Midterm 2 for General Physics I

Date: Dec 1, 2013

(1) Please do not flip the sheet until instructed.
(2) Write as neat as possible, many thanks!
(3) Make the logical flows in your answers clear.
(4) Good luck for all hard-working students!

Lecturer: Hsiu-Hau Lin

Midterm for General Physics I (Fall, 2013)

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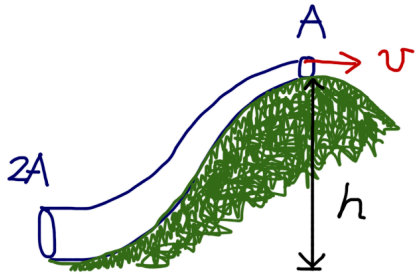
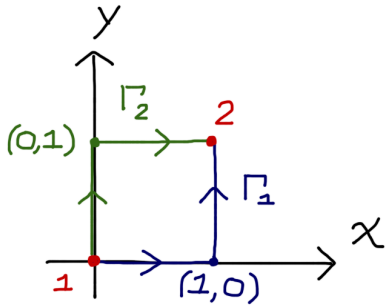
(Dated: Dec 1, 2013)

1. Bernoulli equation (20%) Jill lives on the hilltop of height h and Jack tries to use a smooth-varying pipe as shown in the figure to pump water for his sister. The cross-sectional area on Jill's end is A while that on Jack's end is $2A$ and Jill wishes to have running water coming out at speed v. Assume that the atmosphere pressure P_0 is uniform. What is the pressure ΔP (in additional to P_0) Jack must apply to fulfill the need?



2. Rolling without slipping (20%) An spherical object, of radius R, mass M and moment of inertial gMR^2, is originally at rest and released at the top of an inclined plane of height h. The sphere rolls down the plane without slipping. **(A)** Write down the equations of motion for the center of mass and rotation around it. **(B)** Use energy conservation to find the speed of the center of mass $v_{\rm CM}$ after rolling down the inclined plane.

3. Conservative force (20%) Consider particle dynamics in the two dimensional (x, y) plane. A force $\boldsymbol{F}(\boldsymbol{r}) = (-y, x)$ depends only on position but not on velocity. Compute the work done by the force from point **1** (0,0) to point **2** (1,1) along two different contours Γ_1 and Γ_2 as below. Is the force $\boldsymbol{F}(\boldsymbol{r})$ conservative? If the force is conservative, what is the corresponding potential energy $U(\boldsymbol{r})$?



4. Gravitational red shift (20%) As derived in class, time dilation in weak gravitational field is described by

$$\Delta t' = \Delta t(1 - \Phi/c^2),$$

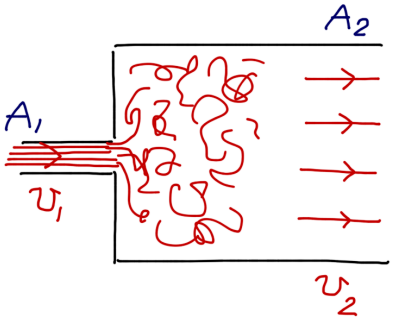
where $\Phi = -GM/r \ll c^2$ is the gravitational potential. Assume the Earth is a sphere of radius R and mass M. Jack stands on the ground of the Earth and emits a light signal of frequency ν_1 to Jill in space at distance $2R$ from the center of the Earth. For simplicity, both Jack and Jill are at rest relative to the Earth. What is the frequency ν_2 of the light signal received by Jill?

5. Conservation of energy (20%) Write a short essay (less than two pages in your answer booklet) on *"Conservation of energy in a many-particle system"*. Starting from the equations of motion, explain how energy can be sorted into different categories and derive the conservation of energy for a many-particle system. Use one or two examples to demonstrate how energies from different categories can transform into each other.

6. Turbulent flows (Bonus 20%) A fluid of density ρ flows at speed v_1 passing an abruptly widened pipe from cross-sectional area A_1 to A_2. The jet mixes with the surrounding fluid and becomes turbulent. However, after the mixing, the fluid will flow on with a uniform velocity v_2 as shown below. If the pipe *were* smoothly widening, the Bernoulli equation gives

$$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2).$$

For simplicity, the gravitational potential is ignored here.



Without referring to the turbulent complications during mixing, use momentum idea to compute the pressure difference $\Delta P = P_2 - P_1$ in the pipe. (*hint:* Think about the difference between elastic and inelastic collisions.)