科目：通訊系統(通訊原理)(500F)

校系所組：中央大學通訊工程學系（甲組）
中央大學電機工程學系（電子組）
交通大學電子研究所（乙組）
交通大學電信工程研究所（甲組）
清華大學電機工程學系（乙組）
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一. (20%) Let $X(t)$ be a zero-mean wide-sense stationary random process with autocorrelation function $E\{X(t+\tau)X(t)\} = R_X(\tau)$ and power spectral density (PSD) $S_X(f)$.

(一) (4%) Let $Y(t) = X(t)\cos(2\pi f_c t + \theta)$, where f_c is a known constant, and θ is a random variable uniformly distributed over $(\pi, -\pi)$ that is independent of $X(t)$. Find the PSD of $Y(t)$.

(二) (5%) Following (一), if $X(t)$ is a Gaussian random process, is $Y(t)$ also a Gaussian random process? Why ?

(三) (6%) If $X(t)$ is passed through a finite-time integrator to obtain $Z(t) = \frac{1}{T}\int_{t-T}^{t} X(\tau)d\tau$, find the PSD of $Z(t)$.

(四) (5%) Following (三), if $X(t)$ is a white noise process with PSD $S_X(f) = N_0/2$, find the average output power $E\{|Z(t)|^2\}$, where $E\{\cdot\}$ is the expectation operator.

二.(20%) Assume a voltage-controlled oscillator (VCO) is used as an FM modulator, whose voltage to frequency transfer coefficient is k (Hz/V). Let $m(t)$ be the input of the VCO and $s(t)$ be the modulated FM signal, given as

$$m(t) = A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)$$
$$s(t) = \cos[2\pi f_c t + \theta(t)]$$

where f_c is the carrier frequency and $f_c >> f_1, f_2$.

(一) (6%) Derive $\theta(t)$.

(二) (7%) If $A = 3V, B = 5V, k = 15KHz/V$ and $f_1 = 2KHz, f_2 = 6KHz, f_c = 100MHz$, find the deviation ratio (D) of the FM signal.

(三) (7%) Assume the above FM signal passes through a band-pass filter (BPF) whose bandwidth is defined by the Carson's rule. After the BPF, the FM signal is amplified with a nonlinear amplifier. The input-output relationship of the amplifier is given as

$$y = a_1 x + a_2 x^2 + a_3 x^3$$

where x is the amplifier input, y is the output, and a_1, a_2, a_3 are non-zero constants. At the amplifier output, find the bandwidth of the FM signal at the carrier frequency f_c and explain your answer.

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三.(20%) Consider a system with two equally probable signals $s_1(t)$ and $s_2(t)$ given by

$$s_1(t)=\begin{cases}A, & \text{for } 0<t<T \\ 0, & otherwise\end{cases}, \qquad s_2(t)=\begin{cases}\text{-}A, & \text{for } 0<t<T \\ 0, & otherwise.\end{cases}$$

The received signal is defined as

$$r(t)=s_i(t)*h(t)+n(t), \quad i=1 \text{ or } 2,$$

where * denotes linear convolution, $h(t)$ is the impulse response of a linear time-invariant channel, and $n(t)$ is white Gaussian noise of zero mean and two-sided power spectral density $S_N(f)=N_0/2$.

(一) (4%) Suppose $h(t)=\delta(t)$, where $\delta(t)$ denotes the ideal impulse function. Draw the block diagram of matched filter receiver and find its average probability of symbol error.

(二) (6%) Redo (一) if $h(t)=\delta(t)+\delta\left(t-\frac{T}{2}\right)$.

(三) (5%) Suppose $h(t)=\delta(t)+\beta\delta(t-\frac{T}{2})$, where β is a random variable with probabilities $\text{Prob}(\beta=1)=\text{Prob}(\beta=-1)=1/2$. Find the average probability of symbol error if the optimum receiver in (一) is used for detection.

(四) (5%) Following (三), find the average probability of symbol error if the optimum receiver in (二) is used for detection.

四. (20%) Consider binary ($a_k=\pm 1$ equally likely) continuous-phase FSK modulation in an AWGN channel with two-sided noise power spectral density $S_N(f)=N_0/2$, assuming the bit duration is T_b sec, the bit energy is E_b, the frequency spacing between the two transmitted tones is equal to $1/T_b$ Hz, and the carrier frequency is f_c Hz.

(一) (6%) Derive and plot the power spectrum density of the FSK signal.

(二) (7%) Derive the optimum coherent receiver and draw its block diagram.

(三) (7%) Derive the optimum non-coherent receiver and draw its block diagram.

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五. (20%)

(一) (6%) Consider a direct sequence spread spectrum (DSSS) QPSK system using only one spreading code *c(t)* with chip time T_c and a period of N chips. Suppose there also exists a single tone jamming interference in the channel. Plot the block diagrams of the transmitter and receiver, assuming that spreading is done after the QPSK modulation. Also, explain why a DSSS system has the anti-jamming capability.

(二) (7%) Consider a source symbol X taking values in the set $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ with probabilities 0.25, 0.1, 0.2, 0.1, 0.25, 0.05, 0.05, respectively. Please construct a ternary source code with symbol taking values from {0, 1, 2} for X, which has the minimal average codeword length.

(三) (7%) Consider two binary random variables X and Y which take values from {0, 1} and have the following joint probabilities:

$$\Pr(Y=0, X=0) = 0.72, \quad \Pr(Y=1, X=0) = 0.08$$
$$\Pr(Y=0, X=1) = 0.08, \quad \Pr(Y=1, X=1) = 0.12$$

We can see that $\Pr(Y=0) = 0.8$ and $\Pr(Y=1) = 0.2$. Now Bob wants to estimate the value of Y from the observation of X. Knowing that $\Pr(Y=0) > \Pr(Y=1)$, Bob thus concludes that a simple way to estimate Y is to set $Y = 0$ all the time. Does Bob provide the best way to estimate Y? If yes, justify your answer. If not, find the best way to estimate Y.