10V

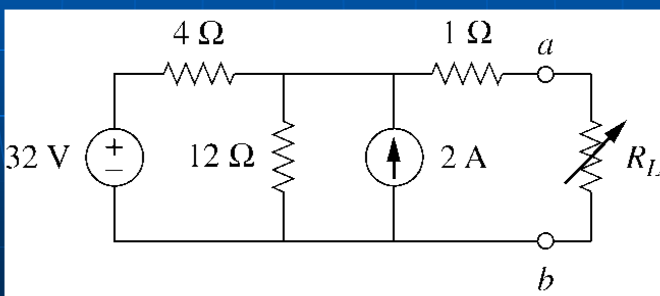
Let independent source be zero



4.7 Thevenin's Theorem

Example 4.7.3

- Find the Thevenin's equivalent circuit of the circuit shown below, to the left of the terminals a-b. Then find the current through R_L = 6, 16, and 36 Ω.



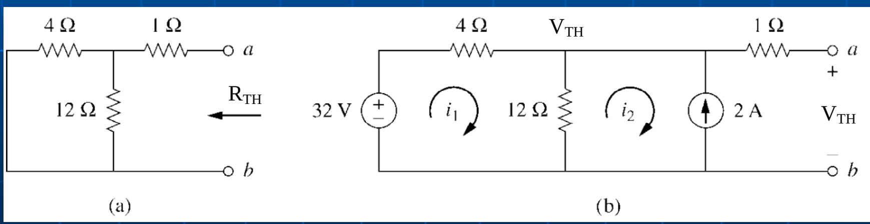
4.7 Thevenin's Theorem

Example 4.7.3 (cont.)

R_{TH} : 32V voltage source $\rightarrow$ short

2A current source $\rightarrow$ open

$$R_{TH} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\Omega$$



4.7 Thevenin's Theorem

Example 4.7.3 (cont.)

V_{TH} :

Mesh analysis

$$-32 + 4i_1 + 12(i_1 - i_2) = 0 , i_2 = -2A$$

$$\therefore i_1 = 0.5A$$

$$V_{TH} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30V$$

4.7 Thevenin's Theorem

Example 4.7.3 (cont.)

To get i_L :

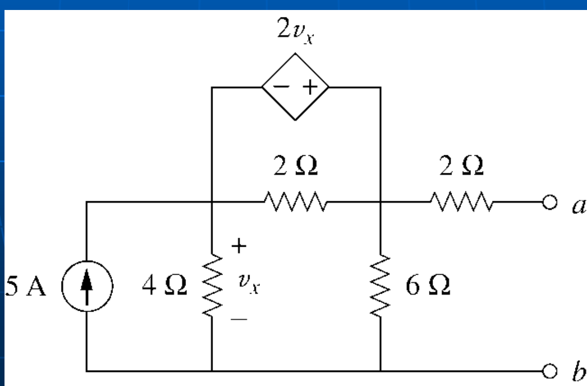
$$i_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{30}{4 + R_L}$$

$$R_L = 6 \rightarrow I_L = 30/10 = 3A$$
$$R_L = 16 \rightarrow I_L = 30/20 = 1.5A$$
$$R_L = 36 \rightarrow I_L = 30/40 = 0.75A$$

4.7 Thevenin's Theorem

Example 4.7.4

Find the Thevenin's equivalent of the following circuit with terminals a-b.



4.7 Thevenin's Theorem

Example 4.7.4 (cont.)

(independent + dependent source case)

To find R_{TH} from Fig.(a)

independent source $\rightarrow$ 0

dependent source $\rightarrow$ unchanged

Apply $v_o = 1V \ , \ R_{TH} = \dfrac{v_o}{i_o} = \dfrac{1}{i_o}$

4.7 Thevenin's Theorem

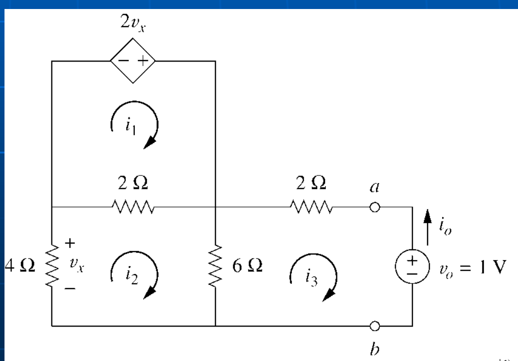
Example 4.7.4 (cont.)

For loop 1 , $-2v_x + 2(i_1 - i_2) = 0 \ or \ v_x = i_1 - i_2$

But

$-4i_2 = v_x = i_1 - i_2$

$\therefore i_1 = -3i_2$



(a)

4.7 Thevenin's Theorem

Example 4.7.4 (cont.)

Loop 2 and 3:

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solving these equations gives

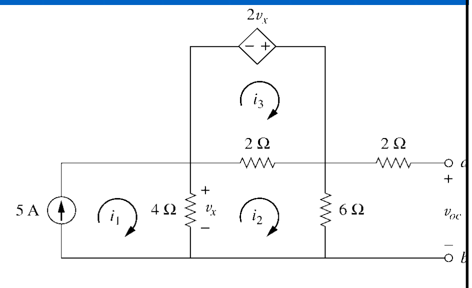
$$i_3 = -\frac{1}{6}A$$

$$\text{But } i_o = -i_3 = \frac{1}{6}A$$

$$\therefore R_{TH} = \frac{1V}{i_o} = 6\Omega$$

4.7 Thevenin's Theorem

Example 4.7.4 (cont.)



(b)

To find V_{TH} from Fig.(b)

Mesh analysis

$$i_1 = 5$$

$$-2v_x + 2(i_3 - i_2) = 0 \Rightarrow v_x = i_3 - i_2$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0 \Rightarrow 12i_2 - 4i_1 - 2i_3 = 0$$

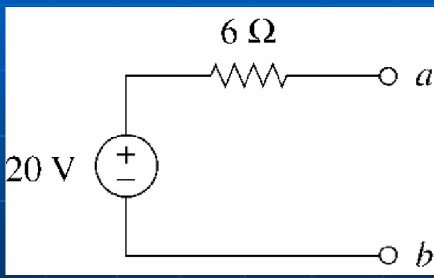
4.7 Thevenin's Theorem

Example 4.7.4 (cont.)

But $4(i_1 - i_2) = v_x$

$\therefore i_2 = 10/3.$

$V_{TH} = v_{oc} = 6i_2 = 20V$



4.7 Thevenin's Theorem

Example 4.7.5

Determine the Thevenin's equivalent circuit :

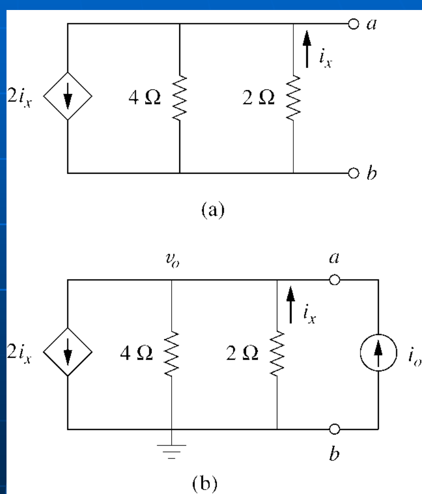
<u>Solution</u>:

(dependent source only)

$$V_{TH} = 0 \ , \ R_{TH} = \frac{v_o}{i_o}$$

Nodal analysis

$$i_o + i_x = 2i_x + \frac{v_o}{4}$$



4.7 Thevenin's Theorem

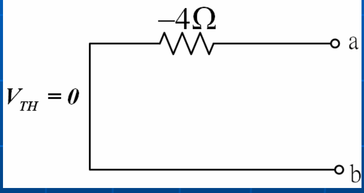
Example 4.7.5 (cont.)

But

$$i_x = \frac{0 - v_o}{2} = -\frac{v_o}{2}$$

$$i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = -\frac{v_o}{4}$$

$$or \quad v_o = -4i_o$$



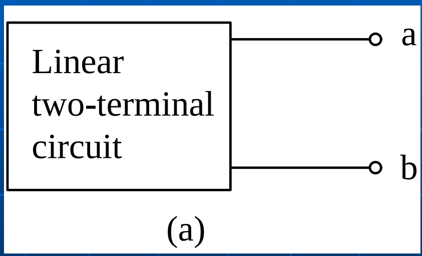
Thus

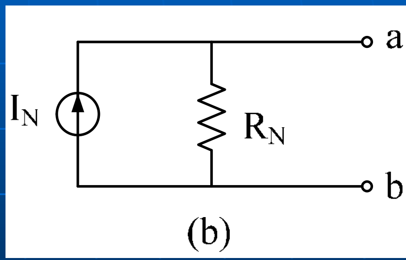
$$R_{TH} = \frac{v_o}{i_o} = -4\Omega$$: Supplying Power !

4.8 Norton's Theorem

- **Norton's theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

4.8 Norton's Theorem



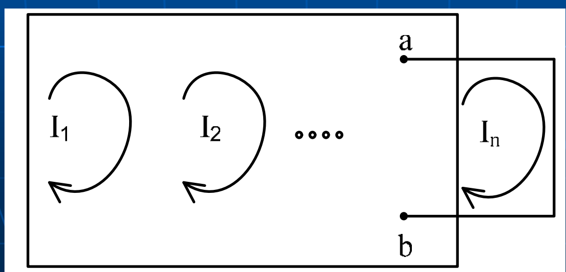


4.8 Norton's Theorem

Proof：

By using Mesh Analysis as an example

Assume the linear two terminal circuit is a planar circuit and there are n meshes when a b terminals are short circuited.



4.8 Norton's Theorem

Mesh equation for case 1 as an example

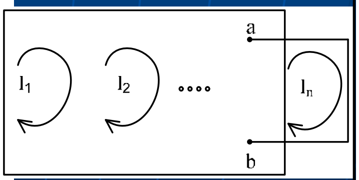
$$\begin{pmatrix} R_{11} & \cdots\cdots & R_{1n} \\ \vdots & & \vdots \\ \vdots & \ddots & \vdots \\ R_{n1} & \cdots\cdots & R_{nn} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{pmatrix} = \begin{pmatrix} V_{1S} \\ V_{2S} \\ \vdots \\ V_{ns} \end{pmatrix}$$

Hence the short circuit cuurent

$$I_n = \frac{1}{\Delta}\sum_{k=1}^{n} \Delta_{kn}\ V_{ks}$$

where $\Delta = \det\left[R_{ik}\right]$

Δ_{kn} *is the cofactor of* R_{kn}



4.8 Norton's Theorem

Now connect an external resistance R_o to a , b terminals , then all the mesh currents will be changed to J_1, J_2, · · · · J_n , respectively.

$$\begin{pmatrix} R_{11} & \cdots\cdots & R_{1n}+0 \\ \vdots & & R_{2n}+0 \\ \vdots & \ddots & \vdots \\ R_{n1} & \cdots\cdots & R_{nn}+R_o \end{pmatrix} \begin{pmatrix} J_1 \\ J_2 \\ \vdots \\ J_n \end{pmatrix} = \begin{pmatrix} V_{1S} \\ V_{2S} \\ \vdots \\ V_{ns} \end{pmatrix}$$

Note that

$$\det\begin{pmatrix} R_{11} & \cdots\cdots & R_{1n}+0 \\ \vdots & & R_{2n}+0 \\ \vdots & \ddots & \vdots \\ R_{n1} & \cdots\cdots & R_{nn}+R_o \end{pmatrix} = \Delta + \det\begin{pmatrix} R_{11} & \cdots & 0 \\ \vdots & & \vdots \\ \vdots & \ddots & \vdots \\ R_{n1} & \cdots & R_o \end{pmatrix}$$

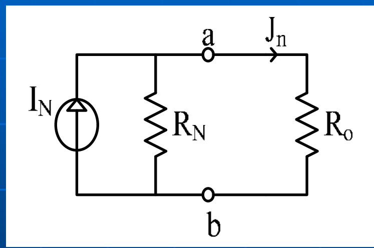
$$= \Delta + R_o\Delta_{nn}$$

4.8 Norton's Theorem

Hence, one has

$$J_n = \frac{\det\begin{pmatrix} R_{11} & \dots & V_{1s} \\ \vdots & \ddots & \vdots \\ R_{n1} & \cdots & V_{ns} \end{pmatrix}}{\Delta + R_o\Delta_{nn}} = \frac{\sum_{k=1}^{n}\Delta_{kn}V_{ks}}{\Delta + R_o\Delta_{nn}}$$

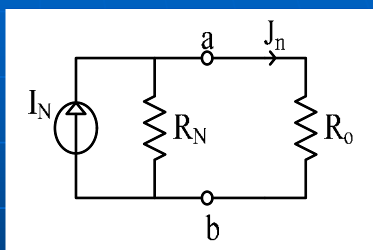
$$= \frac{\frac{1}{\Delta}\sum_{k=1}^{n}\Delta_{kn}V_{ks}}{1 + R_o\frac{\Delta_{nn}}{\Delta}}$$



4.8 Norton's Theorem

$$= \frac{I_n}{1 + R_o\frac{\Delta_{nn}}{\Delta}}$$

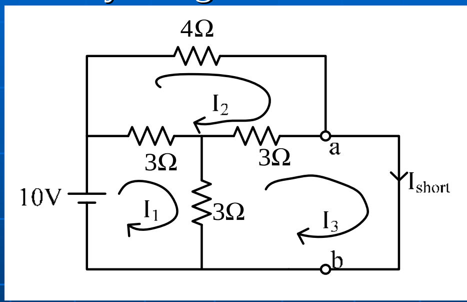
$$= \frac{R_N}{R_o + R_N} I_n$$



$$\textit{where} \quad R_N = \frac{\Delta}{\Delta_{nn}} \quad , \quad I_N = I_n$$

4.8 Norton's Theorem

Example 4.8.1 By using the above formula



Find the short circuit current I_3

$$\begin{bmatrix} 3+3 & -3 & -3 \\ -3 & 3+3+4 & -3 \\ -3 & -3 & 3+3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10V \\ 0 \\ 0 \end{bmatrix}$$

4.8 Norton's Theorem

Example 4.8.1 (cont.)

$$\begin{bmatrix} 3+3 & -3 & -3 \\ -3 & 3+3+4 & -3 \\ -3 & -3 & 3+3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10V \\ 0 \\ 0 \end{bmatrix}$$

$$\det[R_{ik}] = 360 - 27 - 27 - 27 - 90 - 54 - 54 = 108$$

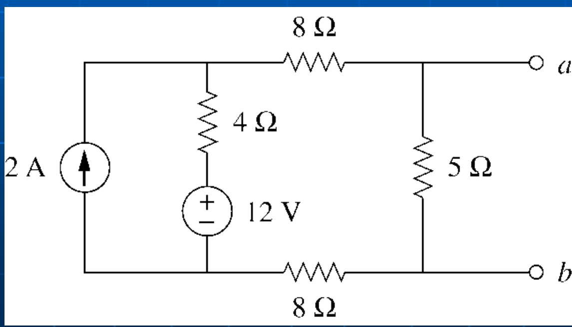
$$I_3 = \frac{1}{108}\det\begin{pmatrix} 6 & -3 & 10 \\ -3 & 10 & 0 \\ -3 & -3 & 0 \end{pmatrix} = \frac{10}{108}(39) = \frac{390}{108} = \frac{65}{18}A = I_N$$

$$R_N = \frac{\Delta}{\Delta_{33}} = \frac{108}{60-9} = \frac{36}{17}\Omega$$

4.8 Norton's Theorem

Example 4.8.2

Find the Norton equivalent circuit of the following circuit



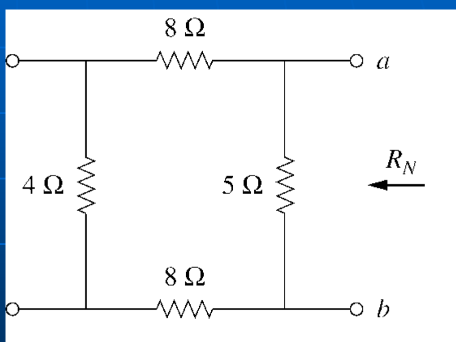
4.8 Norton's Theorem

Example 4.8.2 (cont.)

To find R_N from Fig.(a)

$$R_N = 5 \| (8+4+8)$$

$$= 5 \| 20 = \frac{20 \times 5}{25} = 4\Omega$$



(a)

4.8 Norton's Theorem

Example 4.8.2 (cont.)

To find I_N from Fig.(b)

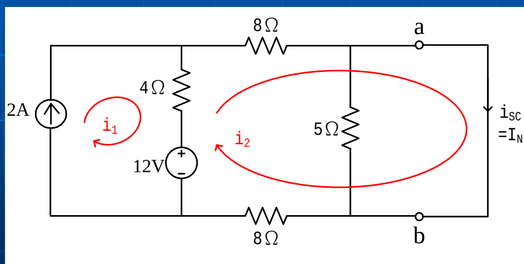
short-circuit terminal a and b

Mesh Analysis:

$i_1 = 2A$

$20i_2 - 4i_1 - 12 = 0$

$\therefore\ i_2 = 1A = I_N$



(b)

4.8 Norton's Theorem

Example 4.8.2 (cont.)

Alternative method for I_N: $I_N = \dfrac{V_{TH}}{R_{TH}}$

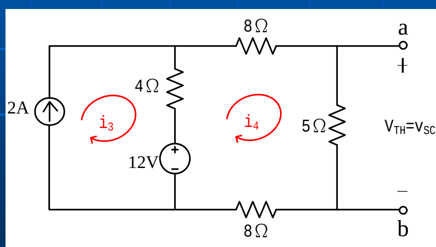
V_{TH}: open−circuit voltage across terminals *a* and *b*

Mesh analysis:

$i_3 = 2A\ ,\ \ 25i_4 - 4i_3 - 12 = 0$

$\therefore i_4 = 0.8A$

$\therefore v_{oc} = V_{TH} = 5i_4 = 4V$

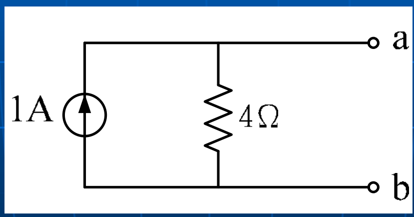


(b)

4.8 Norton's Theorem

Example 4.8.2 (cont.)

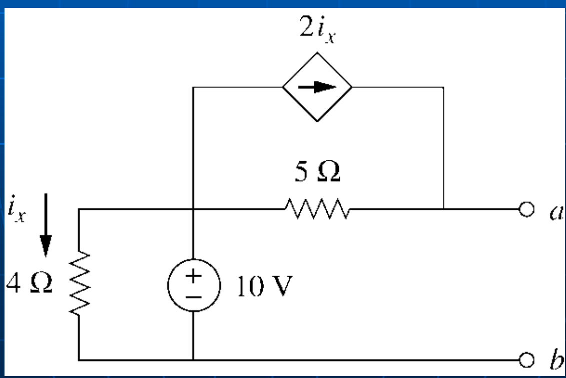
Hence , $I_N = \dfrac{V_{TH}}{R_{TH}} = 4/4 = 1A$



4.8 Norton's Theorem

Example 4.8.3

- Using Norton's theorem, find R_N and I_N of the following circuit.



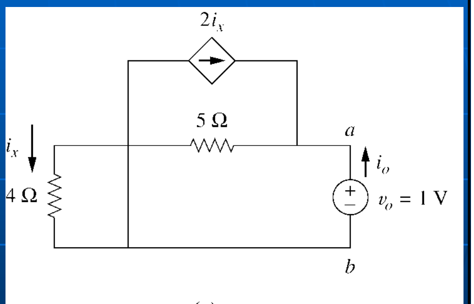
4.8 Norton's Theorem

Example 4.8.3 (cont.)

To find R_N from Fig.(a)

$$Hence\ ,\ i_o = \frac{v_o}{5} = \frac{1}{5} = 0.2\ A$$

$$\therefore\ R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\Omega$$



(a)

4.8 Norton's Theorem

Example 4.8.3 (cont.)

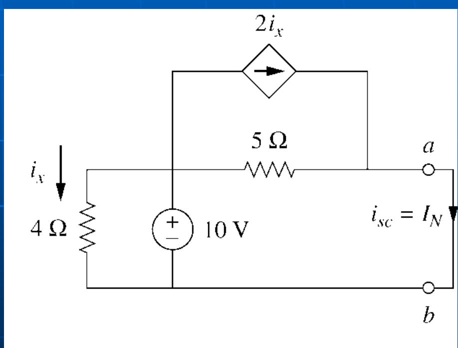
To find I_N from Fig.(b)

$$i_x = \frac{10}{4} = 2.5A$$

$$I_N = \frac{10V}{5\Omega} + 2i_x$$

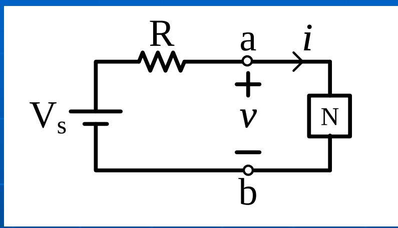
$$= \frac{10}{5} + 2(2.5) = 7A$$

$$\therefore I_N = 7A$$



(b)

4.9 Source Transformation

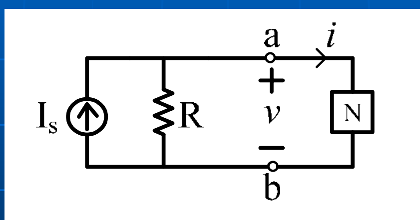


The current through resistor R can be obtained as follows :

$$i = \frac{V_S - v}{R} = \frac{V_S}{R} - \frac{v}{R} @ I_S - \frac{v}{R}$$

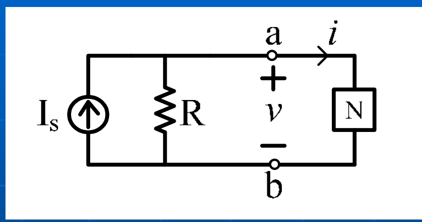
4.9 Source Transformation

From KCL, one can obtain the following equivalent circuit



$$\text{where} \quad I_S @ \frac{V_S}{R}$$

4.9 Source Transformation

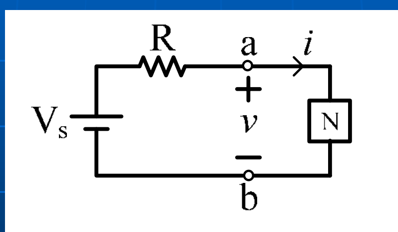


The voltage across resistor R can be obtained as follows :

$$v = (I_S - i)R = I_S R - iR \triangleq V_S - iR$$

4.9 Source Transformation

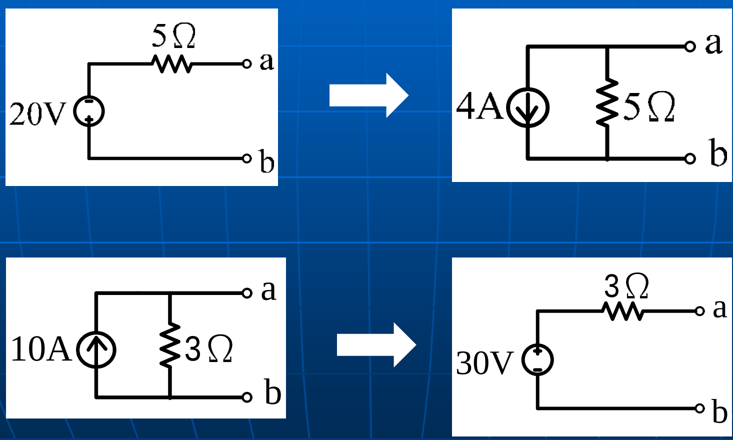
From KVL, one can obtain the following equivalent circuit



$$\text{where} \quad V_S \triangleq R\, I_S$$

4.9 Source Transformation

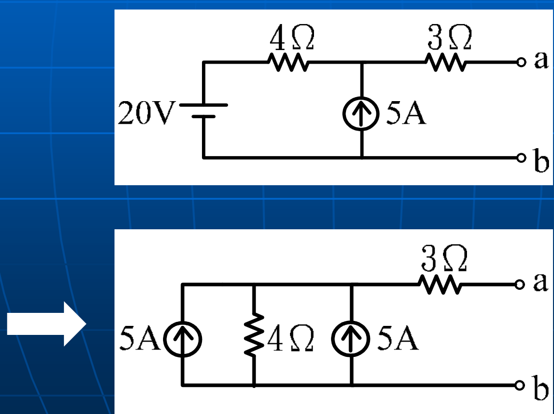
Example 4.9.1



4.9 Source Transformation

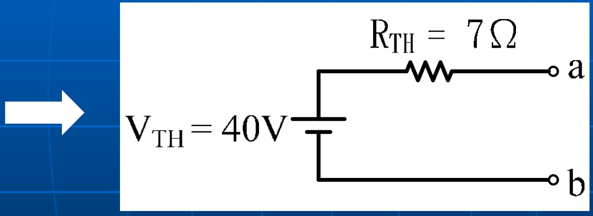
Example 4.9.2

- Find the Thevenin's equivalent

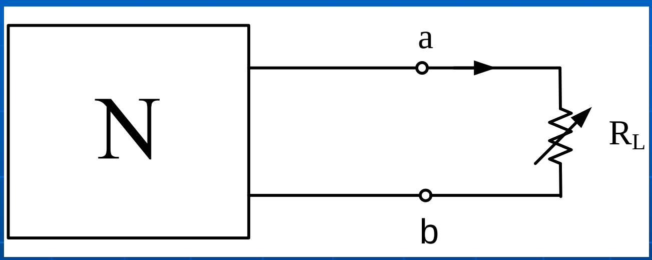


4.9 Source Transformation

Example 4.9.2 (cont.)



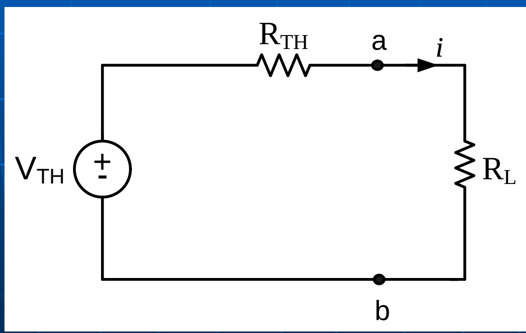
4.10 Maximum Power Transfer Theorem



- Problem : Given a linear resistive circuit N shown as above, find the value of R_L that permits maximum power delivery to R_L .

4.10 Maximum Power Transfer Theorem

Solution : First, replace N with its Thevenin equivalent circuit.



4.10 Maximum Power Transfer Theorem

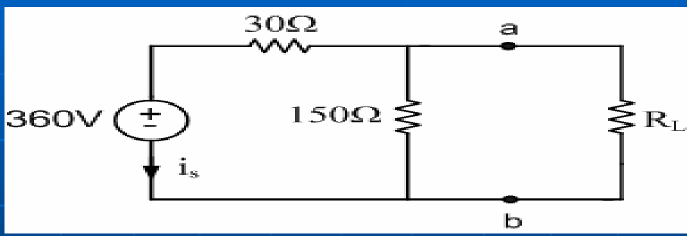
$$p = i^2 R = \left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 R_L$$

$$Let \;\; \frac{dp}{dR_L} = 0 \; ,$$

$$Then \; \mathrm{R_L = R_{TH}} \quad and \quad P_{\max} = \left(\frac{V_{TH}}{2R_L}\right)^2 R_L = \frac{V_{TH}^2}{4R_L}$$

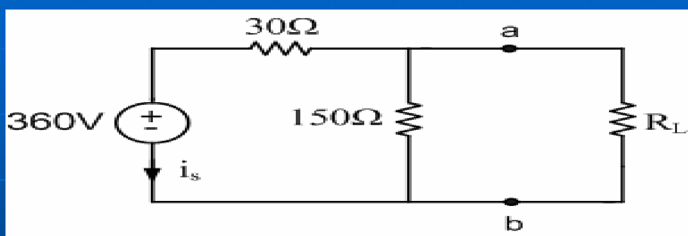
4.10 Maximum Power Transfer Theorem

- Example 4.10.1



(a) Find R_L that results in maximum power transferred to R_L.
(b) Find the corresponding maximum power delivered to R_L, namely P_{max}.
(c) Find the corresponding power delivered by the 360V source, namely P_s and P_{max}/P_s in percentage.

4.10 Maximum Power Transfer Theorem

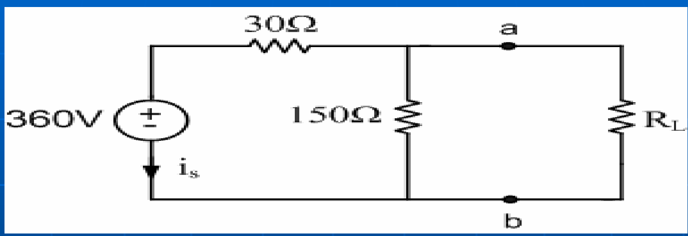


Solution : (a) $V_{TH} = \frac{150}{180}(360) = 300V$

$$R_{TH} = \frac{150 \times 30}{180} = 25\ \Omega$$

$$(b)\ P_{max} = \left(\frac{300}{50}\right)^2 25 = 900W$$

4.10 Maximum Power Transfer Theorem



$$\text{Solution}:(c)\ V_{ab} = \frac{300}{50} \times 25 = 150V$$

$$i_s = \frac{-(360-150)}{30} = -7A$$

$$P_s = i_s(360) = -2520W \ (\text{dissipated})$$

$$\frac{P_{max}}{|P_s|} = \frac{900}{2520} = 35.71\%$$

Summary

- Objective 7 : Understand and be able to use superposition theorem.
- Objective 8 : Understand and be able to use Thevenin's theorem.
- Objective 9 : Understand and be able to use Norton's theorem.

Summary

- Objective 10 : Understand and be able to use source transform technique.
- Objective 11 : Know the condition for and be able to find the maximum power transfer.

Summary

- Problem : 4.60
  4.64
  4.68
  4.77
  4.86
  4.91
- Due within one week.