HH0140 How Can We Date Fossils?

The method of radiocarbon dating was developed in 1947 by Libby (eventually won the 1960 Nobel Prize in chemistry for this work). To understand this method, we need to understand nuclear properties.

⊘ Nuclear binding energy. Nuclei are made up of protons and neutrons. Take carbon as example.

$$\boxed{{}^{12}C \rightarrow \begin{array}{l} \text{proton number } Z = 6 \\ \text{neutron number } N = 6 \end{array}}$$

← also called atomic #.

↑ mass number $Z+N$

The carbon has radioactive isotope ${}^{14}C$, i.e. $Z=6$ and $N=8$. Because the electron number of the atom is also Z, isotopes (same Z, different N) share similar chemical properties.

Nucleons (protons & neutrons) are bound together by strong interactions. Or, in modern terminology, the nuclear attraction is mediated by gluons. The atomic mass (not the nuclear mass) is measured in units of u,

n^0 ∿∿∿ n^0

p^+ ∿∿∿ n^0

p^+ ∿∿∿ p^+

↑ gluons

$$\boxed{\text{mass of } {}^{12}C \equiv 12\,u} \rightarrow 1\,u = 1.66\times10^{-27}\,kg$$

In nuclear physics, Einstein's formula $E=mc^2$ is very helpful. It is quite often to use the equivalent energy of 1 atomic mass unit.

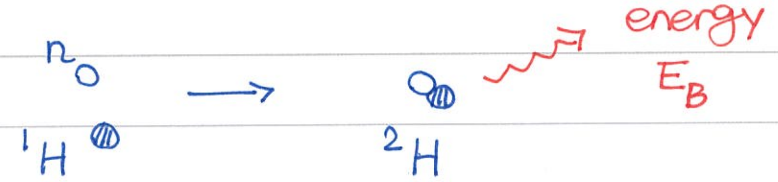
$$E = mc^2 = (1.66\times10^{-27}\,kg)\times(2.9979\times10^{8}\,m/s)^2$$

$$= 931.5\,MeV \leftarrow 1\,MeV = 10^6\,eV.$$

Let us try to estimate the binding energy of the deuteron ^2H. One can look up the masses

$$\boxed{m_n = 1.008665\,u \qquad m(^2H) = 2.014102\,u \qquad m(^1H) = 1.007825\,u}$$

One finds that $m_n + m(^1H) > m(^2H)$. The mass conservation is not true anymore ... It is the mass-energy relation at work here.



$$\boxed{m_n c^2 + m(^1H)c^2 = m(^2H)c^2 + E_B}$$

The binding energy E_B of deuteron ^2H is

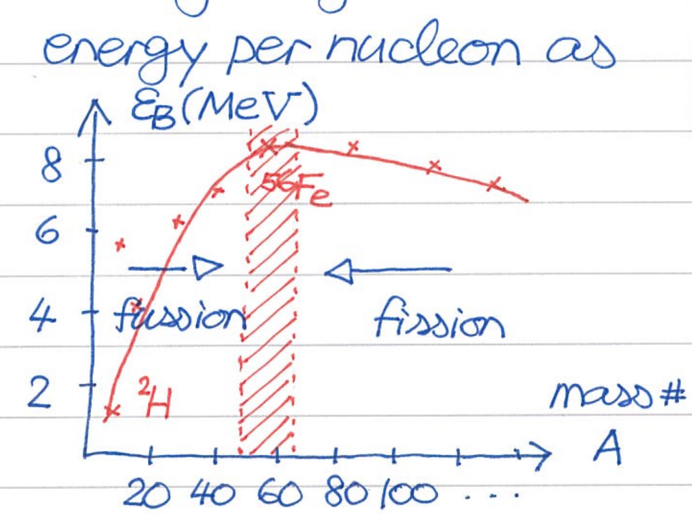
$$\boxed{E_B = \left[m_n + m(^1H) - m(^2H)\right]c^2 = 0.002388\,u \times c^2}$$

$\rightarrow$ $E_B = 2.224$ MeV ← Compared with the binding energy of H atom, just 13.6 eV

The binding energy between n^0 and p^+ in deuteron is 5 orders of magnitude larger than that between e^- and p^+ in hydrogen atom!

We can perform the same calculations to estimate the binding energies of all nuclei. Define the binding energy per nucleon as

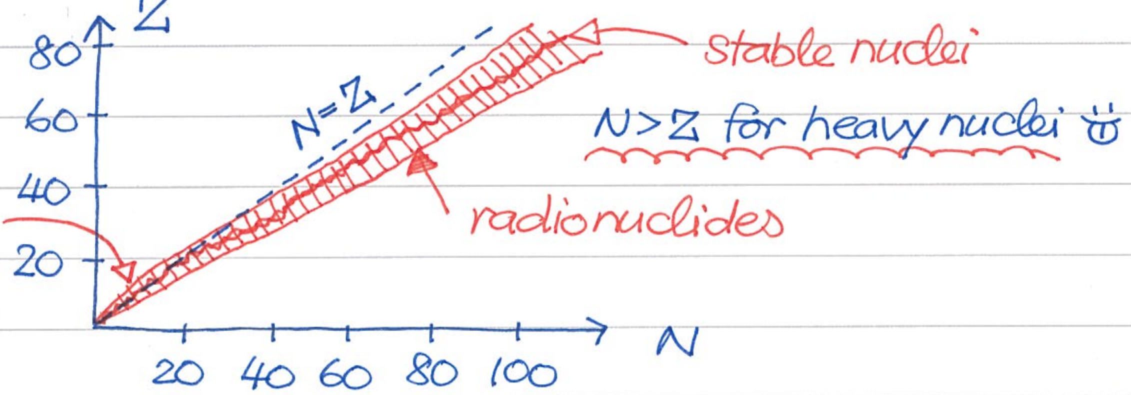
$$\boxed{\varepsilon_B \equiv \frac{E_B}{A} = \frac{E_B}{Z+N}}$$



The binding energy per nucleon reaches maximum in the regime $A \sim 50-80$ which separates fussion and fission reactions.

豪豬筆記

The stability of nuclei also depends on the ratio of Z/N. Plot all stable nuclei and their radioactive isotopes together.



The above trend is reasonable because of the Coulomb repulsion between protons → $N > Z$ for heavy nuclei

⊘ Radioactive decay. Keep in mind that all elements beyond H and He were made in nuclear reactions in the interior of stars or in explosive supernovas. Yes, we are basically "star dusts" Some nuclei are stable while some are radioactive. The age of solar system is about 4.5×10^9 yrs. Most radioactive nuclei have half-lives shorter than this. But, some nuclei have longer half-lives,

$$^{238}U : t_{1/2} = 4.5\times10^9 \text{ yrs}, \quad ^{232}Th : t_{1/2} = 1.4\times10^{10} \text{ yrs}$$

These are the natural radioactivity in our environment.

Let us investigate the survival probability $P(t)$ of some radioactive nucleus. At $t=0$, we start with $P(0)=1$.

$$P(t+dt) = P(t)\underbrace{[1-\gamma dt]}_{\text{Survival prob. in } (t,\,t+dt).} \longrightarrow \boxed{\frac{dP}{dt} = -\gamma P}$$

With the initial condition $P(0)=1$, the solution is

$$\boxed{P(t)=P(0)\,e^{-\gamma t}=e^{-\gamma t}}$$

generalize to the N_0 nuclei @ $t=0$.

Suppose the decay processes of the nuclei are independent. The remaining nuclei at later time,

$$\boxed{N(t)=N_0\,P(t)=N_0\,e^{-\gamma t}}$$

the decay follows a simple exponential.

The half life $t_{\frac{1}{2}}$ is defined in the following way,

$$P(t_{1/2})=\frac{1}{2}\longrightarrow\frac{1}{2}=e^{-\gamma t_{1/2}}, \qquad \boxed{t_{1/2}=\frac{1}{\gamma}\ln 2}$$

In biosphere, the ratio of ^{14}C to ^{12}C is approximately constant,

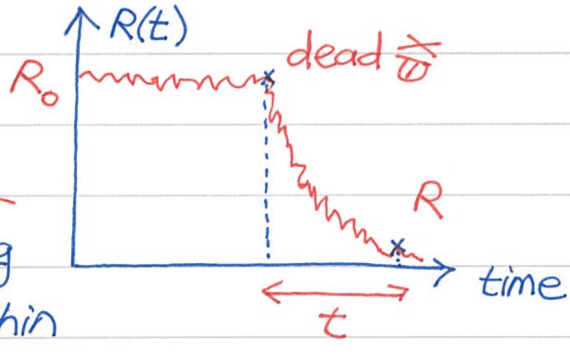
$$R_0\equiv\frac{^{14}C}{^{12}C}=1.3\times10^{-12}$$

Living organisms maintain this ratio by aspiration of CO_2 or by eating plants that have done so. But, when it dies, it stops absorbing ^{14}C and the decay process begins,

$$R=R_0\,e^{-\gamma t}\longrightarrow\frac{R}{R_0}=e^{-\gamma t}\qquad t=-\frac{1}{\gamma}\ln\left(\frac{R}{R_0}\right).$$

Rewrite the answer in half-life $t_{1/2}=5730$ yrs,

$$\boxed{t=\frac{t_{1/2}}{\ln 2}\ln\left(\frac{R_0}{R}\right)}\quad\Rightarrow$$



By measuring the ratio R, one can extract the time t. Carbon dating is quite helpful for fossils within 10^5 years. For longer period of time, we can use

$$^{238}U\longrightarrow{}^{206}Pb\qquad t_{1/2}=4.5\times10^{9}\text{ yrs}$$

$$^{40}K\longrightarrow{}^{40}Ar\qquad t_{1/2}=1.3\times10^{9}\text{ yrs}$$

Q: How can we estimate the age of our solar system?

The decay process discussed here can be classified into three types: α decay, β-decay, γ-decay.

α decay $\boxed{{}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He}$

Slightly more complicated for β-decay,

$$\boxed{\begin{aligned} {}^{A}_{Z}X &\longrightarrow {}^{A}_{Z+1}Y + e^{-} + \bar{\nu} \\ {}^{A}_{Z}X &\longrightarrow {}^{A}_{Z-1}Y + e^{+} + \nu \end{aligned}} \rightarrow \begin{aligned} {}^{14}_{6}C &\longrightarrow {}^{14}_{7}N + e^{-} + \bar{\nu} \\ {}^{12}_{7}N &\longrightarrow {}^{12}_{6}C + e^{+} + \nu \end{aligned}$$

The fundamental process in β-decay is $n \rightarrow p + e^{-} + \bar{\nu}$, the so-called weak interaction. The γ-decay is simple:

$\boxed{{}^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}X + \gamma}$ $\rightarrow$ the nucleus goes from a higher energy state to the lower

⊘ Magic number for nuclei. Just like electrons, nucleons arrange themselves into shells with definite quantum numbers.

$$\boxed{N, Z = 2, 8, 20, 28, 50, 82, 126}$$

← either N or Z will do the magic.

Nuclei with these magic N or Z are particularly stable. Note that these numbers are different from 2, 10, 18, 36, 54, 86 for the closed atomic shells.

Let us try to understand these magic numbers for protons and neutrons. Suppose nucleons inside the nuclei can be treated as particles in harmonic potential

$$\boxed{\begin{aligned} E &= \frac{1}{2m}(P_x^2 + P_y^2 + P_z^2) + \frac{1}{2}k(x^2 + y^2 + z^2) \\ &= \left(\frac{P_x^2}{2m} + \frac{1}{2}kx^2\right) + \left(\frac{P_y^2}{2m} + \frac{1}{2}ky^2\right) + \left(\frac{P_z^2}{2m} + \frac{1}{2}kz^2\right) \end{aligned}}$$

豪豬筆記

According to quantum mechanics, the energy of a SHO is quantized $E=(n+\frac{1}{2})\hbar\omega_0$

$$\rightarrow \boxed{E=\left(n_x+n_y+n_z+\frac{3}{2}\right)\hbar\omega_0} \leftarrow n \equiv n_x+n_y+n_z$$

It is convenient to introduce the principal quantum number $n=n_x+n_y+n_z$ so that $E=n\hbar\omega_0+\text{Const.}$ We can classify the states by n ☺

$n=0$, $(n_x,n_y,n_z)=(0,0,0)$, spin $\frac{1}{2}$ $\rightarrow$ 2 states

$n=1$, $(n_x,n_y,n_z)=(1,0,0),\ (0,1,0),\ (0,0,1)$

multiplied by the spin factor $(2S+1)$ $\rightarrow$ 6 states

$n=2$, $(n_x,n_y,n_z)=(2,0,0),\ (0,2,0),\ (0,0,2)$

$(1,1,0),\ (1,0,1),\ (0,1,1)$

multiplied by $(2S+1)$ factor $\rightarrow$ 12 states

The magic numbers can be obtained by filling up the states according to Pauli exclusion principle.

$$\boxed{N, Z = 2,\ (2+6),\ (2+6+12),\ \ldots = 2,\ 8,\ 20,\ \ldots}$$

It is amazing that the first few magic numbers can be obtained from SHO! But, the larger ones are complicated.

清大東院

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