

國立交通大學 93 學年度碩士班入學考試試題

科目名稱：工程數學(081)<sup>(061)</sup> 考試日期：93 年 4 月 17 日 第 1 節

系所班別：電信工程學系 組別：丙組/甲組 第 1 頁, 共 3 頁

\*作答前, 請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符!!

1. (10 分) Find the QR-decomposition of the following matrix

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 4 \end{bmatrix}$$

2. Suppose that  $T: V \rightarrow V$  is a linear operator and  $B$  is a basis for  $V$ . For any vector  $\mathbf{x}$  in  $V$ ,

$$[T(\mathbf{x})]_B = \begin{bmatrix} x_1 - x_2 + x_3 \\ x_2 - 2x_3 \\ x_1 - x_3 \end{bmatrix} \quad \text{if} \quad [\mathbf{x}]_B = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- (a) (3 分) Find  $[T]_B$ .
- (b) (5 分) Find the *kernel* of  $T$ .
3. (8 分) Let  $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$  be an orthonormal basis for an inner product space  $V$ . Show that if  $\mathbf{w}$  is a vector in  $V$ , then  $\|\mathbf{w}\|^2 = \langle \mathbf{w}, \mathbf{v}_1 \rangle^2 + \langle \mathbf{w}, \mathbf{v}_2 \rangle^2 + \dots + \langle \mathbf{w}, \mathbf{v}_n \rangle^2$ .
4. (10 分) Let  $T: R^2 \rightarrow R^2$  be a linear operator. For any  $\mathbf{z} \in R^2$ ,  $T(\mathbf{z}) = \mathbf{p}$ , where  $\mathbf{p}$  is the projection of  $\mathbf{z}$  on the line  $x = y$ . Find  $[T]$ .
5. Let  $\mathbf{u}, \mathbf{v}$  and  $\mathbf{w}$  are nonzero vectors in  $R^3$ . Answer the following questions and explain your reasoning.
- (a) (4 分) Is it true that  $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ ?
- (b) (3 分) If  $\mathbf{u} \neq \mathbf{v}$ , is it possible that  $\|\mathbf{u}\| = \|\mathbf{v}\|$ ?
- (c) (3 分) If  $\mathbf{u} \neq \mathbf{v}$ , is it possible that  $\|\mathbf{u} - \mathbf{v}\| = 0$ ?
- (d) (4 points) Is  $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + u_2 v_2 - u_3 v_3$  an inner product?

6. Answer each of the following as always true, always false, or neither. **No points if without explaining.**

(a) (3 分) If the probability density function of a random variable  $X$  is symmetric, the expectation of  $X$  is equal to zero.

**Note:** A function  $f(x)$  is symmetric if  $f(x) = f(-x)$  for all  $x$ .

(b) (3 分) The joint density function of a pair of random variables  $X, Y$  is given by

$$f_{X,Y}(x,y) = 8xy, \text{ for } 0 < x < y < 1. \text{ The } X \text{ and } Y \text{ are independent since } f_{X,Y}(x,y)$$

can be rewritten as  $8f_X(x)f_Y(y)$ , where  $f_X(x) = x$ , and  $f_Y(y) = y$ .

(c) (2 分) If two random variables have the same characteristic function, they have the same probability density function.

(d) (2 分) The number of balls which you have to choose until you get the third red balls when you put back each ball before drawing the next is a negative binomial random variable.

(e) (4 分) Consider two events as following,

*Event A.* A coin is tossed 40 times. The number of those tosses which come up heads is fewer than 10.

*Event B.* A coin is tossed until it comes up heads for the 10<sup>th</sup> time. The number of tosses needed is more than 40.

We will have the same value by using the Central Limit Theorem to calculate the probability for each event since the above two events are equivalent.

If your answer is true, justify it. Explain it if your answer is false.

7. (10 分) A thief has been placed in a jail with three doors. The door, *OneDayTrip*, leads into a tunnel which returns him to the jail after one day's travel through the tunnel. The door, *ThreeDayTrip*, leads to a similar tunnel whose traversal requires three days rather than one day. The third door, *OneWayTicket*, leads to freedom. The probability of the thief choosing the door-*OneDayTrip* is 0.25, the door-*ThreeDayTrip* is 0.5, and the door-*OneWayTicket* is 0.25 at each time he makes a choice. Find the expected number of days the thief will be imprisoned.

8. If  $X$  is the amount of money that an application engineer spends on gasoline during a day and  $Y$  is the corresponding amount of money for which she or he is paid back, the joint density of these two random variables is given by

$$f_{X,Y}(x,y) = \begin{cases} (40-x)/100x & \text{for } 20 < x < 40, x/2 < y < x \\ 0 & \text{elsewhere} \end{cases}$$

- (a) (1 分) Find the marginal density of  $X$ .  
(b) (5 分) Find the marginal density of  $Y$ .  
(c) (6 分) Find the expectation of  $Y$  without using the marginal density of  $Y$ .  
(d) (2 分) If the amount of money the application engineer spent was 32 on gasoline today, what the expected money for which he or she will be paid back is.

9. Consider the following probability density function for a random variable  $X$

$$f_X(x) = \begin{cases} x+1 & \text{for } -1 < x < 0 \\ x & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) (7 分) Let  $Y = \log_e(|X|^{-\frac{1}{\alpha}})$ , where  $\alpha$  is a positive constant. Find the probability density function  $f_Y(y)$  of  $Y$ .  
(b) (5 分) Let  $FL(y)$  denote the floor function, that is,  $FL(y)$  is the greatest integer less than or equal to  $y$ . Let  $Z = FL(Y)$ . Find the probability density function  $f_Z(z)$  of  $Z$ .