

1. (13 points) It is a result of statistical mechanics that the internal energy of an ideal gas is

$$U = U(S, V) = \alpha N k_B \left(\frac{N}{V} \right)^{\frac{2}{3}} \exp\left(\frac{2S}{3Nk_B} \right)$$

Where α is a constant, k_B is the Boltzmann constant and the other symbols have their usual meanings. (a) Show that the equation of state $PV=nRT$ follows from this equation. (b) Obtain $G(T, V)$, where G is the Gibbs free energy.

2. (11 points) The low temperature molar specific heat of diamond varies with temperature as :

$$c_v = 1.94 \times 10^3 (T/\theta)^3 \text{ J mol}^{-1} \text{ K}^{-1}$$

Where the Debye temperature $\theta = 1860 \text{ K}$. What are the (a) entropy change, and (b) internal energy change of 1 g of diamond when it is heated at constant volume from 4K to 300K? (The atomic weight of carbon is 12.)

3. (16 points) Some iron parts are being quenched (rapidly cooled) at constant pressure from 510°C to 10°C by immersing them in water, which is maintained at 10°C by a refrigerator. The iron is being fed into the water at a rate of 1 kg per minute. The refrigerator operates in an environment at 35°C . (a) What is the rate of heat given to the water by the iron? (b) What is the minimum power rate, in kilowatts (kW), of the motor required to operate the refrigerator? (c) What is the rate of entropy change in the universe for this process?

DATA For iron: Heat capacity, c_p , is $24 \text{ J mol}^{-1} \text{ K}^{-1}$. Atomic weight is 56 g/mole.

4. (10 points) (a) Derive the relation $\left(\frac{\partial c_p}{\partial P} \right)_T = -T \left(\frac{\partial^2 V}{\partial T^2} \right)_P$ (b) How will the c_p of a solid vary with the pressure at constant temperature?

5. (a) Derive the Gibbs-Duhem equation,

$$X_1 d\bar{G}_1 + X_2 d\bar{G}_2 + \dots = 0$$

(b) What are the major applications of this Gibbs-Duhem equation?

<20 pts>

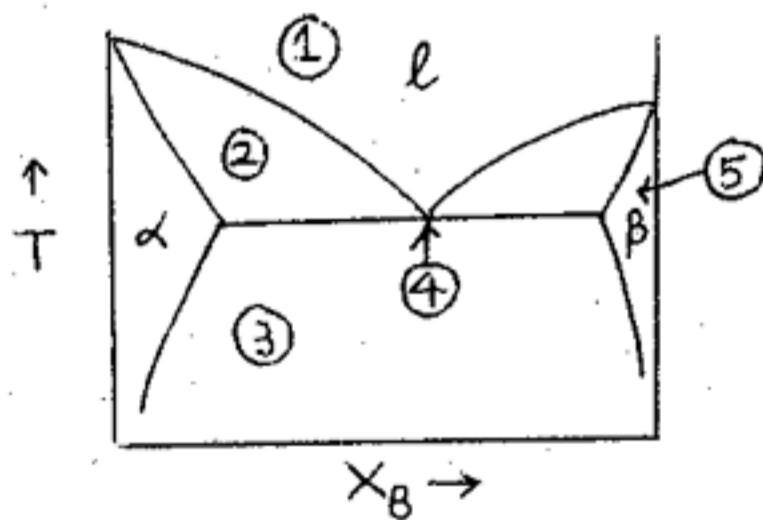
6. (a) What is a regular solution?

(b) How do you experimentally determine whether a solution is a regular solution?

<15 pts>

7. (a) Derive the phase rule, $f = r - \phi + 2$

(b) Determine the degrees of freedom in the regions shown in the following phase diagram:



<15 pts>