＊作答前,請先核對試題、答案卷（試卷）與准考證上之所組別與考試科目是否相符!!

1. (a) (5%) Prove that the autocorrelation function $R_X(\tau)$ of a wide-sense stationary (WSS) process $X(t)$ always peak at zero, namely, $|R_X(\tau)| \le R_X(0)$.

   (b) (5%) Prove that the output $Y(t)$ of a stable linear-time-invariant (LTI) filter (with impulse response $h(t)$) due to a WSS input $X(t)$ is also WSS.

2. For an informational signal $m(t)$, write down the mathematical expression of the corresponding analog transmission passband signal of the following, if the carrier is $A_c \cos(2\pi f_c t)$.

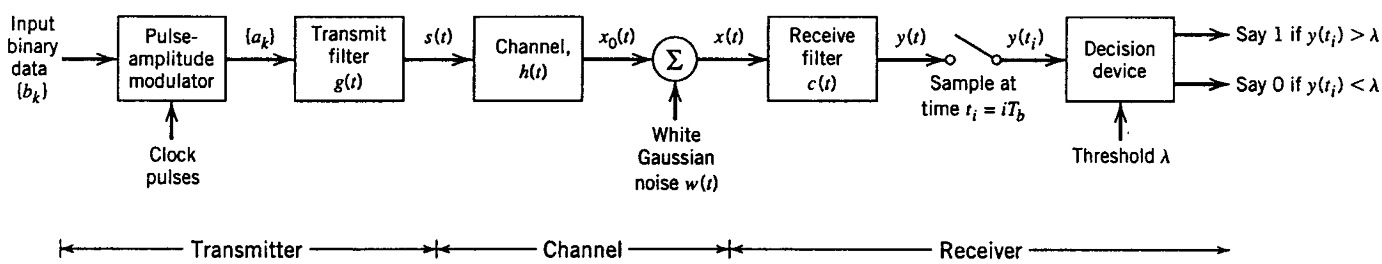
   (a) (2%) Amplitude modulation (DSB-C) with amplitude sensitivity k_a.

   (b) (2%) Frequency modulation with frequency sensitivity k_f.

   (c) (2%) Phase modulation with phase sensitivity k_p.

   (d) (6%) Explain in words on (i) threshold effect; (ii) noise quieting effect; (iii) capture effect for an FM receiver.

3. In the figure below, $b_k \in \{0,1\}$ is the input data at time instant k, $\{a_k = (2b_k - 1)d\}_{k=-\infty}^{\infty}$ are unit impulses with amplitude $\pm d$, $s(t) = \sum_{k=-\infty}^{\infty} a_k \cdot g(t - kT_b)$, and $G(f)$, $H(f)$ and $C(f)$ are transfer functions corresponding to the impulse responses $g(t)$, $h(t)$ and $c(t)$, respectively.



(a) (6%) In absence of noise $w(t)$ (namely, $w(t) = 0$), determine $y(iT_b)$ if $g(t)$ and $c(t)$ are selected such that

$$p(t) = \int_{-\infty}^{\infty} G(f)H(f)C(f)\exp\{j2\pi ft\}dt$$

satisfies that $p(iT_b) = 0$ for all $i \ne 0$, and $p(0) = 1$.

(b) (6%) In presence of the zero-mean additive white Gaussian noise $w(t)$ with power spectral density $\Phi_{ww}(f) = N_0/2$, find the best threshold λ that gives the minimum decision error about a_i upon the reception of $y(iT_b)$. What is the minimum decision error corresponding this best threshold?

(c) (6%) For fixed average transmission power P_{av}, prove that the choice of $|G(f)| = |C(f)| = \sqrt{|P(f)/H(f)|}$ minimizes the "minimum decision error" in (b).

(Hint: Observe $P_{av} = \frac{d^2}{T_b}\int_{-\infty}^{\infty} |G(f)|^2 df = \frac{d^2}{T_b}\int_{-\infty}^{\infty} \frac{|P(f)|^2}{|H(f)|^2 |C(f)|^2} df$ and use Cauchy-Schwarz inequality to find the best $|C(f)|$ that minimizes the "minimum decision error" in (b).)

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4. Consider two orthonormal functions $\{\phi_1(t), \phi_2(t)\}$ given as

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(\omega t) \quad , \quad \phi_2(t) = \sqrt{\frac{2}{T}}\sin(\omega t)$$

where T is the symbol duration. Let the received signal be $x(t) = s_i(t) + w(t)$, where $s_i(t)$ is a 16-QAM signal, expressed as

$$s_i(t) = a_i\phi_1(t) + b_i\phi_2(t),\ i=1,2,\ldots,16,\ a_i, b_i \in (\pm A, \pm 3A),$$

while $w(t)$ is the added white Gaussian noise with two-sided power spectral density $N_0/2$. We can look $x(t)$ as a vector in the signal space:

$$\vec{x} = (x_1, x_2),$$

where $x_j = \int_0^T x(t)\phi_j(t)dt, \qquad j = 1,2.$

(a). (5%) Derive mean and variance of x_1.

(b).(5%) Explain the meaning of maximum a posteriori probability (MAP) decision rule and maximum likelihood (ML) decision rule.

(c). (5%) Let all symbols be sent with equal probability, please derive average symbol error probability of the received signal under optimum detection.

5. A binary FSK signal is given as:

$$s_i(t) = A\cos(2\pi f_i t + \theta_i), \qquad i = 1,2$$

After passing through the channel, the received signal is:

$$y(t) = s_i(t) + w(t)$$

where $w(t)$ is the added white Gaussian noise with two-sided power spectral density $N_0/2$.

(a). (5%) If the frequency f_i and the phase θ_i are known, please plot the optimum receiver structure to detect $y(t)$ and explain why such a structure is optimum.

(b). (5%) In (a), please derive average bit error probability of the optimum receiver.

(c). (5%) If the frequency f_i is known but the phase θ_i is a random variable uniformly distributed within $[0,2\pi]$, please plot the optimum receiver structure to detect $y(t)$ and explain why such a structure is optimum.

6. (a) (5%) Consider a random variable X taking values in the set $\{x_1, x_2, x_3, x_4, x_5\}$ with probabilities 0.25, 0.25, 0.2, 0.15, 0.15, respectively. Please construct a ternary Huffman code with symbol taking values from {0, 1, 2} for X.

(b) (5%) Obtain a ternary Huffman code for another random variable taking values in the set $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ with probabilities 0.25, 0.25, 0.2, 0.1, 0.1, 0.1, respectively.

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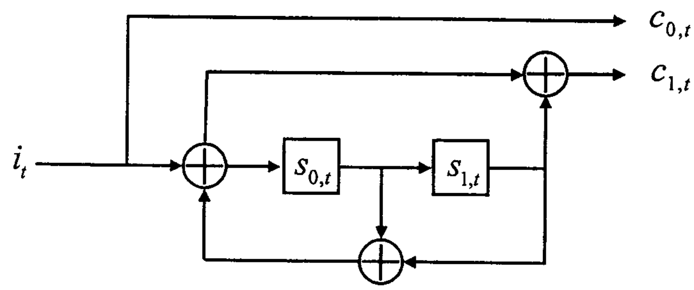
7. Consider a binary convolutional code with the following encoder:



where i_t, $(c_{0,t}, c_{1,t})$, and $(s_{0,t}, s_{1,t})$ stand for the input information bit, output coded bits, and state vector at time t, respectively.

(a) (5%) What is the corresponding generator matrix?

(b) (5%) Denote by $c_{0,t}c_{1,t}/i_t$ the branch label associated with the state transition from $(s_{0,t}, s_{1,t})$ to $(s_{0,t+1}, s_{1,t+1})$. Draw the state diagram.

(c) (10%) For transmission over additive white Gaussian channels with the binary modulation which maps binary "0" → +1 and "1" → 0, let $r_{l,t}$ denote the received symbol corresponding to $c_{l,t}$ for all l, t. Please find the codeword estimate by the maximum-likelihood decoding given the following received sequence:

$$(r_{0,0}, r_{1,0}, r_{0,1}, r_{1,1}, r_{0,2}, r_{1,2}, r_{0,3}, r_{1,3}, r_{0,4}, r_{1,4}) = (-1, -1, 2, -1, -1, +1, 2, 0, -1, 0).$$