1. Given a signal $s(t) = e^{-|t|/\tau}$, let $s_1(t) = \begin{cases} s(t) & , \ |t| \le T \\ 0 & , \ \text{elsewhere} \end{cases}$,
and $s_\delta(t) = s_1(t) \cdot [\sum_{n=-\infty}^{\infty} \delta(t - nT_s)]$.
   (a) (3%) Derive and plot $S(f)$, the Fourier transform of $s(t)$.
   (b) (3%) Derive and plot $S_1(f)$, the Fourier transform of $s_1(t)$.
   (c) (3%) Derive and plot $S_\delta(f)$, the Fourier transform of $s_\delta(t)$.
   (d) (3%) How should you choose T and T_s in order to make $S_\delta(f)$ in baseband as close to $S(f)$ as possible? Why?

2. The characteristic function of a random variable X is defined as $M_x(\lambda) = E[e^{i\lambda X}]$.
   (a) (5%) Find the characteristic function of a Gaussian random variable Y with mean μ and variance σ^2.
   (b) (5%) Apply the result obtained in (a) to find the 2nth center moment of Y defined by $E[(Y-\mu)^{2n}]$.

3. AWGN is assumed in the following.
   (a) (7%) Draw the block diagram of a coherent Double-Sideband(DSB) demodulator and find out the demodulated output SNR.
   (b) (5%) Write out the FM modulated signal for stereophonic FM broadcasting in terms of the carrier frequency f_c, $m_L(t) + m_R(t)$, $m_L(t) - m_R(t)$, subcarrier frequency f_{sc}, and pilot frequency f_p, where $m_L(t)$ and $m_R(t)$ are the left and right channel audio messages respectively.
   (c) (5%) Show that the FM demodulated output noise power spectral density is proportional to f^2.
   (d) (5%) Find the ratio between the demodulated output SNR for message $m_L(t) - m_R(t)$ and the demodulated output SNR for message $m_L(t) + m_R(t)$.
   (e) (5%) Draw the block diagram of a superheterodyne stereophonic FM broadcasting receiver. Explain how to avoid the image signal.

4. One of two equally likely signals is transmitted and received in additive Gaussian noise $n(t)$. The signals are $s_1(t) = 4 = -s_2(t)$, the bit period being 1 msec long. Eight equally-spaced independent samples of the received signal $v(t) = \{s_1(t) \text{ or } s_2(t)\} + n(t)$ are taken and constitute an 8-dimensional vector $\underline{v} = [v_1, v_2, \ldots, v_8] = [\underline{s}_1 \text{ or } \underline{s}_2] + \underline{n}$, where $\underline{s}_1 = [4, 4, \ldots, 4] = -\underline{s}_2$, $\underline{n} = [n_1, n_2, \ldots, n_8]$, and n_i's, $i = 1, 2, \ldots, 8$, are independent zero-mean Gaussian noise with variance$= 2$.
   (a) (4%) For the observed data $\underline{v}$, what is the optimum detector?
   (b) (4%) According to (a), if $\underline{v} = [4.5, 0, -1.5, 2, -6, 10, 1, -4]$, which signal would you decide?
   (c) (7%) What is the probability of error for the optimum detector in (a)?

5. (10%) A sample of a polar signal of amplitude ± 1 is perturbed by a random noise n with probability density function $f(n) = \frac{3}{32}(4 - n^2)$, $|n| \leq 2$. Find the minimum probability of error if the *a priori* probabilities are $P_{+1} = \frac{2}{5} = 1 - P_{-1}$. What is the decision threshold?

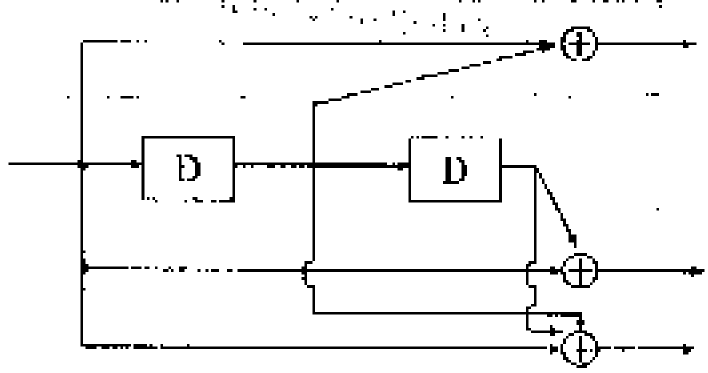
6. Consider coherent BPSK and QPSK transmission systems in AWGN ($S_n(f) = N_0/2$) and the received signals are optimally detected.

   (a) (4%) If both systems transmit at the same data rate R and the same transmission power P, write out their bit error probabilities. Which system is better? Why?

   (b) (4%) If both systems transmit at the same null-to-null bandwidth B and the same transmission power P, write out their bit error probabilities. Which system is better? Why?

   (c) (5%) Derive and plot the power spectral density of a BPSK modulated signal at a data rate of R bits/sec.

7. For the convolutional code encoder shown below,



   (a) (5%) Define the encoder state and depict the corresponding encoder state diagram.

   (b) (5%) What is the resulting codeword if the input sequence = (1 1 0 0 1 0 1 0 1).

8. (5%) Find a **ternary**(*i.e.*, code symbols are 0, 1, and 2) Huffman code for the following source:

Source Alphabet	Probability
A	.275
B	.225
C	.2
D	.175
E	.125