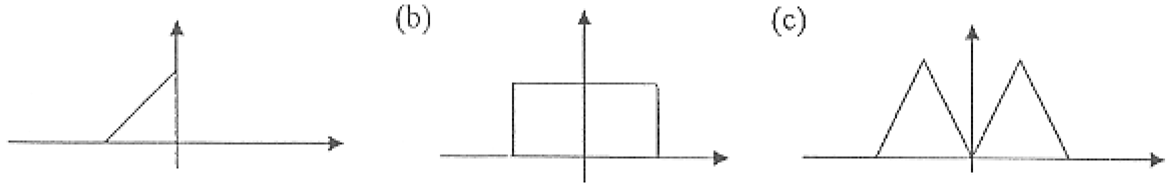
＊＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！

1. (6%) Can the function below be autocorrelation function of a wide-sense stationary (WSS) random process? Justify your answer.

(a) (b) (c)

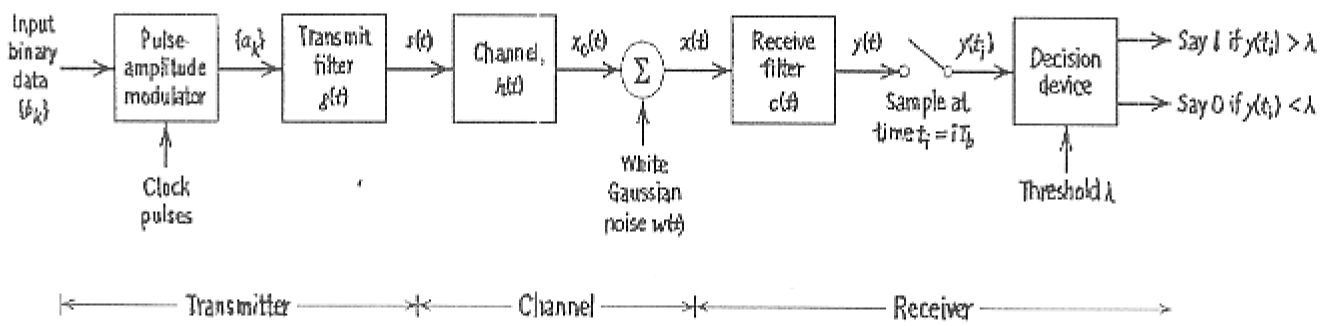


2.(a) (6%) Prove that $S_Y(f) = H(f)H(-f)S_X(f)$ if $Y(t) = \int_{-\infty}^{\infty} h(\tau)X(t-\tau)d\tau$, where $S_X(f)$ and $S_Y(f)$ are respectively the power spectral densities (PSDs) of the real-valued WSS signals $X(t)$ and $Y(t)$, and $H(f)$ is the Fourier transform of the filter impulse response $h(\tau)$.

(b) (4%) Show that the relation in (a) can be reduced to $S_Y(f) = |H(f)|^2 S_X(f)$, if $h(\tau)$ is real.

(c) (4%) Use (b) to prove that the PSD of a real-valued WSS process is always non-negative.

3. In the figure below, $\{a_k\}$ are unit impulses with amplitude ± 1, whereas $G(f)$, $H(f)$ and $C(f)$ are transfer functions corresponding to the impulse responses $g(t)$, $h(t)$ and $c(t)$,



respectively.

(a) (5%) In absence of noise $w(t)$, namely, $w(t) = 0$, describe the Nyquist criterion for zero-ISI in the above **baseband** transmission system.

(b) (5%) Describe the model of the ideal Nyquist channel.

(c) (5%) Consider a rectangular pulse $g(t)$, and a known channel impulse response $h(t)$ as:

$$g(t) = \begin{cases} 1, & 0 \le t < T_b \\ 0, & \text{otherwise} \end{cases}, \text{ and } h(\tau) = \delta(\tau) + \delta(\tau - T_b)$$

where $\delta(\tau)$ is the Dirac delta function. Find the **matched filter impulse response** $c(t)$ that maximizes the signal-to-noise ratio at the output of the sampler in presence of the white noise $w(t)$.

(d) (5%) Does $c(t)$ in (c) satisfy the Nyquist Criterion? Justify your answer.

4.Consider a discrete memoryless source S with source alphabet $\mathbb{S}=\{s_1, s_2, \cdots, s_K\}$ and occurrence probabilities $\{p_1, p_2, \cdots, p_K\}$.

(a) (10%) Denote the entropy of S as H(S). Find the values of $p_1, p_2 \cdots, p_K$ so that H(S) is maximized. Prove your result.

(b) (10%) The second-order extension of this source is another discrete memoryless source T with source alphabet $\mathbb{S}^2=\{t_1, t_2, \cdots, t_M\}$, where $M = K^2$. Denote the occurrence probabilities of T as $\{q_1, q_2, \cdots, q_M\}$ and its entropy as H(T). Derive the relationship between H(S) and H(T).

5.Consider the (7,4) Hamming code defined by the generator polynomial $g(X) = 1 + X + X^3$.

(a) (4%) Find its parity-check polynomial $h(X)$.

(b) (6%) If the received word is represented as $r(X) = X + X^3 + X^6$, determine

(i) the syndrome polynomial $s(X)$ for this received word, and

(ii) the decoded message polynomial $m(X)$.

6.Let $\phi_1(t) = \cos w_1 t + \cos w_2 t$, $\phi_2(t) = \cos w_1 t - \cos w_2 t$, $w_1 = \frac{3\pi}{T}$, $w_2 = \frac{4\pi}{T}$ and T be the symbol duration.

(a) (5%) Are $\phi_1(t)$ and $\phi_2(t)$ orthogonal functions? Please verify your answer.

(b) (10%) If $p_1(t) = a\phi_1(t) + b\phi_2(t)$ and $p_2(t) = a\phi_1(t) - b\phi_2(t)$ are orthonormal basis functions, specify (a, b) accordingly.

7.An FSK signal is given as:

$$s_0(t) = \sqrt{\frac{2E_b}{T}} \cos w_0 t \quad , \quad s_1(t) = \sqrt{\frac{2E_b}{T}} \cos w_1 t$$

where E_b is the bit energy, T is the bit duration and $\{\cos w_0 t, \cos w_1 t\}$ is an orthogonal basis. The received signal $x(t) = s_i(t) + w(t)$, $i = 0, 1$, and $w(t)$ is the added white gaussian noise with two-sided power spectral density $N_o / 2$

(a) (5%) Show the optimum receiver structure to detect x(t) and explain why it is optimum.

(b). (10%) Derive the corresponding bit error probability.