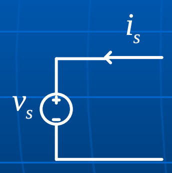
CHAPTER 2

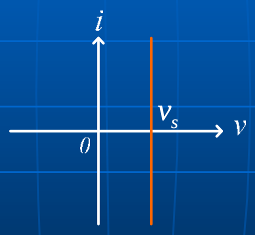
CIRCUIT ELEMENTS
AND
KIRCHHOFF'S LAWS

CONTENTS

2.1 Active Components

- Ideal independent voltage source
- Symbol and model





$i_s = \infty$

$i_s = 100$ A

$i_s = 0$ A

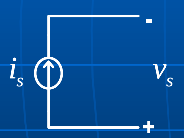
$i_s = -50$ A

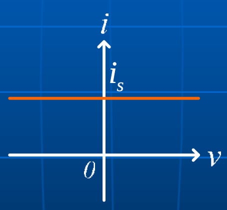
$i_s = -\infty$

- It maintains a prescribed voltage independent of the current through this source.

2.1 Active Components

- Ideal independent current source
- Symbol and model





$v_s = \infty$

$v_s = 100$ V

$v_s = 0$ V

$v_s = -50$ V

$v_s = -\infty$

- It maintains a prescribed current independent of the voltage across this source.

2.1 Active Components

- Ideal dependent voltage and current sources are convenient for modeling transistors
- An ideal dependent (or controlled) voltage source is an active element whose source quantity is controlled by other voltage or current.

2.1 Active Components

- Symbol and model of dependent voltage source

$v_s = m v_x$

voltage-controlled voltage source (VCVS)

$v_s = g i_x$

current-controlled voltage source (CCVS)

v_x and i_x are controlling parameter.

2.1 Active Components

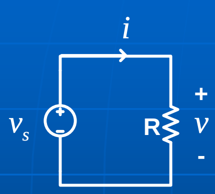
- Symbol and model of dependent current source

$i_s = a v_x$  voltage-controlled current source (VCCS)

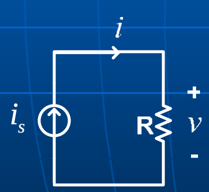
$i_s = b i_x$  current-controlled current source (CCCS)

v_x and i_x are controlling parameter.

2.1 Active Components



$R_s \to 0 \;,\; p = vi \to \infty$



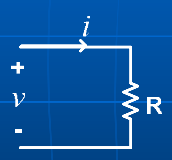
$R_s \to \infty \;,\; p = vi \to \infty$

Ideal sources do not exist in real world.

2.2 Passive Components

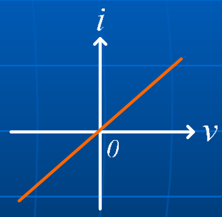
(a) Resistance

- Symbol and model



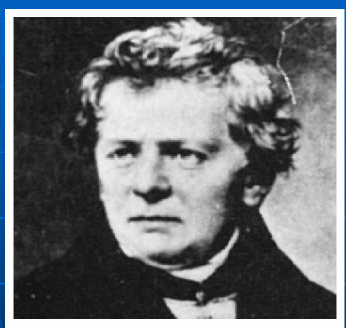
$v = Ri$

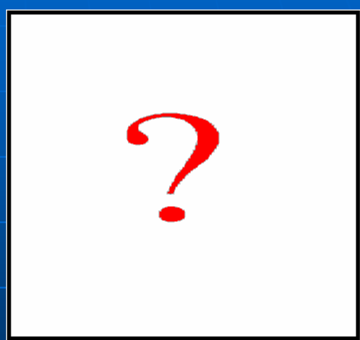
$i = Gv$



R: resistance , in ohms (Ω)
G: conductance , in siemens (S)
Ohm's Law

2.2 Passive Components





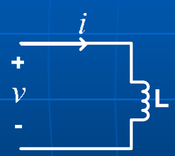
Georg Simon Ohm
1787-1854
Germany

???
19XX-20XX
R.O.C. Taiwan

2.2 Passive Components

(b) Inductor

- Symbol and model



$$v = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

$$= \frac{1}{L}\int_{-\infty}^{0} v\,dt + \frac{1}{L}\int_{0}^{t} v\,dt$$

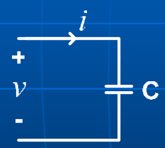
$$= i_0 + \frac{1}{L}\int_{0}^{t} v\,dt$$

L: inductance , in henry (H)

2.2 Passive Components

(c) Capacitor

- Symbol and model



$$i = C\frac{dv}{dt}$$

$$v = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$= \frac{1}{C}\int_{-\infty}^{0} i\,dt + \frac{1}{C}\int_{0}^{t} i\,dt$$

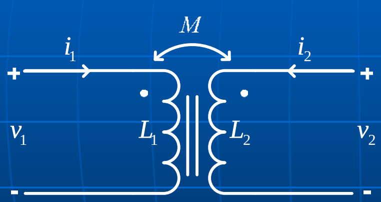
$$= v_0 + \frac{1}{C}\int_{0}^{t} i\,dt$$

C: capacitance , in Farad (F)

2.2 Passive Components

(d) Coupling Inductor

- Symbol and model



$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

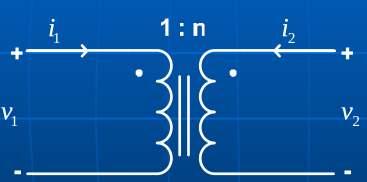
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{L_1 L_2 - M^2} \int_{-\infty}^{t} \begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} dt$$

M: mutual inductance , in henry (H)
L_1: primary self inductance , in henry (H)
L_2: secondary self inductance , in henry (H)

2.2 Passive Components

(e) Ideal Transformer

- Symbol and model



$$\frac{v_2}{v_1} = n$$

$$\frac{i_1}{i_2} = -n$$

n: turn ratio , positive real number

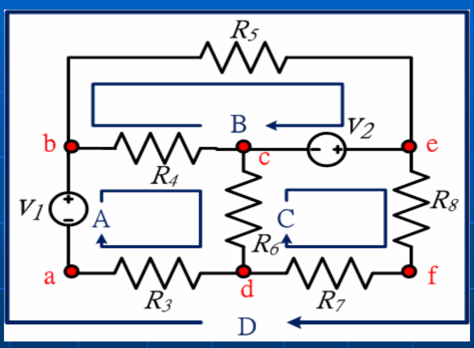
- It is convenient for modeling practical transformers or coupling inductors.

2.3 Node , Branch and Loop

- A branch represents a single two-terminal element.
- A node is a point where two or more elements join.
- A loop is a closed path in a circuit without passing through any intermediate node more than once.

2.3 Node , Branch and Loop

- Example

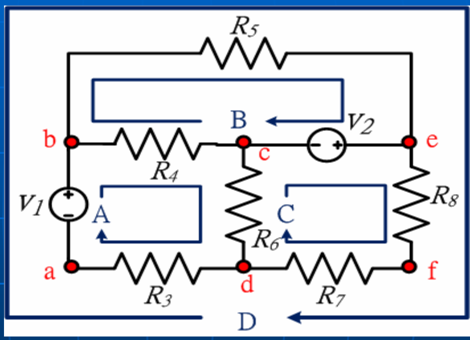


node : a , b , c , d , e , f 6 nodes

branch : V_1 , V_2 , R_3 , R_4 , R_5 , R_6 , R_7 , R_8 8 branches

2.3 Node , Branch and Loop

- Example



loop A : a => b => c => d => a
loop B : b => e => c => b
loop C : c => e => f => d => c
loop D : a =>b =>e =>f =>d => a

2.4 Kirchhoff's Laws

- Kirchhoff's current law (KCL)

  The algebraic sum of all the current leaving any node in a circuit equals zero

$$\sum_{n=1}^{N} i_n = 0$$, for any node

N : number of elements connected to this node
i_n : the *n*th element current leaving this node

2.4 Kirchhoff's Laws

- The algebraic sum of all the current entering any node in a circuit equals zero

$$\sum_{n=1}^{N} (-i_n) = 0$$, for any node

- If $N = L \cup M$, then

$$\sum_{L} i_n = \sum_{M} i_m$$, for any node

L : set of n with i_n leaving this node

M : set of m with i_m entering this node

2.4 Kirchhoff's Laws

- KCL is based on the law of conservation of charge.

- KCL can be applied to a super node (closed boundary)

2.4 Kirchhoff's Laws

- Kirchhoff's voltage law (KVL)
  The algebraic sum of all the element voltage drops around any loop in a circuit equals zero.

$$\sum_{m=1}^{M} V_m = 0$$, for any loop

2.4 Kirchhoff's Laws

- The algebraic sum of all the element voltage rises around any loop in a circuit equals zero.

$$\sum_{m=1}^{M} (-V_m) = 0$$, for any loop

2.4 Kirchhoff’s Laws

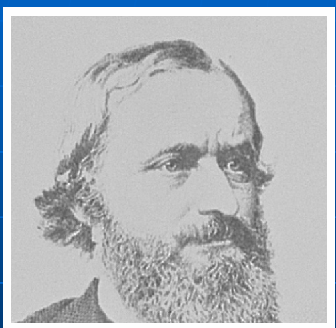
- If $M = K + J$, then

$$\sum_{K} i_k = \sum_{J} i_j$$

M = set of k with V_k being a voltage drop in this loop
J = set of j with V_j being a voltage rise in this loop

- KVL is based on the law of conservation of energy.

2.4 Kirchhoff’s Laws

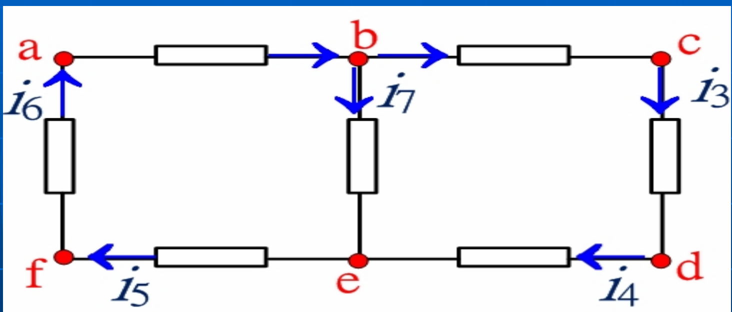


Gustav Robert Kirchhoff
1824-1887
Germany

???
19XX-20XX
R.O.C. Taiwan

2.4 Kirchhoff's Laws

- Example 1



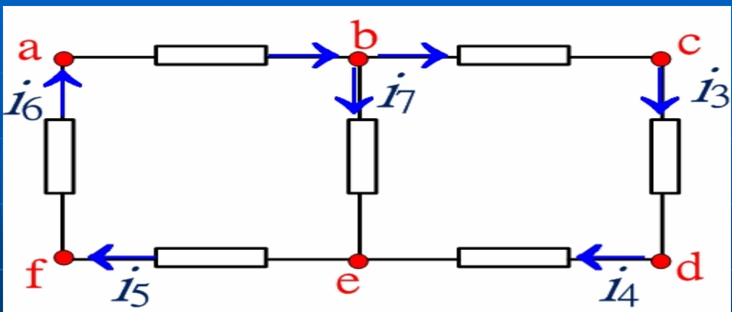
$$\sum_k i_k(leaving) = 0 \text{ , } KCL \text{ in time domain.}$$

Node b : $(-i_1) + i_7 + i_2 = 0$

Node e : $(-i_7) + (-i_4) + i_5 = 0$

2.4 Kirchhoff's Laws

- Example 1 (cont.)



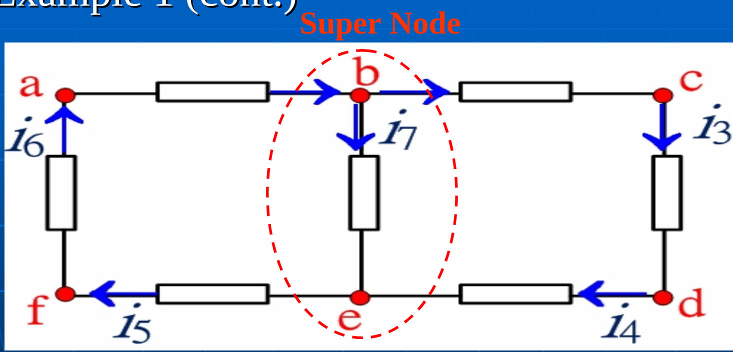
$$\sum_j i_j(entering) = 0$$

Node b : $i_1 + (-i_7) + (-i_2) = 0$

Node e : $i_7 + i_4 + (-i_5) = 0$

2.4 Kirchhoff's Laws

- Example 1 (cont.)



$$\sum_m i_m(leaving) = \sum_n i_n(entering)$$

Node b : $i_7 + i_2 = i_1$

Node e : $i_5 = i_7 + i_4$

Super Node : $i_5 + i_2 = i_1 + i_4$

2.4 Kirchhoff's Laws

- Example 2

$\sum_{n=1}^{N} i_n(t) = 0$, for any node

Assume

$$i_n(t) = I_n \cos(wt + f_n) = \mathrm{Re}[I_n e^{j(wt+f_n)}] = \mathrm{Re}[I_n e^{jf_n} e^{jwt}] \quad , \text{ n=1,2,3,}\cdots\text{,N}$$

Re: real part

Define $\dot{I}_n = I_n e^{jf_n} = I_n \cos f_n + jI_n \sin f_n$

Then $i_n(t) = \mathrm{Re}[\dot{I}_n e^{jwt}]$

2.4 Kirchhoff's Laws

- Example 2 (cont.)

$$\sum_{n=1}^{N} Re[\vec{I}_n e^{jwt}] = 0$$

$\because$ Re operator & $\sum$ operator are commutative.

$$\text{Re}[(\sum_{n=1}^{N} \vec{I}_n) e^{jwt}] = 0$$

$$\therefore \sum_{n=1}^{N} \vec{I}_n = 0 \quad , \; complex\ equation$$

KCL in phasor domain

2.4 Kirchhoff's Laws

- Example 3

$$\sum_{n=1}^{N} i_n(t) = 0$$, for any node

Let $I_n(s) \triangleq \int_0^{\infty} i_n(t) e^{-st} dt$

Then $$\int_0^{\infty} \sum_{n=1}^{N} i_n(t)\, dt = 0$$

$\int$ operator and $\sum$ operator are commutative

2.4 Kirchhoff's Laws

- Example 3 (cont.)

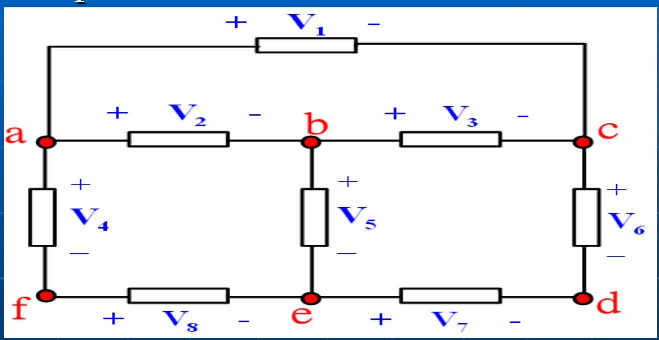
$$\sum_{n=1}^{\infty}\left(\int_0^{\infty} i_n(t)e^{-st}dt\right)=0$$

$$\sum_{n=1}^{\infty} I_n(s)=0\ ,\ scalar\ equation$$

KCL in s domain

2.4 Kirchhoff's Laws

- Example 4



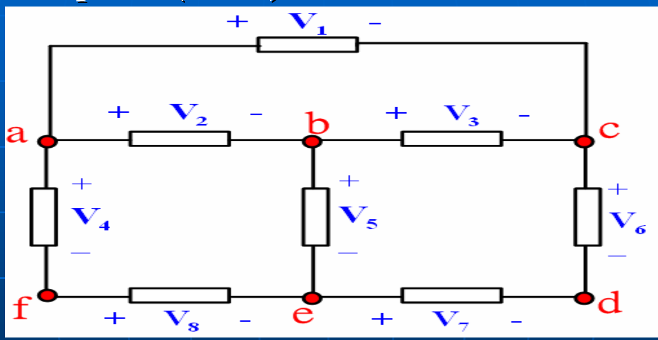
$$\sum_{K} V_k(drop)=0\ ,\ KVL\ in\ time\ domain$$

loop a , b , c , d , e , f , a

$V_2 + V_3 + V_6 + (-V_7) + (-V_8) + (-V_4) = 0$

2.4 Kirchhoff's Laws

- Example 4 (cont.)



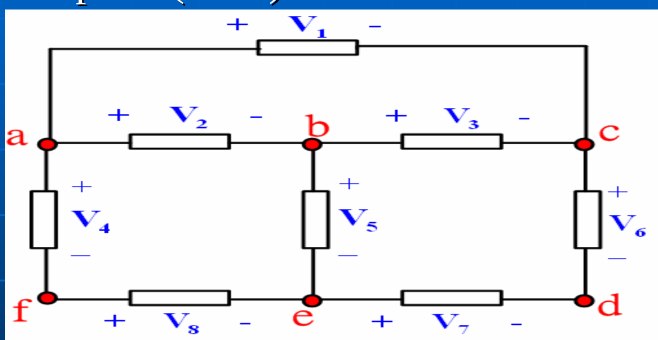
$$\sum_{K} V_k(rise) = 0$$

loop a , b , c , d , e , f , a

$(-V_2) + (-V_3) + (-V_6) + V_7 + V_8 + V_4 = 0$

2.4 Kirchhoff's Laws

- Example 4 (cont.)



$$\sum V_n(drop) = \sum V_m(rise)$$

loop a , b , c , d , e , f , a

$V_2 + V_3 + V_6 = V_7 + V_8 + V_4$

2.4 Kirchhoff's Laws

- Example 5

$$\sum_{k=1}^{M} v_k(t) \ (drop) = 0$$, for any node

Assume
$$v_k(t) = V_k \cos(wt + f_k)$$
$$\therefore v_k(t) = \mathrm{Re}\left[V_k e^{jf_k} e^{jwt}\right]$$

Let
$$\vec{V}_k = V_k e^{jf_k} = V_k \cos f_k + jV_k \sin f k$$

Then
$$v_k(t) = \mathrm{Re}[\vec{V}_k e^{jwt}]$$

2.4 Kirchhoff's Laws

- Example 5 (cont.)

Then
$$\sum_{k=1}^{M} \mathrm{Re}\left[\vec{V}_k e^{jwt}\right] = 0$$
$$\Rightarrow \mathrm{Re}\left[\left(\sum_{k=1}^{M} \vec{V}_k\right) e^{jwt}\right] = 0$$
$$\Rightarrow \sum_{k=1}^{M} \vec{V}_k = 0 \text{ , complex equation}$$

KVL in phasor domain

2.4 Kirchhoff's Laws

- Example 6

$$\sum_{n=1}^{N} v_k(t) = 0$$

Define $V_K(s) = \int_0^{\infty} v_k(t)\, e^{-st} dt$

$$\int_0^{\infty} \sum_{k=1}^{M} v_k(t)\, e^{-st} dt = 0$$

$$\sum_{k=1}^{M} \int_0^{\infty} v_k(t)\, e^{-st} dt = 0$$

$$\sum_{k=1}^{M} V_K(s) = 0 \;,\; scalar\ eqution$$

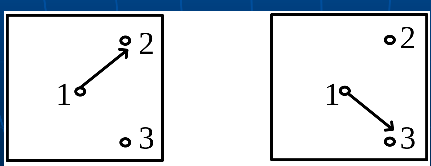
$KVL\ \ in\ \ s\ \ domain$

2.4 Kirchhoff's Laws

- Example 7 (Chapter Problem 2.33)

Design an electric wiring system to control a single appliance from two or more locations.

A three-way switch
：single pole double throw switch
(單極双投開關)

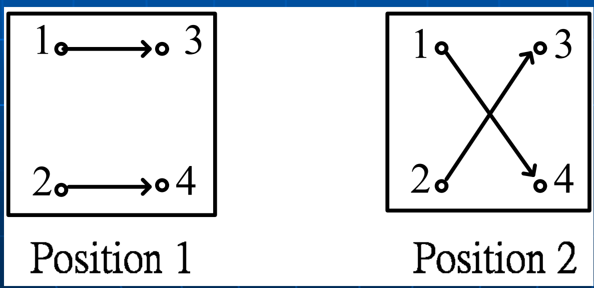


Position 1 Position 2

2.4 Kirchhoff's Laws

- Example 7 (cont.)

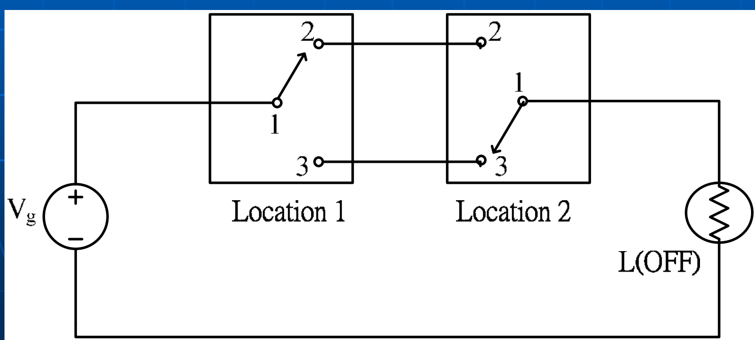
A four-way switch
：double pole double throw switch
(双極双投開關)



2.4 Kirchhoff's Laws

- Example 7 (cont.)

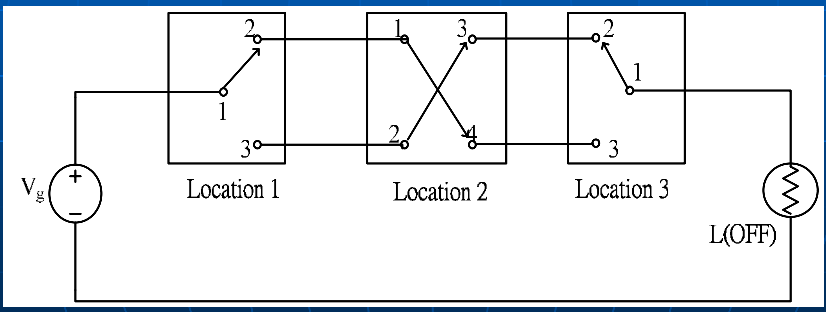
(a) Two three-way switches can be connected between two locations.



2.4 Kirchhoff's Laws

- Example 7 (cont.)

(b) One four-way switch is required for each location in excess of two.



Summary

- Objective 1：Understand ideal circuit components.
- Objective 2：Be able to state and use component models.
- Objective 3：Be able to state and use KCL and KVL.

Assignment : Chapter Problems

- Problem 2.3
  2.9
  2.26
  2.29

- Due within one week.

THE END