



豪豬筆記

HH0143 After the BIG BANG.

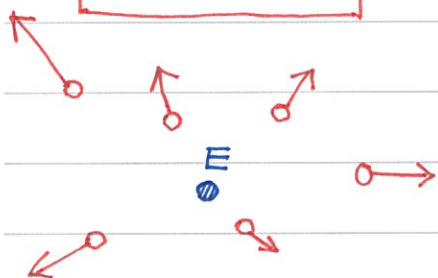
It may seem very abstract to ask when and how our universe comes to existence. But, we have a pretty theory — BIG BANG theory now with supporting experimental evidences.

① Hubble's law. Through extensive observations, Hubble found that the galaxies are moving away from one another. Taking Earth as the origin, the recessional velocity v is proportional to the distance d ,

$$v = H d$$

H : Hubble parameter

$$H^{-1} \approx 12 \sim 15 \times 10^9 \text{ years}$$



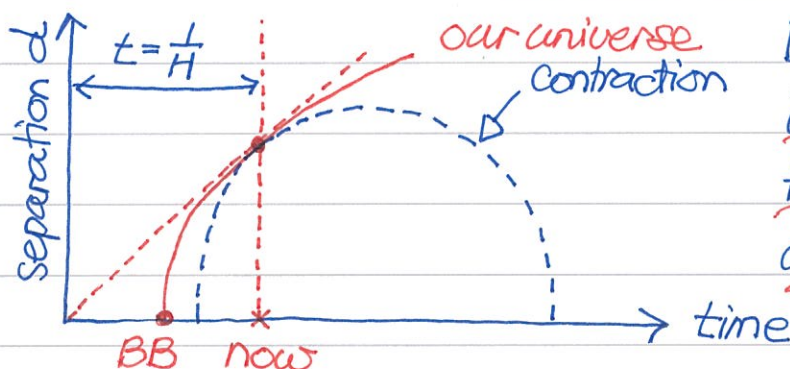
ALL stars are flying away from E ☺

The Hubble parameter has the dimension of T^{-1} . The recessional velocity can be measured easily by the Doppler shift but the distance is harder to determine.

Hubble's law can be easily explained by the BIG BANG theory. If we assume that the universe has been expanding at the same rate since its formation, the distance of a typical galaxy d is related to the age of universe t ,

$$d = v t$$

$$t = \frac{1}{H} \approx 12 \sim 15 \times 10^9 \text{ years}$$



Due to gravitational int, it is more likely that the expansion slows down as the universe expands.



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⑦ CMB - Cosmic microwave background.

In 1965, Penzias and Wilson found the strange and annoying "hiss" background no matter where they pointed their microwave antenna.

Detail analysis shows that it agrees with Planck's radiation law @ $T \approx 2.725 \text{ K}$.

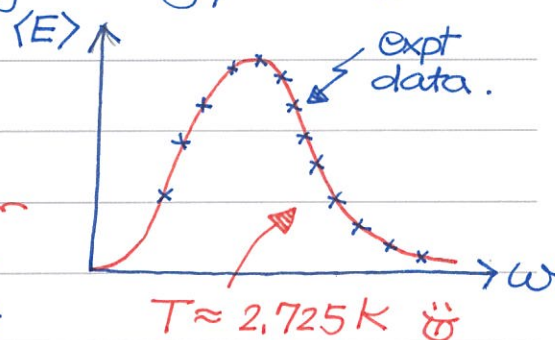
In HH0130, we derived the average energy of photons (with frequency ω) in thermal equilibrium,

$$U(\omega) = \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}$$

In 3D, the modes of photons with frequency ω is proportional

to $4\pi k^2 \sim \omega^2$. Thus, the average energy at different frequencies take the form,

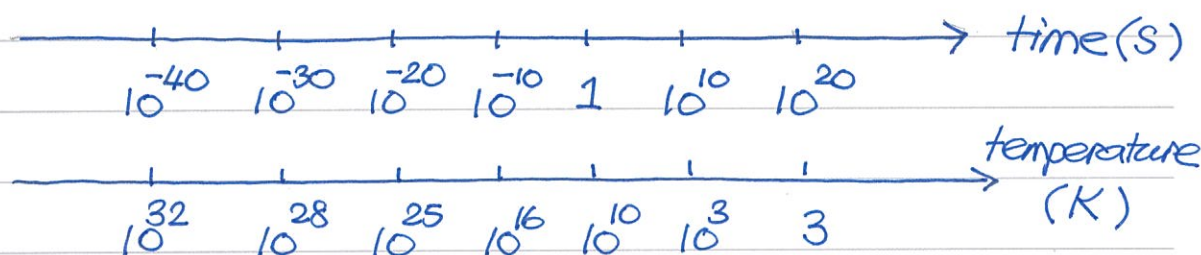
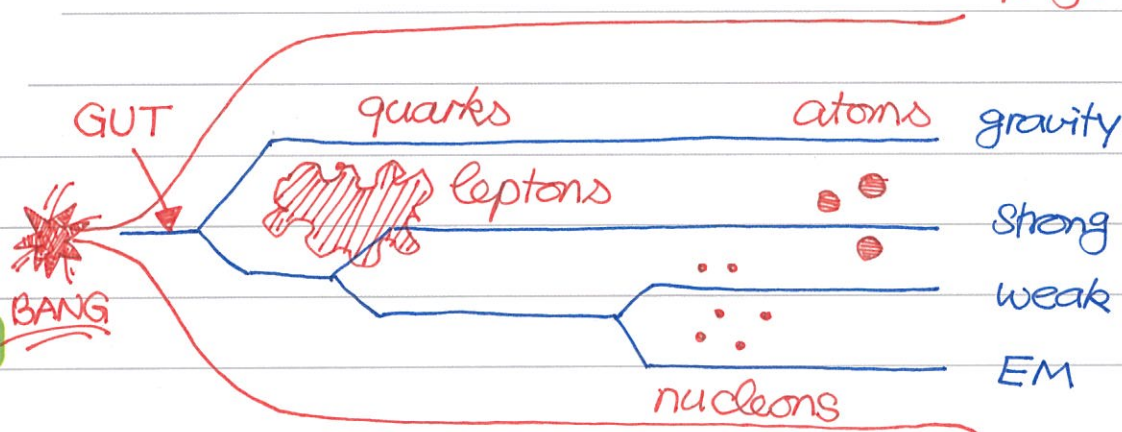
$$\langle E \rangle \propto 4\pi k^2 U(\omega) \propto \frac{\omega^3}{e^{\hbar\omega/kT} - 1}$$



Measurements of the CMB intensity in various directions show that it is almost isotropic. The CMB does not seem to come from any particular source but instead fills the entire universe more or less uniformly ☺

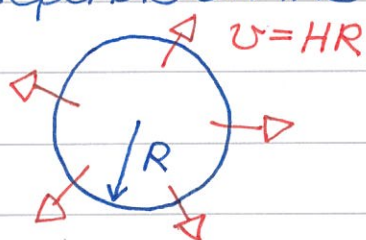
⑦ BIG BANG Cosmology. The theory in best agreement with the Hubble law and the CMB radiation is the BIG BANG cosmology. According to the theory, the universe began around 10^{10} years ago in a state of extreme density and temperature. As the universe expands, its temperature cools down — materials in different forms start to emerge ☺





The above graph is a brief history of our universe. At the very beginning, all 4 interactions were united (the so called Grand Unification Theory, GUT). As the temperature cools down, quarks + leptons $\rightarrow$ nucleons $\rightarrow$ atoms. Once the atoms form, the thermalized photons become decoupled @ $T \approx 3000$ K ($t \approx 700,000$ years old). Radiation no longer dominated our universe and different forms of neutral matters started to show up: atoms, molecules, gas clouds, stars and galaxies. The decoupled photons cool down and explain the CMB. Amazing, isn't it? The CMB tells us a piece of ancient history about our universe.

Will the universe keep expanding forever? The fate depends on the density of the universe.



Choose the Gauss surface as shown and compute the escape velocity by energy conservation.





At critical density $\rho = \rho_c$, the velocity $v = HR$ happens to be the escape velocity,

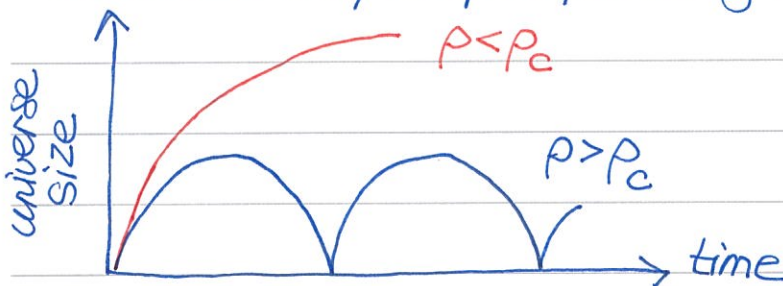
$$\frac{1}{2}mv^2 = \frac{GmM}{R}, \quad M = \frac{4}{3}\pi R^3 \rho_c$$

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Solve for the critical density ρ_c ,

$$\rightarrow \rho_c = \frac{3v^2}{8\pi GR^2} = \frac{3H^2}{8\pi G} \approx 10^{-29} \text{ g/cm}^3$$

The luminous matter in galaxies gives a universe density about $5 \times 10^{-33} \text{ g/cm}^3$. Inclusion of dark matter increases the total density to about $0.1 \rho_c$. So it seems that our universe may keep expanding forever ☹️!!!



Some scientists propose exotic particles to make up the missing mass so that our universe has the chance to be closed.

Some scientists argue that a tiny neutrino rest mass (just 20 eV or so) will do the job to "close" the universe. The question of open/closed universe is still open at this point ☹️

① Cosmological constant Λ and dark energy. A big surprise was found in 1998 — the expansion of our universe is not slowing down but is accelerating! To explain this unexpected acceleration, physicists propose some mysterious energy of the vacuum — the dark energy. We shall start the story from Einstein back in 1917 — cosmological constant Λ .





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To maintain a static universe, Einstein added a term to his famous equation:

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{spacetime curvature}} = \underbrace{\frac{8\pi G}{c^4} T_{\mu\nu}}_{\text{attraction}} - \underbrace{\Lambda g_{\mu\nu}}_{\text{repulsion.}}$$

After Hubble observed the expansion of our universe, Einstein removed the Λ term and called it "the greatest blunder" in his scientific career ☹☹☹

For non-zero Λ , even in the absence of matter ($T_{\mu\nu} = 0$), it gives rise to some spacetime curvature and thus can be viewed as some sort of energy density of the vacuum. For positive Λ , it speeds up the expansion of the universe. At the beginning of the BIG BANG, the matter is dense and Λ is not important. But, because dark energy from Λ remains constant even when space expands, it becomes important when the matter density becomes thin. The $\Lambda \neq 0$ term gives rise to noticeable accelerating expansion of our universe. ☺



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