國立交通大學九十二學年度碩士班入學考試試題

科目名稱：通訊原理(062)　　考試日期：92 年 4 月 20 日 第 3 節

系所班別：電信工程學系　　組別：甲組　　

*作答前,請先核對試題、答案卷（試卷）與准考證上之所組別與考試科目是否相符!!

1. Consider the following system,

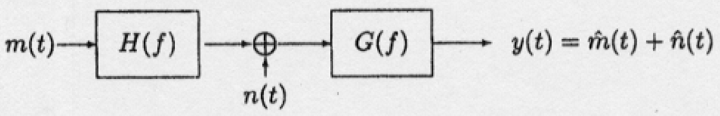
$$x(t) = \cos(2\pi f_0 t + \theta) \longrightarrow \boxed{\text{sampler}} \longrightarrow \boxed{\text{LPF}} \longrightarrow y(t)$$

where $x(t)$ is a sinusoidal input signal to an ideal sampler(no anti-aliasing filter is used) with sampling frequency f_s, followed by an ideal lowpass filter with cutoff frequency $f_s/2$. For example, if $x(t) = \cos(20\pi t + \frac{\pi}{4})$ and $f_s = 100$, then the output $y(t) = \cos(20\pi t + \frac{\pi}{4})$.

(a) (4%) If $f_0 = 100, \theta = \frac{\pi}{6}$, and $f_s = 150$, find $y(t)$.

(b) (8%) Suppose $f_0 < 1000$ is an unknown parameter to be estimated by two students. Student A uses a sampling rate $f_s = 150$ and finds that frequency of the output signal $y(t)$ is 50. Student B uses another sampling frequency $f_s = 240$ and finds the output signal's frequency is 20. Please determine the input signal's frequency f_0.

2. Consider the following system,



where $m(t)$ and $n(t)$ are two independent wide-sense stationary zero-mean Gaussian random processes with power spectral densities $S_m(f)$ and $S_n(f)$, respectively, and $H(f)$ and $G(f) = \frac{1}{H(f)}$ are the frequency responses of two linear time-invariant filters. Assume

$$S_m(f) = \begin{cases} 1, & |f| \le 3 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad S_n(f) = \begin{cases} 1, & |f| \le 1 \\ 4, & 1 \le |f| \le 2 \\ 9, & 2 \le |f| \le 3 \\ 0, & \text{otherwise} \end{cases}$$

$$H(f) = \begin{cases} \alpha, & |f| \le 1 \\ \beta, & 1 \le |f| \le 2 \\ \gamma, & 2 \le |f| \le 3 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad G(f) = \begin{cases} 1/\alpha, & |f| \le 1 \\ 1/\beta, & 1 \le |f| \le 2 \\ 1/\gamma, & 2 \le |f| \le 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) (4%) Suppose $m(t)$ is sampled at a rate of $f_s = 6$. Find the 1-dimensional probability density function(*pdf*) for $m(t)|_{t=0}$ and the 2-dimensional *pdf* for $[m(\frac{-1}{6}), m(\frac{1}{6})]$, respectively.

(b) (4%) Suppose $\alpha = \beta = \gamma = 1$, find the output SNR $\frac{E[\hat{m}^2(t)]}{E[\hat{n}^2(t)]}$.

(c) (3%) Explain the purpose of using $H(f)$ and $G(f)$.

(d) (7%) Find the optimum α^*, β^*, and γ^*, so that the output SNR can be maximized, under the constraint $\alpha^2 + \beta^2 + \gamma^2 = 3$, *i.e.*, the transmitted signal power is the same as that in (b). [Hint: Cauchy-Schwartz Inequality: $||\vec{a} \cdot \vec{b}||^2 \le ||\vec{a}||^2 ||\vec{b}||^2$ with equality holds when $\vec{a} = k \cdot \vec{b}$.]

3. Consider binary digital baseband data transmission in an AWGN channel (noise PSD $S_n(f) = \frac{N_0}{2}$) with ideal Nyquist pulse shaping function $p(t)$ (roll-off factor = 0) at a bit rate of R b/s, where $R = \frac{1}{T}$ and T is the bit period.

(a) (4%) Write down the equation and plot the waveform of $p(t)$, assuming the average transmission power is P.

(b) (4%) If the baseband channel has an impulse response of $h(t) = \delta(t) + \delta(t - T)$, find and plot the transfer function of a zero forcing equalizer.

(c) (5%) Following the channel assumption in (b), show that the precoding technique can be used at the transmitter side such that we can have a receiver without equalization. Draw the whole system block diagram and explain how it works.

(d) (4%) Following (c), find out the average bit error probability.

4. Consider binary FSK signalling in an AWGN channel (noise PSD $S_n(f) = \frac{N_0}{2}$) with two signalling tones

$$s(t) = A\cos(2\pi f_c t \pm \frac{\pi t}{T}),\ \ 0 \le t \le T,$$

where the $\pm$ sign depends on whether the transmitted bit is 0 or 1.

(a) (4%) Find out two basis functions and draw the signal space plot.

(b) (5%) Find out and plot the power spectral density of the FSK signal.

(c) (4%) Draw the block diagram of an optimum coherent receiver and find its bit error probability.

(d) (5%) Draw the block diagram of an optimum noncoherent receiver and find its bit error probability.

5. Consider a slowly flat fading channel. The received signal can be expressed as

$$x(t) = a_i R\cos 2\pi f_c t + n(t),\ \ 0 \le t \le T,$$

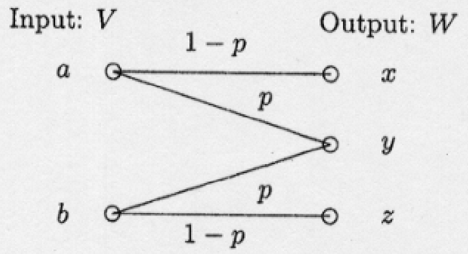
where a_i is a binary random data taking values of -1 or $+1$ with equal probability, R is a Rayleigh distributed random variable with *pdf*, $f(r) = \frac{r}{\sigma^2}e^{-\frac{r^2}{2\sigma^2}}$, $r \ge 0$, and $n(t)$ is an additive white Gaussian noise with power spectral density $\frac{N_0}{2}$.

(a) (4%) Find the bit error probability for a specified value of R.

(b) (5%) Find the average bit error probability over all values of R.

(c) (4%) Discuss the effects of this Rayleigh random amplitude R and give methods to reduce its effect.

6. Consider a channel with transition probability $P(x|a) = P(z|b) = 1-p$ and $P(y|a) = P(y|b) = p$ as shown in the figure:



(a) (4%) Determine the average mutual information $I(V;W)$.

(b) (5%) Determine the channel capacity.

7. The generator of a convolutional encoder is described by $g_1 = [1\ 1\ 0], g_2 = [1\ 0\ 1], g_3 = [1\ 1\ 1]$.

(a) (4%) Draw the encoder block diagram.

(b) (4%) Draw the state diagram.

(c) (5%) Find the transfer function from the modified state diagram and determine the free distance.