國 立 清 華 大 學 命 題 紙

九十三學年度 通訊工程研究所 碩士班研究生招生考試

科目 通訊系統 科號 3403 共 4 頁第 1 頁 *請在試卷【答案卷】內作答

1. Consider a wideband FM signal $s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$, where the carrier frequency f_c is much larger than the input signal bandwidth f_m. $s(t)$ can be rewritten in the form $s(t) = \mathrm{Re}\left[\tilde{s}(t)\exp(j2\pi f_c t)\right]$, where $\tilde{s}(t)$ is the complex envelop of $s(t)$.

   (a) $\tilde{s}(t)$ can be expanded in the form of a complex Fourier series $\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t)$. Find the complex Fourier coefficient c_n. [Hint: $J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp\left[j(\beta \sin x - nx)\right]dx$.] (5%)

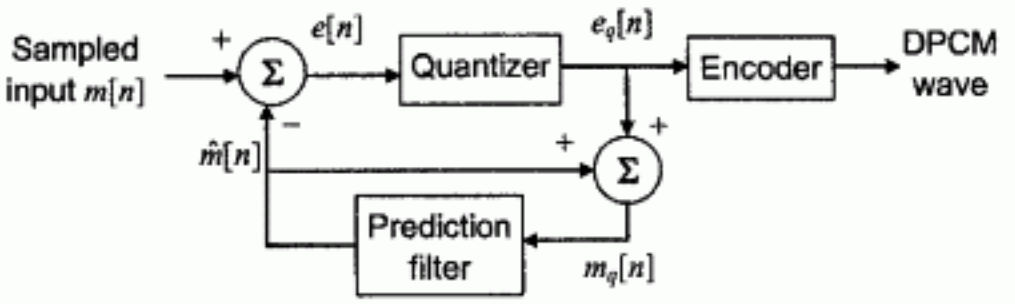
   (b) According to the results of (a), find the spectrum $S(f)$ of $s(t)$. (5%)

   (c) According to the results of (b), it is assumed that the estimated transmission bandwidth of $s(t)$ is B_T and the number of significant side frequencies is N. If the modulation index β remains unchanged and the input signal bandwidth is increased by a factor of two, i.e. $f_m' = 2f_m$, what is the estimated transmission bandwidth? (3%)

   (d) According to (c), what is the number of significant side frequencies? (2%)

   [Note: You must give detail derivations, otherwise you get no points.]

2. For a differential pulse-code modulation (DPCM) transmitter, the prediction error $e[n]$ of the input signal $m[n]$ is quantized and transmitted as shown in the following figure. The prediction error is denoted as $e[n] = m[n] - \hat{m}[n]$, where $\hat{m}[n]$ is the prediction of $m[n]$.



   (a) Design a DPCM receiver with function blocks. (5 %)

   (b) Assume that the channel is noiseless and the quantization error is $q[n]$, i.e. $e_q[n] = e[n] + q[n]$. Find the noise component in the output signal of the DPCM receiver. (5 %)

   (c) Assume that the variances of $m[n]$, $e[n]$ and $q[n]$ are σ_M^2, σ_E^2 and σ_Q^2, respectively. Find the processing gain produced by the differential quantization scheme as compared with PCM system. (5%)

3. A binary, '0' or '1', message transmitted through a noisy communication channel is received incorrectly with probability ε_0 and ε_1, respectively. Errors in different symbol transmissions are independent.

   (a) Suppose that the string of symbols '1011' is transmitted. What is the probability that all the symbols in the string are received correctly? (1%)

   (b) In an effort to improve reliability, each symbol is transmitted three times and the received symbol is decoded by majority rule. In other words, a '0' (or '1') is transmitted as '000' (or '111', respectively), and it is decoded at the receiver as a '0' (or '1') if and only if the received three-symbol string contains at least two '0's (or '1's, respectively). What is the probability that a transmitted '0' is correctly decoded? (2%)

   (c) Suppose that the channel source transmits a '0' with probability p and transmits a '1' with probability $(1-p)$, and that the scheme of part (b) is used. What is the probability that a '0' was transmitted given that the received string is '101'? (2%)

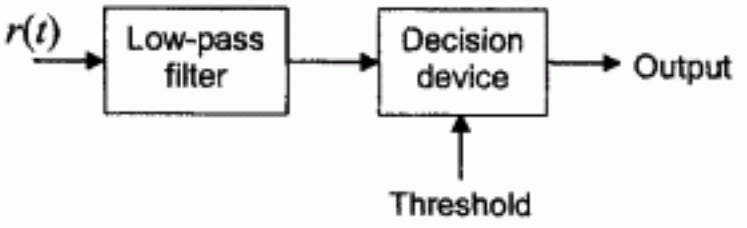
4. A discrete memoryless source emits binary equiprobable symbols at a rate of 1000 symbols per second. The symbols are sent over a bandlimited channel using 4PAM modulation. In particular, the transmitted signal $u(t)$ is given by

$$u(t)=\sum_k A_k \operatorname{sinc}\left(\frac{t}{T}-k\right)$$

where $\operatorname{sinc}(x)=\dfrac{\sin(\pi x)}{\pi x}$, T=0.002, A_k takes values in $\{-3d/2, -d/2, d/2, 3d/2\}$ and we use the mapping

$$00 \rightarrow 3d/2,\ 01 \rightarrow d/2,\ 10 \rightarrow -d/2,\ 11 \rightarrow -3d/2.$$

   (a) Assume the received signal is $r(t)=u(t)+n(t)$, where $n(t)$ is the additive white Gaussian noise with variance equal to 1. The receiver model is shown in the following figure. What is the average symbol error probability of this system as a function of d? (5%)



   (b) Suppose that a sample sequence $\{A_k : 1 \le k \le 500\}$ of transmitted symbols includes 115 appearances of $3d/2$, 130 appearances of $d/2$, 120 appearances of $-d/2$, and 135 appearances of $-3d/2$. Find the energy in the corresponding transmitted waveform $u(t)$ as a function of d. (7%)

   (c) What is the bandwidth of the waveform $u(t)$ in part (b)? (3%)

5. Assume we have three received signals $r_i(t) = \alpha_i m(t) + n_i(t), \quad i = 1,2,3$, where each α_i is a complex number, $m(t)$ is the message signal, and $n_i(t)$ is additive white Gaussian noise with variance σ_i^2. Our goal is to form a linear combination of $r_i(t)$, $i = 1,2,3$, i.e. finding β_i, $i = 1,2,3$ such that $\sum_{i=1}^{3} \beta_i r_i(t)$ has the largest signal to noise ratio (SNR). Please follow the steps below to achieve this goal.

(a) Assume the power of $m(t)$ is 1. Express the SNR of $\sum_{i=1}^{3} \beta_i r_i(t)$ in terms of α_i, β_i and σ_i, $i = 1,2,3$. (6%)

(b) With the following inequality $\left|a_1^* b_1 + a_2^* b_2 + a_3^* b_3\right|^2 \le \left(|a_1|^2 + |a_2|^2 + |a_3|^2\right)\left(|b_1|^2 + |b_2|^2 + |b_3|^2\right)$, where a_i's and b_i's are complex numbers, find the maximum value of SNR obtained in part (a). (7%)

(c) In the above inequality, the equality holds when $(a_1, a_2, a_3) = k(b_1, b_2, b_3)$, where k is any constant. Use this information to find the optimal β_i, $i = 1,2,3$ such that the maximum SNR in part (b) is achieved. (7%)

6. Consider a coherent M-ary frequency-shift keying (MFSK) digital modulated system used to transmit a block of $k = \log_2 M$ bits/signal waveform. The M signal waveforms are expressed as

$$s_i(t) = \sqrt{\frac{2E}{N_0}} \cos(2\pi f_c t + 2\pi i \Delta f t), \text{ for } i = 1 \sim M \text{ and } t \in [0, T]$$

(a) What is the minimum value of frequency separation $\Delta f = |f_{i+1} - f_i|$ such that $s_i(t)$ and $s_{i+1}(t)$ are orthogonal, i.e. $\int_0^T s_i(t) s_{i+1}(t) dt = 0$. (3%)

(b) Assuming that the carrier frequency $f_c >> \Delta f$, please determine the correlation coefficient of $s_i(t)$ and $s_j(t)$ defined by $\rho_{ij} = \dfrac{\int_0^T s_i(t) s_j(t) dt}{\int_0^T s_i^2(t) dt}$. (4%)

(c) For $M = 2$, please determine the value of Δf that minimizes the average probability of symbol error. (4%)

(d) For the value of Δf in (c), please determine the required increase in E/N_0 such that this BFSK system has the same noise performance as a coherent BPSK system. (4%)

[Hint: The average symbol error probabilities for BFSK and BPSK systems are $P_e = \frac{1}{2}\text{erfc}(\sqrt{E(1-\rho)/N_0})$ and $P_e = \frac{1}{2}\text{erfc}(\sqrt{E/N_0})$, respectively, where $\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)\, dz$.]

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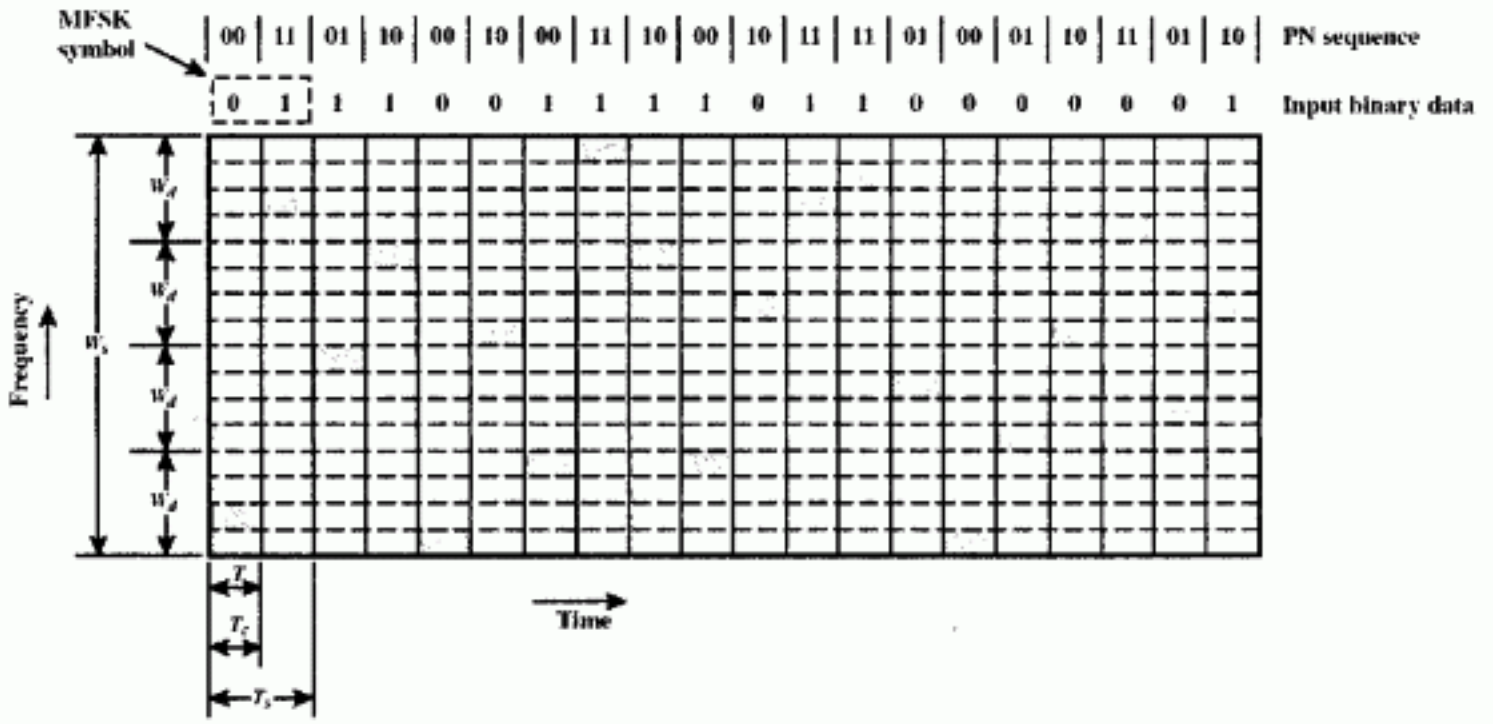
7. Consider the following frequency-hop spread spectrum (FHSS) using MFSK as shown in the following figure. The MFSK uses $M = 2^L$ different frequencies, each of bandwidth f_d, to encode the digital input L bits at a time. The input data bit rate R is 1 Mbps. For this FHSS, the MFSK signal is translated to a new frequency every T_c seconds by modulating the MFSK signal with the FHSS carrier signal. The duration of a bit is T (=1/R) seconds and the duration of a signal element is T_s seconds. Each signal element is a discrete frequency tone, and the total MFSK bandwidth is $W_d = M f_d$. Suppose there are 2^k channels allocated for the FH signal. The total FHSS bandwidth is $W_s = 2^k W_d$.



(a) What is the length (i.e. bits) of the PN sequence per hop? What is the hopping period of the system (in μs)? The system makes use of a form of FSK for digital modulation. What form of FSK is it? (3%)

(b) Is this a slow or fast FHSS system? You need to explain it briefly to get credits. (3%)

(c) Given input bit stream "0 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1" and

PN sequence "1 1 1 0 0 1 0 0 0 0 0 1 1 0 1 1 1 1 1 0 0 1 0 0 0 0 0 1 1 0 1 1",

please draw the frequency-time diagram using this FHSS system. (3%)

(d) Use the result you have obtained in (c), please re-draw the frequency-time diagram using the dehopped frequency. What is the processing gain of this FHSS system? (3%)

(e) Let the probability of error for a single user operating in the system be P_e. Please find the probability of error for N users operating in the system. Assume every user is independently frequency hopped. (3%)