*作答前，請先核對試題、答案卷（試卷）與准考證上之所組別與考試科目是否相符!!

1. (5%) Draw the block diagram of a superheterodyne receiver and explain its advantages.
2. (4%) Draw the block diagram of a phase-lock loop and show that it can be used as a demodulator of an FM signal.
3. (9%) A baseband message $m(t)$ with Fourier transform $M(f)$ and bandwidth B is sampled using a flat-top sampler. The sampled pulse width is one half of its sampling period T_s. (a) Write out the equation of sampler output. (b) Determine the Fourier transform of the sampler output. (c) What is the method to completely recover $m(t)$ from the sampler output.
4. (12%) A real baseband signal $m(t)$ has its Fourier transform $M(f)$ being equal to a rectangular window function in a bandwidth of B.

   (a) Plot the Fourier transform of $m(t)+j\hat{m}(t)$ where $\hat{m}(t)$ is the Hilbert transform of $m(t)$

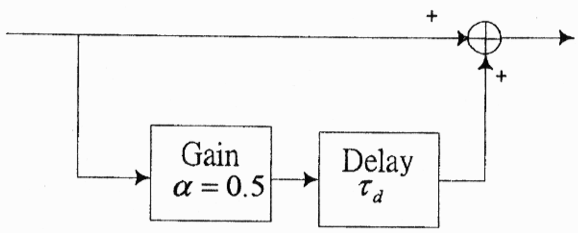
   (b) What is the Hilbert transform of $m(t)\cos 2\pi f_c t$ where $f_c >> B$

   (c) What is the Fourier transform of $[m(t)+j\hat{m}(t)]e^{2\pi f_c t}$ where $f_c >> B$

   (d) Plot the Fourier transform of $m(t)+0.2m^2(t)$.
5. (6%) For a linear system as shown below

   (a) what are its impulse response and transfer function?

   (b) how do you construct a receiver to completely recover a signal $m(t)$ from the output of the linear system when $m(t)$ is its input?



6. (a) (4%) Define the Gaussian Q-function. Find Q(0).

   (b) (4%) Derive the characteristic function of a Gaussian random variable with mean M and variance σ^2. (hint: **characteristic function of random variable X,** $\Phi_X(\upsilon) \equiv E\left[e^{j\upsilon X}\right]$)

   (c) (4%) Use the result of (b) to show that the sum of two independent Gaussian random variables is still a Gaussian random variable.

   (d) (4%) Use the result of (c) to show that the output sample of a linear time-invariant system is a Gaussian random variable when the input to the system is a Gaussian noise.

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7. Consider an FM signal x(t) in the presence of AWGN, assuming $x(t)$ has an amplitude of A_c, modulating signal is $\cos(2\pi f_m t)$, peak frequency deviation is $f_d = 9 f_m$, carrier frequency is f_c, and the AWGN has a two-sided power spectral density of $S(f)=N_0/2$.

(a) (4%)Specify the predetection bandpass filter. Derive the **predetection** SNR.

(b) (4%)Write down the equation of the received signal plus noise after passing through the predetection filter.

(c) (4%) **Derive** the **postdetection** SNR of an FM discriminator receiver, assuming the predetection SNR is 20 dB.

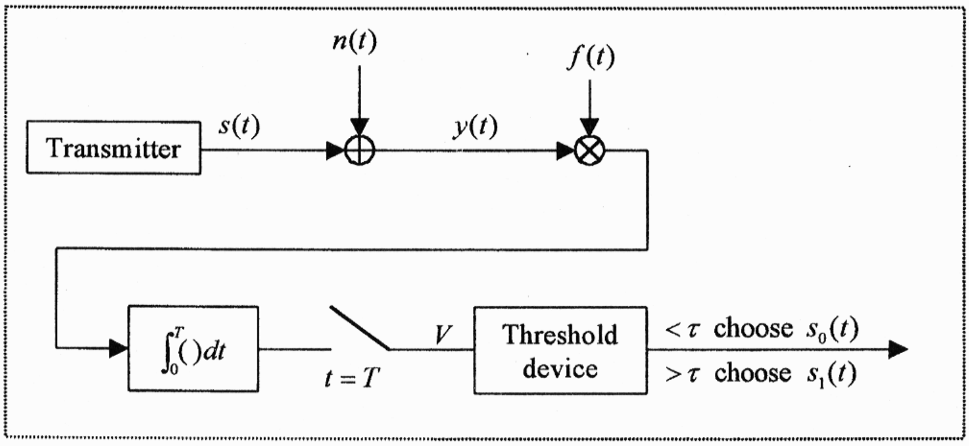
(d) (4%) Draw a phasor diagram and explain the generation mechanism of a negative noise spike (click).

8. A binary digital communication system employs the two signals

$$\begin{cases} s_0(t) = 0, & 0 \le t < T; \\ s_1(t) = g(t), & 0 \le t < T, \end{cases}$$

for transmitting the 0 or 1 information. This is called *on-off signaling*.

(a) (6%) The demodulator cross-correlates the received signal $y(t)$ with $f(t)$ and samples the output of the correlator at $t = T$ (see the figure below).



Determine the optimal threshold that minimizes the probability of error for an AWGN channel (assuming $n(t)$ is a zero-mean white Gaussian noise with two-sided power spectral density $N_0/2$ and the two signals are equally probable).

(b) (6%) For fixed $g(t)$ and a given constraint on transmitted power (i.e., $\int_0^T g^2(t) \le S$), what is the function $f(t)$ that yields the lowest error probability? Justify your answer. (Hint:by Schwartz inequality)

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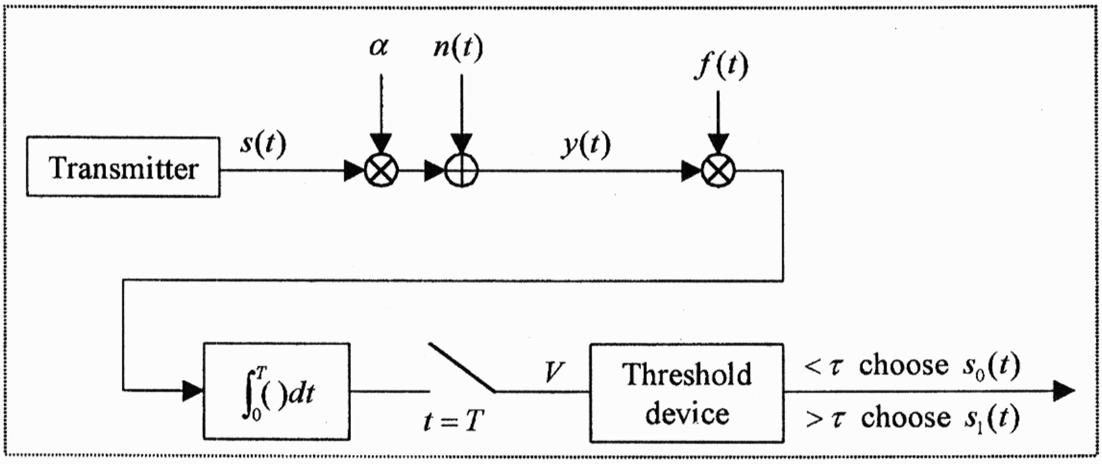
(c) (4%.) Suppose the channel is a flat fading channel as shown below, where $\Pr\{\alpha=1\}=1-\Pr\{\alpha=0\}=p$, and $n(t)$ is a zero-mean white Gaussian noise with two-sided power spectral density $N_0/2$. Determine the optimal threshold that minimizes the probability of error, assuming that the two signals are equally probable.



9. Assume that a data stream $d(t)$ consists of a random sequence of +1 and −1 each of T seconds in duration. The autocorrelation function of such a sequence is:

$$R_d(\tau)=\begin{cases}1-\left|\dfrac{\tau}{T}\right|, & |\tau|\le T\ ; \\ 0, & \text{otherwise.}\end{cases} \qquad (1)$$

(a) (6%) Find the power spectral density of an ASK-modulated signal given by

$$s_{ASK}(t)=d(t)\cos(2\pi f_c t+\theta),$$

where θ is uniformly distributed over $[0,\ 2\pi)$. (Hint: Fourier transform pair is $H(f)=\int_{-\infty}^{\infty}h(t)e^{-j2\pi ft}dt$ and $h(t)=\int_{-\infty}^{\infty}H(f)e^{j2\pi ft}dt$. In addition,

$$\int_{-\infty}^{\infty}h(t)\cos(2\pi f_c t)e^{-j2\pi ft}dt=\frac{1}{2}[H(f-f_c)+H(f+f_c)].)$$

(b) (6%) Compute the power spectral density of a PSK-modulated signal given by

$$s_{PSK}(t)=\sin\left[2\pi f_c t+\theta+\frac{\pi}{2}d(t)\right],$$

where θ is uniformly distributed over $[0,2\pi)$. (Hint: Expand the PSK signal into its carrier and modulation components.)

(c) (4%) By sampling the random sequence $d(t)$ with sampling period T, a discrete-random sequence $d_1,d_2,d_3,\ldots$ of +1 and −1 is formed. Please calculate the entropy of d_i.