

## 第 1 講 Parametrized Curves, Regular Curves and Arc Length

- L1\_A** 1. Definition: Parametrized Curves  
2. Examples: Parametrized Differentiable Curves

- L1\_B** 1. Note: Difference Between Curve and Trace  
2. Definition: Regular Curves  
3. Example: Helix

- L1\_C** 1. Example: Helix (cont.)  
2. Definition: The Arc Length of a Regular Curve

## 第 2 講 The Local Theory of Curves Parametrized by Arc Length

- L2\_A** 1. Note: Properties of Arc Length  
2. Recall: Inner Product and Wedge Product  
3. Definition: Parametrized by Arc Length

- L2\_B** 1. Example: Logarithmic Spiral  
2. Derivation: Curvature of a Curve Parametrized by Arc Length

- L2\_C** 1. Definition: Curvature  
2. Examples: Straight Line and Circle  
3. Definition: Frenet Frame

## 第 3 講 The Local Theory of Curves Parametrized by Arc Length (cont.)

- L3\_A** 1. Definition: Torsion of a Curve Parametrized by Arc Length  
2. Derivation: Frenet Formula

- L3\_B** 1. Definition: Normal Plane and Rectifying Plane  
2. Example: Helix

- L3\_C** 1. Theorem: A Curve with Positive Curvature Is a Plane Curve  
if and only if Its Torsion Is Identically Zero

## 第 4 講 The Local Theory of Curves Parametrized by Arc Length (cont.)

- L4\_A** 1. Derivation: Curvature and Torsion of a Curve  
NOT Parametrized by Arc Length

- L4\_B** 1. Derivation: Curvature and Torsion of a Curve  
NOT Parametrized by Arc Length (cont.)  
2. Example: Curve in  $R^2$

- L4\_C** 1. Definition: Rigid Motion  
2. Theorem: Fundamental Theorem of the Local Theory of Curves

## 第 5 講 The Local Theory of Curves Parametrized by Arc Length (cont.)

- L5\_A** 1. Proof: Fundamental Theorem of the Local Theory of Curves

- L5\_B** 1. Proof: Fundamental Theorem of the Local Theory of Curves (cont.)

- L5\_C** 1. Proof: Fundamental Theorem of the Local Theory of Curves (cont.)

## 第 6 講 Isoperimetric Inequality

**L6\_A** 1. Definition: Closed Curves, Simple Closed Curves and Positive Oriented Simple Closed Curves

2. Motivation: Dido's Problem

**L6\_B** 1. Theorem: Isoperimetric Inequality

2. Recall: Green's Theorem

**L6\_C** 1. Proof: Isoperimetric Inequality

## 第 7 講 Regular Surface

**L7\_A** 1. Review: Isoperimetric Inequality

2. Definition: Regular Surface

**L7\_B** 1. Derivation: Operation of  $d\vec{X}_q$

**L7\_C** 1. Note: Regularity Condition of  $\vec{X}$

2. Example: Unit Sphere (Method I)

## 第 8 講 Regular Surface (cont.)

**L8\_A** 1. Example: Unit Sphere (Method II)

**L8\_B** 1. Proposition: The Graph of a Differentiable Function Is a Regular Surface

2. Definition: Critical Points and Critical Values

**L8\_C** 1. Definition: Regular Values

2. Proposition: The Inverse Image of a Regular Value Is a Regular Surface

3. Example: Unit Sphere (Method III)

4. Recall: Inverse Function Theorem

## 第 9 講 Regular Surface (cont.)

**L9\_A** 1. Proof: The Inverse Image of a Regular Value Is a Regular Surface

**L9\_B** 1. Derivation: Quadric Surfaces

**L9\_C** 1. Proposition: A Regular Surface Is Locally the Graph of a Differentiable Function

## 第 10 講 Change of Parameters

**L10\_A** 1. Example: Hyperboloid

2. Definition: Connected Surfaces

**L10\_B** 1. Proposition: Change of Parameters

**L10\_C** 1. Proof: Change of Parameters

2. Definition: Differentiable Functions on Surfaces

## 第 11 講 Differentiable Functions on Surfaces

- L11\_A** 1. Review: Differentiable Functions on Surfaces  
2. Examples: Height Function and Distance Square Function

- L11\_B** 1. Corollary: Differentiable Functions Between Surfaces  
2. Definition: Diffeomorphic and Diffeomorphism  
3. Examples: Symmetric Map and Rotation Map  
4. Example: A Sphere Is Diffeomorphic to an Ellipsoid

- L11\_C** 1. Derivation: Surface of Revolution

## 第 12 講 Parametrized Surfaces and Tangent Vectors

- L12\_A** 1. Definition: Parametrized Surfaces and Regular Parametrized Surfaces  
2. Example: The Tangent Surface

- L12\_B** 1. Proposition: A Regular Parametrized Surface Is Locally a Regular Surface  
2. Definition: Tangent Vectors

- L12\_C** 1. Proposition: Relation Between  $T_p(S)$  and  $d\vec{X}_q(\mathbb{R}^2)$   
2. Proposition: The Tangent Plane to  $f^{-1}(a)$  Is the Kernel of  $df_p$

## 第 13 講 The Tangent Plane and the Differential of a Map

- L13\_A** 1. Review: The Tangent Plane  
2. Proof: The Tangent Plane to  $f^{-1}(a)$  Is the Kernel of  $df_p$   
3. Example: Sphere

- L13\_B** 1. Example: Sphere (cont.)  
2. Proposition: Differential of a Map

- L13\_C** 1. Example: Rotation Map on Unit Sphere  
2. Definition: Local Diffeomorphism  
3. Proposition: The Differential Is a Local Isomorphism  
Implies a Local Diffeomorphism

## 第 14 講 The Tangent Plane and the Differential of a Map (cont.)

- L14\_A** 1. Definition: Critical Points  
2. Example: The Tangent Plane at a Critical Point  
3. Derivation: Differential of Composite Maps

- L14\_B** 1. Derivation: Differential of Composite Maps (cont.)  
2. Definition: Orthogonality of Two Surfaces

## 第 15 講 The First Fundamental Form and the Area of a Surface

- L15\_A** 1. Definition: The First Fundamental Form  
2. Example: Unit Sphere

- L15\_B** 1. Example: Unit Sphere (cont.)  
2. Application: Rhumb Line

- L15\_C** 1. Application: Rhumb Line (cont.)  
2. Definition: The Area of a Surface

## 第 16 講 **The First Fundamental Form and the Area of a Surface (cont.)**

- L16\_A** 1. Definition: The Area of a Surface (cont.)  
2. Example: Unit Sphere

- L16\_B** 1. Definition: Locally Isometric  
2. Example: A Plane Is Locally Isometric to a Cylinder

## 第 17 講 **Orientation of a Surface**

- L17\_A** 1. Definition: Orientable Surface  
2. Example: Surface Covered by One Parametrization

- L17\_B** 1. Example: Surface Covered by Two Parametrizations  
2. Proposition: A Surface Is Orientable if and only if  
It Has a Differentiable Unit Normal Vector Field

- L17\_C** 1. Proposition: A Surface Is Orientable if and only if  
It Has a Differentiable Unit Normal Vector Field (cont.)  
2. Example: Mobius Band

## 第 18 講 **The Gauss Map and Its Fundamental Properties**

- L18\_A** 1. Proposition: Regular Surface Given by the Inverse Image  
of a Regular Value Is Orientable  
2. Definition: Gauss Map

- L18\_B** 1. Definition: Gauss Map (cont.)  
2. Examples: Plane and Hyperbolic Paraboloid

- L18\_C** 1. Examples: Hyperbolic Paraboloid (cont.) and Unit Sphere

## 第 19 講 **The Gauss Map and Its Fundamental Properties (cont.)**

- L19\_A** 1. Motivation: The Curvature of a Surface Is Characterized by  
the Differential of Gauss Map  
2. Example: Cylinder

- L19\_B** 1. Definition: Self-Adjoint Linear Map  
2. Proposition: The Differential of Gauss Map Is a Self-Adjoint Linear Map  
3. Definition: The Second Fundamental Form

- L19\_C** 1. Definition: Normal Curvature  
2. Derivation: The Geometric Meaning of the Second Fundamental Form

## 第 20 講 The Gauss Map and Its Fundamental Properties (cont.)

- L20\_A** 1. Review: The Geometric Meaning of the Second Fundamental Form  
2. Proposition: Meusnier  
3. Example: Unit Sphere

- L20\_B** 1. Definition: Normal Section  
2. Derivation: Define the Normal Curvature by Normal Section

- L20\_C** 1. Definition: Principal Curvatures and Principal Directions  
2. Definition: Line of Curvature  
3. Proposition: Olinde Rodrigues  
4. Derivation: Euler Formula

## 第 21 講 The Gauss Map and Its Fundamental Properties (cont.)

- L21\_A** 1. Definition: Gauss Curvature and Mean Curvature  
2. Definition: Elliptic, Hyperbolic, Planar and Parabolic Point

- L21\_B** 1. Definition: Umbilical Point  
2. Example: Study the Gauss Map on  $\{2z = x^2 + y^2\}$  at  $(0,0,0)$

- L21\_C** 1. Proposition: A Connected Surface with Every Point Being Umbilical Point  
Is a Piece of Plane or Sphere

## 第 22 講 The Gauss Map and Its Fundamental Properties (cont.)

- L22\_A** 1. Proposition: A Connected Surface with Every Point Being Umbilical Point  
Is a Piece of Plane or Sphere (cont.)

- L22\_B** 1. Proposition: A Connected Surface with Every Point Being Umbilical Point  
Is a Piece of Plane or Sphere (cont.)  
2. Definition: Asymptotic Direction and Asymptotic Curve  
3. Example: Straight Line

- L22\_C** 1. Example: Straight Line (cont.) and Curve with Positive Curvature  
2. Observation: There Is NO Asymptotic Direction at an Elliptic Point  
3. Definition: Dupin Indicatrix

## 第 23 講 The Gauss Map in Local Coordinates

- L23\_A** 1. Derivation: Equations of Weingarten

- L23\_B** 1. Derivation: Equations of Weingarten (cont.)  
2. Gauss Curvature in terms of the First and Second Fundamental Form

- L23\_C** 1. Mean Curvature in terms of the First and Second Fundamental Form  
2. Principal Curvatures in terms of the First and Second Fundamental Form  
3. Proposition: Smoothness of Gauss Curvature, Mean Curvature and  
Principal Curvatures

## 第 24 講 The Gauss Map in Local Coordinates (cont.)

**L24\_A** 1. Review: The Formula of Gauss Curvature, Mean Curvature and Principal Curvatures

2. Example: Torus

**L24\_B** 1. Example: Torus (cont.)

**L24\_C** 1. Example: Helicoid

## 第 25 講 The Gauss Map in Local Coordinates (cont.)

**L25\_A** 1. Proposition: The Position of a Surface in the Neighborhood of an Elliptic Point or a Hyperbolic Point with respect to the Tangent Plane

## 第 26 講

**L26\_A** 1. Review: Dupin Indicatrix and Its Graph

2. Example: Monkey Saddle

**L26\_B** 1. Examples:  $z = y^3$  Rotated About  $z = 1$  and Cylinder

2. Derivation: Gauss Curvature of a Surface of Revolution

**L26\_C** 1. Derivation: Gauss Curvature of a Surface of Revolution (cont.)

## 第 27 講 The Gauss Map in Local Coordinates (cont.)

**L27\_A** 1. Review: Gauss Curvature of a Surface of Revolution

2. Derivation: Differential Equation of the Asymptotic Curves

**L27\_B** 1. Proposition: The Coordinate Curves Are Asymptotic Curves if and only if  $e = g = 0$

2. Example: Asymptotic Curves

**L27\_C** 1. Example: Asymptotic Curves (cont.)

2. Proposition: The Coordinate Curves Are Asymptotic Curves if and only if  $f = F = 0$

## 第 28 講 The Gauss Map in Local Coordinates (cont.)

**L28\_A** 1. Proposition: The Coordinate Curves Are Asymptotic Curves if and only if  $f = F = 0$  (cont.)

**L28\_B** 1. Preview: Local Version of Gauss Bonnet Theorem