An increase in the price of $X, P_x \uparrow$

$P_{x_1} \to P_{x_2}, \quad P_{x_2} > P_{x_1}$

Assume $P_y = 1$ and $m$ are fixed.

$m' = e(P_{x_2}, P_y, u_1)$

$m = e(P_{x_1}, P_y, u_1) = e(P_{x_2}, P_y, u_2)$

$m'' = e(P_{x_1}, P_y, u_2)$

$CV = e(P_{x_2}, P_y, u_1) - m = e(P_{x_2}, P_y, u_1) - e(P_{x_2}, P_y, u_2)$

distance between $u_1$ and $u_2$ in terms of new prices

$EV = e(P_{x_1}, P_y, u_2) - m = e(P_{x_1}, P_y, u_2) - e(P_{x_1}, P_y, u_1)$

distance between $u_2$ and $u_1$ in terms of old prices
EX: \(X \& Y\) are perfect complement.

\[ u(x,y) = \min\left\{ \frac{x}{2}, \frac{y}{3} \right\} \]

\( (P_{x1}, P_y, m)=(0.5,1,100) \)
\( (P_{x2}, P_y, m)=(1,1,100) \)

most efficient \(x,y\) ratio: \( \frac{x}{2} = \frac{y}{3} \Rightarrow y=1.5x \)

\( e_1 \) satisfies \( \begin{cases} y=1.5x \\ 0.5x+y=100 \end{cases} \Rightarrow \begin{cases} x=50 \\ y=75 \end{cases} \)

\[ u(x,y) = \min\left\{ \frac{x}{2}, \frac{y}{3} \right\} \Rightarrow u_1 = 25 \]

\( e_2 \) satisfies \( \begin{cases} y=1.5x \\ x+y=100 \end{cases} \Rightarrow \begin{cases} x=40 \\ y=60 \end{cases} \)

\[ u(x,y) = \min\left\{ \frac{x}{2}, \frac{y}{3} \right\} \Rightarrow u_2 = 20 \]

since \( e_3 = e_1 = (50,75) \Rightarrow m^*=1*50+1*75=125 \)
\( CV=125-100=25 \)

since \( e_4 = e_2 = (40,60) \Rightarrow m^"=0.5*40+1*60=80 \)
\( EV=80-100=-20 \)

\[ \triangleright \text{ ordinary demand functions} \]

\[ \max x,y \left( \min\left\{ \frac{x}{2}, \frac{y}{3} \right\} \right) \]
s.t. \( P_x x + P_y y = m \)
\[
\begin{align*}
\begin{cases}
\frac{x}{2} = \frac{y}{3} & \Rightarrow \quad y = 1.5 x \\
P_x x + P_y y = m & \\
P_x x + 1.5 P_y y = m
\end{cases}
\]
\((P_x + 1.5 P_y)x = m\)

\[
x^* = \frac{m}{P_x + 1.5 P_y} = \frac{2m}{2P_x + 3P_y}
\]

\[
y^* = \frac{1.5 m}{P_x + 1.5 P_y} = \frac{3m}{2P_x + 3P_y}
\]

- **Indirect utility function**

  \[
  V(P_x, P_y, m) = u(x^*, y^*) = \frac{m}{2P_x + 3P_y}
  \]

- **Compensated demand functions**

  \[
  \min x, y \quad P_x x + P_y y
  \]

  \[
  s.t. \min\{\frac{x}{2}, \frac{y}{3}\} = u
  \]

  \[
  \begin{cases}
  \frac{x}{2} = \frac{y}{3} = u & \Rightarrow \quad x^h = 2u \\
  y^h = 3u
  \end{cases}
  \]

  independent of \( P_x, P_y \)

- **Expenditure function**

  \[
  e(P_x, P_y, u) = P_x x^h + P_y y^h = P_x \cdot 2u + P_y \cdot 3u = (2P_x + 3P_y)u
  \]

  \[
  u_1 = V(P_{x_1}, P_y, m) = V(0.5, 1, 100) = \frac{100}{2 + 0.5 + 3 + 1} = 25
  \]

  \[
  u_2 = V(P_{x_2}, P_y, m) = V(1, 1, 100) = \frac{100}{2 + 1 + 3 + 1} = 20
  \]

  \[
  CV = e(P_{x_2}, P_y, u_1) - m = e(1, 1, 25) - 100 = (2*1 + 3*1)*25 - 100 = 25
  \]

  \[
  EV = e(P_{x_1}, P_y, u_2) - m = e(0.5, 1, 20) - 100 = (2*0.5 + 3*1)*20 - 100 = -20
  \]
EX:

\[ u(x, y) = x^2 y \]

\((P_{x_1}, P_y, m) = (0.5, 1, 90)\)

\((P_{x_2}, P_y, m) = (1, 1, 90)\)

- **ordinary demand functions**

\[
\begin{align*}
\text{max } & x, y \quad x^2 y \\
\text{s.t. } & P_x x + P_y y = m \\
x^* & = \frac{2}{2+1} m = \frac{2}{3} m P_x \\
y^* & = \frac{1}{2+1} m = \frac{1}{3} m P_y
\end{align*}
\]

- **Indirect utility function**

\[
V(P_x, P_y, m) = x^*^2 y^* = (\frac{2}{3} P_x)^2 (\frac{1}{3} P_y)
\]

\[u_1 = V(0.5, 1, 90) = 432000\]

\[x_1 = 120, y_1 = 30\]

\[u_2 = V(1, 1, 90) = 108000\]

\[x_2 = 60, y_2 = 30\]

- **Compensated demand functions**

\[
\begin{align*}
\text{min } & x, y \quad P_x x + P_y y \\
\text{s.t. } & x^2 y = u \\
\text{Foc } & MRS_{xy} = \frac{P_x}{P_y} \\
x^2 y & = u \\
\text{LHS of } & MRS_{xy} \frac{Mux}{Muy} = \frac{2xy}{x^2} = \frac{2y}{x}
\end{align*}
\]

\[
\begin{align*}
\Phi & = > \frac{2y}{x} = \frac{P_x}{P_y} \quad \Rightarrow y = \frac{P_x x}{2P_y} \\
\phi & = > x^2 \frac{P_x}{2P_y} x = u \\
& \frac{P_x}{2P_y} x^3 = u \\
x^3 & = \frac{2P_y}{P_x} u \\
x^h & = \sqrt[3]{\frac{2P_y}{P_x} u} \\
y^h & = \frac{P_x}{2P_y} \sqrt[3]{\frac{2P_y}{P_x} u} = \sqrt[3]{\frac{P_x^2}{4P_y^2} u}
\end{align*}
\]
Expenditure function

\[ e(P_X, P_Y, u_2) = P_X x + P_Y y = P_X \frac{2 \sqrt{P_Y}}{P_X} u + P_Y \frac{\sqrt{P_X^2}}{4P_Y} u = \frac{1}{2} 2P_X^2 P_Y u + \sqrt{0.25P_X^2 P_Y u} \]

\[ e(P_X, P_Y, u_1) = e(1, 1, 432000) = \frac{3}{2} \sqrt{864000} + \sqrt{108000} \]

\[ = 3 \sqrt{108000} = 90 \sqrt{4} \]

\[ e(P_X, P_Y, u_2) = e(0.5, 1, 108000) = \frac{3}{2} \sqrt{0.5 \times 108000} + \sqrt{\frac{1}{16} 108000} \]

\[ CV = 90 \sqrt{4} - 90 \]

\[ EV = 45 \sqrt{2} - 90 \]

\((P_x, P_y, m) \rightarrow x^*, y^* \) old equilibrium

\[ V(P_{x1}, P_y, m) = u(x^*, y^*) = u_1 \]

\((P_{x2}, P_y, m) \rightarrow x', y' \) new equilibrium

\[ V(P_{x2}, P_y, m) = u(x', y') = u_2 \]

\( e(P_X, P_Y, u) \) is the expenditure function

\[ \text{min } x, y \quad P_X x + P_Y y \quad \text{(} x^h, y^h \text{)} \quad \text{compensated demand function} \]

\[ \text{s.t. } u(x, y) = u \quad x^h = x(P_X, P_Y, u) \]

\[ y^h = y(P_X, P_Y, u) \]

\[ e(P_X, P_Y, u) = P_X x^h + P_Y y^h \]

\[ e(P_{x1}, P_y, u_2) = e(P_{x1}, P_y, u_1) = m \]

\[ e(P_{x2}, P_y, u_1) = e(P_{x2}, P_y, u_2) = m \]
EX:
\[ u(x,y) = \sqrt{x} + y \]
\[ \max_{x,y} \sqrt{x} + y \]
\[ \text{s.t. } P_x x + P_y y = m \]

\[ \text{Foc } MRS_{xy} = \frac{P_x}{P_y} \]
\[ P_x x + P_y y = m \]
\[ \phi \]

\[ \text{LHS } MRS_{xy} = \frac{M_{ux}}{M_{uy}} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{0.5}} \]

\[ \rightarrow \frac{1}{2x^{0.5}} = \frac{P_x}{P_y} \rightarrow x^* = \frac{P_y^2}{4P_x^2} \]

Note that in the process of finding compensating demand function, we have the FOC:

\[ MRS_{xy} = \frac{P_x}{P_y} \]
\[ \sqrt{x} + y = u \]

\[ \phi \]

\[ \Rightarrow x^* = \frac{P_y^2}{4P_x^2} \]

PE = SE

In this example, \( x_2 = x_3 \)

\[ \Rightarrow \text{PE} = \text{SE}, \text{there is no income effect.} \]

Figure72:
\[ \phi \Rightarrow P_x \cdot \frac{P_y^2}{4P_x^2} + P_y y = m \]

\[ P_y y = m - \frac{P_y^2}{4P_x^2} \]

\[ y^* = \frac{m}{P_y} - \frac{P_y}{4P_x} \]

\[ \text{Figure 73:} \]

\[ P_{x1} = 0.5, P_y = 1, m = 90 \]
\[ P_{x2} = 1, P_y = 1, m = 90 \]

\[ V(P_x, P_y, m) = \sqrt{\frac{P_y^2}{4P_x^2}} + \frac{m}{P_y} - \frac{P_y}{4P_x} \]
\[ = \frac{P_y}{2P_x} + \frac{m}{P_y} - \frac{P_y}{4P_x} \]
\[ = \frac{m}{P_y} + \frac{P_y}{4P_x} \]

\[ u_1 = V(P_{x1}, P_y, m) = V(0.5, 1, 90) = 90.5 \]
\[ x^* = 1, y^* = 90.5 \]

\[ u_2 = V(P_{x2}, P_y, m) = V(1, 1, 90) = 90.25 \]
\[ x' = 0.25, y' = 89.75 \]

\[ e(P_x, P_y, u) = ? \]

\[ \min_{x,y} P_x x + P_y y \]

\[ \text{s.t. } \sqrt{x} + y = u \]
FOC. \( MRS_{xy} = \frac{p_x}{p_y} \phi \)
\[
\begin{align*}
\sqrt{x} + y &= u \quad \phi' \\
\phi &\Rightarrow x^h = x^* = \frac{p_y^2}{4p_x^2} u \text{不影響}
\end{align*}
\]
\[
\phi' \Rightarrow \frac{p_y}{2p_x} + y = u
\]
\[
y^h = u - \frac{p_y}{2p_x}
\]
\[
e(P_x, P_y, u) = P_x \cdot \frac{p_y^2}{4p_x^2} + P_y (u - \frac{p_y}{2p_x})
\]
\[
= \frac{p_y^2}{4p_x} + P_y u - \frac{p_y^2}{2p_x}
\]
\[
= P_y u - \frac{p_y^2}{4p_x}
\]
CV = \( e(P_{x2}, P_y, u_1) - m \)
\[
= e(1, 1, 90.5) - m \\
= (90.5 - 0.25) - 90 = 0.25
\]
EV = \( e(P_{x1}, P_y, u_2) - m \)
\[
= e(0.5, 1, 90.25) - m \\
= (90.25 - 0.5) - 90 = -0.25 \quad \text{CV = -EV}
\]

\* quasi-linear function

quasi–linear in y utility function
\[
u(x, y) = y + f(x)
\]
CV = -EV

\* Quasi-linear utility function

\[
u(x, y) = y + f(x)
\]
given \( u(x, y) = u_i \)
an indifference curve
\[
\{(x, y) | u(x, y) = u_i\}
\]
\[
=\{(x, y) | y + f(x) = u_i\}
\]
\[
=\{(x, y) | y = u_1 - f(x)\}
\]
u(x, y) = \( u_2 \)
another indifference curve \( \{(x, y) | y = u_2 - f(x)\} \)
the vertical distance between those two difference curve
(suppose $u_2 > u_1$)

$y_2 - y_1 = u_2 - u_1$ (f(x) canceled)

given a $X$

$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{f'(x)}{1} = f'(x)$  no $y$

⇒ given a $X$, $f(x) = MRS_{xy}$ is independent of $Y$.

Since $MRS_{xy} = f'(x)$

From the FOC, $MRS_{xy} = \frac{P_x}{P_y}$

$$f'(x) = \frac{P_x}{P_y} \Rightarrow x^* = x^h \text{ is a function of } P_x \text{ and } P_y$$
(no m, no income effect)

\[ PE = SE \]
\[ IE = 0 \]

\* **Consumer Surplus, CS**

A consumer is willing to pay \( P \) for \( X \) units of \( X \) units of \( X \) units of \( X \)

The consumer pays \( P_x * X \) for \( X \) units of \( X \)

\[
CS = \int_0^{x_1} p_x \, dx - p_{x_1} X_1
\]

an increase of price of \( X \) from \( P_{x_1} \) to \( P_{x_2} \), \( P_{x_2} > P_{x_1} \)

\[
\Delta CS = \frac{1}{2} (P_{x_1} - P_{x_2}) (X_2 - X_1)
\]

Figure 76: Change in consumer surplus with an increase of price of \( x \)
\* Relationship among CV, EC & ∆CS

\[ a \sim b \sim c \sim d \]

\[(0, y_0) \sim (1, y_1) \sim (2, y_2) \sim (3, y_3) \]

the consumer is willing to pay \((y_0 - y_1)\)

for the 1st unit of \(X\)

2nd \((y_1 - y_2)\)

Figure 78:

\[ P_y = 1, y \text{ : other expenditure.} \]

\[
\text{MRS}_{xy} \text{ at } x=1 = y_0 - y_1 \\
\text{at } x=2 = y_1 - y_2 \\
\text{at } x=3 = y_2 - y_3
\]

in equilibrium, \(\text{MRS}_{xy} = \frac{P_x}{P_y}\)

\[ P_y = 1 \implies \text{MRS}_{xy} = P_x \]

(or \(MV_x\))
Figure 79:

\[ \Delta CS = ? \]

based on \( X^* \)

\[ \Delta CS \text{ base on } x^h(u_1) = ? \quad \text{CV} \]

\[ \Delta CS \text{ base on } x^h(u_2) = ? \quad \text{EV} \]

Figure 80:

結論: \( P_{x1} \rightarrow P_{x2}, \quad P_{x2} > P_{x1} \)

\( \Delta CS = \) compensated demand function.

not an ordinary demand function.

given \( u(x, y) = u_1 \)

\( X \) is normal \( \Rightarrow \) \( CV > \Delta CS > EV \)

Quasi-linear \( u(x, y) \) \( \Rightarrow \) \( CV = \Delta CS = EV \)
**Revealed Preference**

The theory of revealed preference.

Bundle \((x_1, y_1)\) is revealed preferred to bundle \((x_2, y_2)\)
if
\[ \begin{align*}
\&(1) \text{ both bundles are affordable/} \\
\&(2) (x_1, y_1) \text{ is chosen (but not } (x_2, y_2))
\end{align*} \]

\[ \left( (\& P_x x_1 + P_y y_1 \geq P_x x_2 + P_y y_2 ) \right) \]

**The principle of the revealed preference.**

Suppose a consumer is rational and \((x_1, y_1)\) is revealed preferred to \((x_2, y_2)\), then \((x_1, y_1)\) must be preferred to \((x_2, y_2)\)

(Axiom)

The weak axiom of revealed preference. (WARP)

Suppose 「(\((x_1, y_1)\) is revealed preferred to \((x_2, y_2)\)」 statement A

then 「(\((x_1, y_1)\) cannot be revealed preferred to \((x_2, y_2)\)」 statement B

statement A is true => according to the principle of the revealed preference,
we have \((x_1, y_1) > (x_2, y_2)\)

statement B is true =>
an inconsistency in the consumer’s preference.

**EXAMPLE**

某人將全部所得用於買 A 物及 B 物。

當 A 物價格 \(P_A\) 為 2 元，而 B 物價格也為 2 元時，他以 80 元的所得購買 20 單位的 A 物，當 \(P_A = 4, P_B = 2\) 時，他以 120 元的所得購買 25 單位的 A 物。

請問其消費行為是否符合經濟理性？試加比較說明之。

(Definition) \((x_1, y_1)\) is revealed preferred to \((x_2, y_2)\)

if (1) both \((x_1, y_1)\) and \((x_2, y_2)\) are affordable
(2) \((x_1, y_1)\) is chosen

if \((x_1, y_1)\) is chosen at \((P_{x_1}, P_{x_2})\), \(P_x x_1 + P_y y_1 \geq P_x x_2 + P_y y_2\)

**The principle of the revealed preference**

If consumer is rational

\(\Rightarrow\) Then \((x_1, y_1)\) is revealed preferred to \((x_2, y_2)\)

implies \((x_1, y_1)\) is preferred to \((x_2, y_2)\)

\((x_1, y_1)\) is revealed preferred to \((x_2, y_2)\)
then \((x_2, y_2)\) cannot be revealed preferred to \((x_1, y_1)\)

\[
P_{x_1}x_1 + P_{y_1}y_1 \geq P_{x_1}x_2 + P_{y_1}y_2
\]

and \((x_1, y_1)\) is chosen

at \((P_{x_2}, P_{y_2})\), \((x_2, y_2)\) is chosen, but not \((x_1, y_1)\)

\[
P_{x_2}x_1 + P_{y_2}y_1 \geq P_{x_2}x_2 + P_{y_2}y_2
\]

Example

(a) \((P_{x_1}, P_{y_1}) = (20, 1)\) \((x_1, y_1) = (2, 40)\) \(m_1 = 80\)

\((P_{x_2}, P_{y_2}) = (20, 4)\) \((x_2, y_2) = (3, 25)\) \(m_2 = 160\)

(b) \((P_{x_1}, P_{y_1}) = (20, 1)\) \((x_1, y_1) = (3, 20)\) \(m_1 = 80\)

\((P_{x_2}, P_{y_2}) = (20, 4)\) \((x_2, y_2) = (2, 30)\) \(m_2 = 160\)

sol.

\(a\)

\[
P_{x_1}x_1 + P_{y_1}y_1 = 20 \times 2 + 1 \times 40 = 80
\]

\[
P_{x_1}x_2 + P_{y_1}y_2 = 20 \times 3 + 1 \times 25 = 85
\]

\(\Rightarrow\) \((x_1, y_1)\) is not revealed preferred to \((x_2, y_2)\) …(1)

\[
P_{x_2}x_2 + P_{y_2}y_2 = 20 \times 3 + 4 \times 25 = 160
\]

\[
P_{x_2}x_1 + P_{y_2}y_1 = 20 \times 2 + 4 \times 40 = 200
\]

\(\Rightarrow\) \((x_2, y_2)\) is not revealed preferred to \((x_1, y_1)\) …(2)

(1), (2) \(\Rightarrow\) doesn’t violate the WARP.

sol.

\(b\)

\[
P_{x_1}x_1 + P_{y_1}y_1 = 20 \times 3 + 1 \times 20 = 80
\]

\[
P_{x_1}x_2 + P_{y_1}y_2 = 20 \times 2 + 1 \times 30 = 70
\]

\(\Rightarrow\) \((x_1, y_1)\) is revealed preferred to \((x_2, y_2)\) …(1)

\[
P_{x_2}x_2 + P_{y_2}y_2 = 20 \times 2 + 4 \times 30 = 160
\]

\[
P_{x_2}x_1 + P_{y_2}y_1 = 20 \times 3 + 4 \times 20 = 140
\]

\(\Rightarrow\) \((x_2, y_2)\) is revealed preferred to \((x_1, y_1)\) …(2)

(1), (2) \(\Rightarrow\) violate the WARP.

\[
\begin{array}{c}
\text{a not RP to b} \\
\text{b not RP to a}
\end{array}
\]

WARP ok!

\[
\begin{array}{c}
\text{c RP to d} \\
\text{d RP to c}
\end{array}
\]

violate to WARP
Figure 81:

- At $(P_{x1}, P_{y1})$, $e$ is chosen.
- $f$ is affordable, $e$ is RP to $f$.

- At $(P_{x2}, P_{y2})$, $f$ is chosen.
- $e$ is not affordable, $f$ is not RP to $e$.

Figure 82:

- $\Rightarrow e, f$ don’t violate WARP.

- At $(P_{x1}, P_{y1})$, $g$ is chosen.
- $h$ is not affordable, $g$ is not RP to $h$.

- At $(P_{x2}, P_{y2})$, $h$ is chosen.
- $g$ is affordable, $h$ is RP to $g$.
- $h \geq g$.

- $\Rightarrow g, h$ don’t violate WARP.
(a, b) \{ \text{ok!} \\
(a, c) \{ \text{ok!} \\
(a, d) \{ \text{not} \\
(a, e) \{ \text{ok}

\text{Figure 83:}

Income is fixed at } m
\text{ at } (P_{x1}, P_{y1}) \text{ the consumer chooses } a
\text{ at } (P_{x2}, P_{y2}) \text{ the consumer chooses } b
a \rightarrow b \text{ price effect of a decrease in price of } X
(P_{x1} \rightarrow P_{x2}, P_{x2} < P_{x1})

\text{Figure 89:}

How to decompose PE into SE and IE?
slutsky substitution effect.
If a consumer would have chosen } c' \text{ after a slutsky income subsidy,
note that at } P_{x1}, P_{y}, \text{ and } m, a \text{ and } c' \text{ are affordable, } a \text{ is chosen}
=> a \text{ is RP to } c'
\text{ at } P_{x2}, P_{y}, \text{ and } m, a \text{ and } c' \text{ are both affordable, } c' \text{ is chosen}
=> c' \text{ is RP to } a
\text{check } a, c \text{ is OK with WARP}
At \ (P_{x1}, P_{y}, m), a \text{ is chosen but } c \text{ is not affordable}
=> a \text{ is not RP to } c.
At \ (P_{x2}, P_{y}, m), c \text{ is chosen} , a \text{ and } c \text{ are both affordable}
$\Rightarrow c$ is RP to $a$
$\Rightarrow$ doesn’t violate WARP

$P_x \downarrow$, slutsky substitution effect

$\quad a \rightarrow c$

$x_3 > x_1$ ($X$ is cheaper, $X$ substitutes for $Y$)

After slutsky subsidy,

$P_{x2}x_1 + P_{y2}y_1 = P_{x2}x_3 + P_{y2}y_3 = m' \ldots \phi$

original bundle $a$ is $c$ is chosen after affordable at new price slutsky subsidy

$\Rightarrow c$ is RP to $a$

$a$ cannot RP to $c$ $\Rightarrow c$ is not affordable at $(P_x, P_y)$

$m = P_{x1}x_1 + P_{y1}y_1 > P_{x1}x_3 + P_{y1}y_3 \ldots \omega$

$\phi-\omega$

$(P_{x2} - P_{x1})x_1 + (P_{y2} - P_{y1})y_1 > (P_{x2} - P_{x1})x_3 + (P_{y2} - P_{y1})y_3$

$(P_{x2} - P_{x1})(x_3 - x_1) + (P_{y2} - P_{y1})(y_3 - y_1) < 0$

$P_{y2} = P_{y1} = P_y$ fixed

$\Rightarrow (P_{x2} - P_{x1}) (x_3 - x_1) < 0$

$\Rightarrow P_{x2} < P_{x1}$ $\Rightarrow x_3 - x_1 > 0$

$\Rightarrow x_3 > x_1$

\* Tax

Tax on gasoline ($X$)

$\$t$ tax on each unit of $X$

$P_x \rightarrow P_x + t$ $\Rightarrow$ equilibrium $e_1 \rightarrow e_2$

consumer is worse off $e_1$ is revealed preferred to $e_2$

(e$_1$ & $e_2$ are affordable before imposing a $\$t$ unit tax)

$\$t^* X$ $\Rightarrow$ a tax return to the consumer

$\Rightarrow$ new equilibrium $e_3$
A: \( P_x x + P_y y = m \)

B: \((P_x + t)x + P_y y = m\)

The budget line after tax return: \((P_x + t)x + P_y y = m + tx\)

\[ \Rightarrow P_x x + P_y y = m \quad \text{same as A} \]

new income: \(m' = m + tx\)

new price: \(P_x + t \text{ & } P_y\)

slope of the budget line (after tax and tax return)

\[ \Rightarrow \frac{P_x + t}{P_y} \quad \text{(a steeper budget line)} \]

\((P_x + t)x + P_y y = m + tx \Rightarrow \text{both } e_1 \text{ and } e_3 \text{ are affordable at } (P_x)\)

\[ \Rightarrow e_1 \text{ is revealed preferred to } e_3 \]

\[ \Rightarrow \text{consumer is worse off at } e_3 \]

Suppose new equilibrium were at \(e_3'\)

the new budget line is D.

the expenditure of \(e_1\) at \((P_x + t)\) is less than \(m'\)

\[ \Rightarrow e_3' \text{ is revealed preferred to } e_3 \text{ (both are on A, and } e_1 \text{ is chosen before tax)} \]

Based on new budget line C

\[ \Rightarrow e_1 \text{ is not affordable after tax and tax return} \]

\[ \Rightarrow e_3 \text{ is not revealed preferred to } e_1 \]
conclusion:  $X \downarrow$ after a tax and tax return

\* Price Index and welfare

Based period: 0
Current period: \( t \)
At period 0:  \( P_{x0}, P_{y0} \)
\( X_0, Y_0 \)
At period \( t \):  \( P_{xt}, P_{yt} \)
\( X_t, Y_t \)

Compare welfare between periods 0 and \( t \)
The consumer is better off in period 0 \((X_0,Y_0)\) is RP to \(X_t,Y_t\)
if \( P_{x0}x_0 + P_{y0}y_0 > P_{x0}x_t + P_{y0}y_t \) …(1)

The consumer is better off in period \( t \) \((X_t,Y_t)\) is RP to \(X_0,Y_0\)
if \( P_{xt}x_t + P_{yt}y_t > P_{xt}x_0 + P_{yt}y_0 \) …(2)

\[
\frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_0 + P_{y0}y_0} < \frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_t + P_{y0}y_t}
\]

Paasche price index  巴氏指數

\[
\frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_0 + P_{y0}y_0} \leq \frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_t + P_{y0}y_t}
\]

Index \( p \geq \frac{m_t}{m_0} \Rightarrow \) the consumer is better off in period 0.

\[
\frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_0 + P_{y0}y_0} \geq \frac{P_{xt}x_0 + P_{yt}y_0}{P_{x0}x_0 + P_{y0}y_0}
\]

Lasperes price index  拉氏指數

\[
\frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_0 + P_{y0}y_0} \geq \frac{P_{xt}x_0 + P_{yt}y_0}{P_{x0}x_0 + P_{y0}y_0}
\]

CPI is one of Lasperes price index

\[
\frac{m_t}{m_0} \leq \frac{P_{xt}x_t + P_{yt}y_t}{P_{x0}x_0 + P_{y0}y_0} \Rightarrow \) the consumer is better off in period \( t \).