1.(a) (4%) Consider the system $y(t) = -\int_{-\infty}^{\infty} x(-\tau)h(t+\tau)d\tau$ where $x(t)$ and $y(t)$ represent the input and output signals respectively. Suppose $x(t) = \text{sinc}^2(t)$, $h(t) = \text{sinc}(t)$ and $\text{sinc}\,x \equiv \sin \pi x / \pi x$.

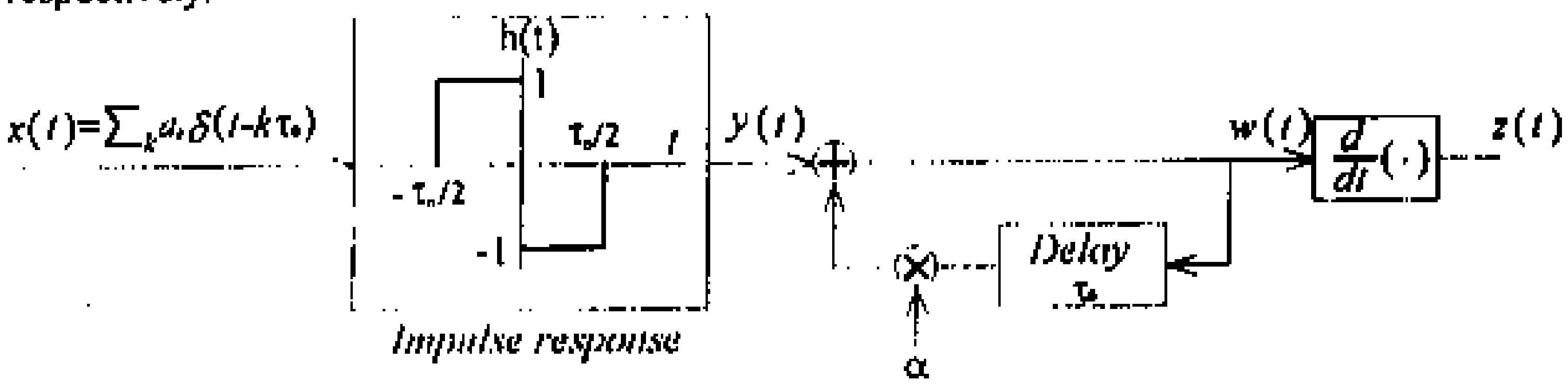
(1) Is this system linear time-invariant? Justify your answer

(2) Find and sketch the Fourier transform of the output.

(b) (4%) Redo (a) if $y(t) = \int_{-\infty}^{\infty} [x(\tau) + x(\tau - 1)]h(\tau - t)d\tau$

(c) (5%) Redo (a) if $y(t) = \int_{-\infty}^{\infty} x(\tau)h(2t+\tau)d\tau$

2.(12%) Information digits $a_k$ which can take on values 1 or -1 independently with equal probability are processed as shown below ($0 < \alpha << 1$). Derive the power spectral densities of $y(t)$, $w(t)$ and $z(t)$, respectively.

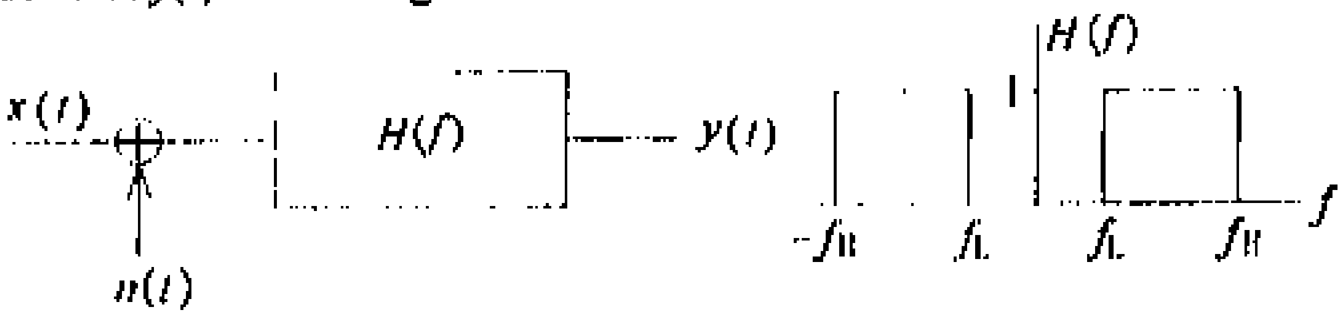



3. Let $m_1(t), m_2(t)$ : baseband signals with same bandwidth $W$ and same power $P_m$

$n(t)$ : Additive White Gaussian Noise(AWGN) with two-sided power spectral density $N_0/2$

$\omega_c$ carrier frequency$=2\pi f_c$, $f_c >> W$.

(a) (5%) Let $x(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$. Find $f_L$ and $f_H$ (as shown in the following figure) such that $y(t)$ has the largest SNR. What is the SNR?




(b) (5%) Redo (a) for $x(t) = m_1(t)\cos\omega_c t + \hat{m}_1(t)\sin\omega_c t$, where $\hat{m}_1(t)$ is the Hilbert transform of $m_1(t)$.

(c) (5%) Redo (a) for $x(t) = m_1(t) + m_2(t)\cos\omega_0 t$, where $\omega_0 = 4\pi W$.

(d) (5%) Redo (a) for $x(t) = A\cos[\omega_c t + k_f\int_{-\infty}^{t} m_1(\tau)d\tau]$, where $A$ and $k_f$ are constants.

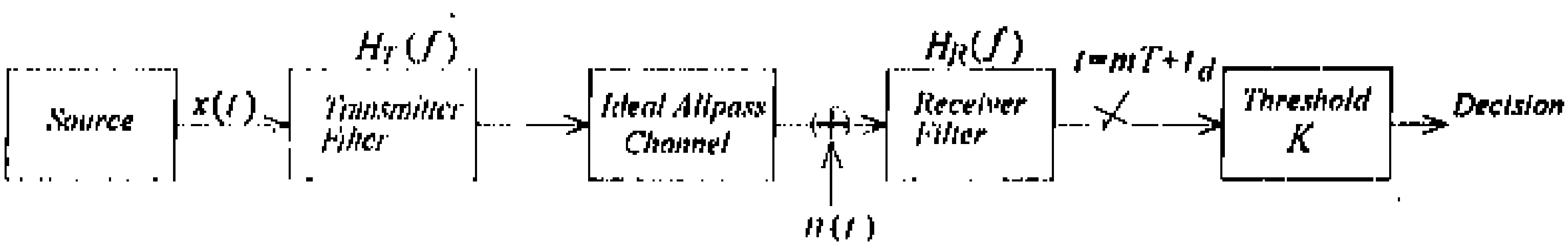
(e) (5%) Draw the block diagram of a stereo FM receiver for the FM signal $x(t) = A\cos\{\omega_c t + k_f\int_{-\infty}^{t}[m_1(\tau) + m_2(\tau)\cos\omega_0\tau]d\tau\}$, where $\omega_0 = 4\pi W$. Find the ratio of demodulated SNR for $m_1(t)$ and demodulated SNR for $m_2(t)$.

4.(a) (4%) Draw the block diagram of an optimum DPSK receiver. Explain its operation.

(b) (4%) Mathematically prove that the optimum DPSK receiver can be used to detect a DPSK signal and the detection operation is not sensitive to the local carrier phase offset.

(c) (5%) Analyze the probability of error performance of the optimum DPSK receiver in additive white Gaussian noise with two-sided power spectral density $N_0/2$.

5.Given the following binary data transmission system.




where $x(t)=\sum_{k=-\infty}^{\infty} a_k\delta(t-kT)$, $a_k=\pm 1$, and $a_k$'s are independent and equally likely. Assume $n(t)$ is additive white Gaussian noise with two-sided power spectral density $N_0/2$.

(a) (4%) Describe how the Nyquist's pulse shaping criterion can be used in designing $H_T(f)$ and $H_R(f)$.

(b) (4%) Describe how $t_d$ and decision threshold $K$ should be chosen.

(c) (4%) Determine the optimum $H_T(f)$ and the optimum $H_R(f)$ such that minimum probability of error can be achieved by the system.

6.(10%) A binary Hamming code C has parity-check matrix

$$H=\begin{bmatrix}1&1&1&1&0&0&0\\1&1&0&0&1&1&0\\1&0&1&0&1&0&1\end{bmatrix}$$

(a) Find a generator matrix for C. (b) Decode the two received words $\mathbf{r}_1=(1110000)$ and $\mathbf{r}_2=(1111000)$.

7.(10%) Sketch the block diagrams for the modulator and demodulator of a coherent 8-PSK system.

8.(5%) Consider the six-symbol source $\{x_1, x_2, \cdots, x_6\}$ whose a priori probability distribution is $\{0.275, 0.25, 0.2, 0.125, 0.1, 0.05\}$. Construct a binary Huffman code for this source.