CH1

4.

$$\bigcup_{i=1}^{n} A_i = A_1 + \sum_{k=2}^{n} A_1^c A_2^c \dots A_{k-1}^c A_k$$

$$then, p(\bigcup_{i=1}^{n} A_i) = p(A_1) + p(\sum_{k=2}^{n} A_1^c A_2^c \dots A_{k-1}^c A_k)$$

$$also, \because A_1^c A_2^c \dots A_{k-1}^c A_k \Rightarrow p(A_1^c A_2^c \dots A_{k-1}^c A_k) \leq p(A_k)$$

$$so, p(\bigcup_{i=1}^{n} A_i) = p(A_1) + p(\sum_{k=2}^{n} A_1^c A_2^c \dots A_{k-1}^c A_k) \leq p(A_1) + \sum_{k=2}^{\infty} p(A_i) = \sum_{k=1}^{\infty} p(A_k)$$

46.(a)

$$P(Red) = \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{7} = \frac{31}{70}$$

46.(b)

$$P(Head|Red) = \frac{P(H \cap Red)}{P(Red)} = \frac{\frac{3}{10}}{\frac{31}{70}} = \frac{21}{31}$$

60.

$$P(A \cup C|B) = \frac{P((A \cup C) \cap B)}{P(B)} = \frac{P((A \cap B) \cup (C \cap B))}{P(B)}$$

$$= \frac{P(A \cap B) + P(C \cap B) - P(A \cap B \cap C)}{P(B)} = P(A|B) + P(C|B) - P(A \cap C|B)$$

68.

$A \perp B$,$B \perp C$,then $A \perp C$ is wrong.

Example:

$P(a) = \frac{1}{2}$, $A = 0, 1$

$P(b) = \frac{1}{2}$, $B = 0, 1$

$P(c) = \frac{1}{2}$, $C = 0, 1$

$P(a, b) = \frac{1}{4}$, $A = 0, 1, B = 0, 1$

$P(b, c) = \frac{1}{4}$, $B = 0, 1, C = 0, 1$

$P(a, c)$

$$= 0, (a, c) = (0, 0), (1, 1)$$

$$= \frac{1}{2}, (b, c) = (1, 0), (0, 1)$$

and

$$P(a, b) = P(a) \times P(b) \Rightarrow A \perp B$$

$$P(b, c) = P(b) \times P(c) \Rightarrow B \perp C$$

but $P(a,c) \neq P(a) \times P(c) \nRightarrow A \perp C$ QED.
78.(a)

$$P(AA) = \frac{1}{2}$$
$$P(Aa) = \frac{1}{2}$$
$$P(aa) = 0$$

78.(b)
$P(AA) = p$,$P(Aa) = 2q$,$P(aa) = r$
Second generation

$$P_2(A) = P(A|AA)P(AA) + P(A|Aa)P(Aa) + P(A|aa)P(aa) = p + q$$
$$P_2(a) = P(a|AA)P(AA) + P(a|Aa)P(Aa) + P(a|aa)P(aa) = q + r$$
$$P_2(AA) = P_2(A)P_2(A) = (p+q)^2 = p_1$$
$$P_2(Aa) = 2P_2(A)P_2(a) = 2(p+q)(q+r) = 2q_1$$
$$P_2(aa) = P_2(a)P_2(a) = (q+r)^2 = r_1$$

Third generation

$$P_3(A) = p_1 + q_1$$
$$P_3(a) = q_1 + r_1$$
$$P_3(AA) = (p_1 + q_1)^2 = (p+q)^2$$
$$P_3(Aa) = 2(p_1 + q_1)(q_1 + r_1) = 2(p+q)(q+r)$$
$$P_3(aa) = (q_1 + r_1)^2 = (q+r)^2$$

$\therefore$ The second and third generations have the same probability.
CH2
6(a).
X =number of heads before first tail.
$P(X=0) = \frac{1}{2}$,$P(X=1) = \frac{1}{4}$,$P(X=2) = \frac{1}{8}$,$P(X=3) = \frac{1}{16}$,$P(X=4) = \frac{1}{32}$

$$F(x) = \frac{1}{2}, X = 0$$
$$= \frac{3}{4}, 0 < X \leq 1$$
$$= \frac{7}{8}, 1 < X \leq 2$$
$$= \frac{15}{16}, 2 < X \leq 3$$
$$= \frac{31}{32}, 3 < X \leq 4$$

$$= 1, 4 \le X$$

6(b).
X =number of heads following first tail.
$P(X=0)=\frac{5}{16}$,$P(X=1)=\frac{3}{8}$,$P(X=2)=\frac{1}{4}$,$P(X=3)=\frac{1}{16}$,$P(X=4)=0$

$$F(x) = \frac{5}{16}, 0 \le X \le 1$$
$$= \frac{11}{16}, 1 < X \le 2$$
$$= \frac{15}{16}, 2 < X \le 3$$
$$= 1, 3 \le X$$

6(c).
Y =number of heads minus the number of tails.
$P(Y=0)=P(2H2T)=\frac{3}{8}$,$P(Y=2)=P(3H1T)=\frac{1}{4}=P(Y=-2)$
$P(Y=4)=P(4H)=\frac{1}{16}=P(Y=-4)$

$$F(y) = 0, Y \le -4$$
$$= \frac{1}{16}, -4 < Y \le -2$$
$$= \frac{5}{16}, -2 < Y \le 0$$
$$= \frac{11}{16}, 0 < Y \le 2$$
$$= \frac{15}{16}, 2 < Y \le 4$$
$$= 1, 4 \le Y$$

6(d).
Z =number of tails times the number of heads.
$P(Z=0)=P(4H)+P(4T)=\frac{1}{8}$,$P(Z=3)=2P(Y=2)=\frac{1}{2}$
$P(Z=4)=P(Y=0)=\frac{3}{8}$

$$F(z) = 0, Z < 0$$
$$= \frac{1}{8}, 0 \le Y < 3$$
$$= \frac{5}{8}, 3 \le Y < 4$$
$$= 1, 4 \le Z$$

33.
(1)
$$\lim_{x \mapsto -\infty} F(x) = 0, \lim_{x \mapsto \infty} F(x) = 1, 0 \le F(x) \le 1$$

(2) 且$F(x) \propto 1-\frac{1}{e^{\alpha x^{\beta}}}$ 爲非遞減函數, 所以 $F(x)$爲一 CDF.

$$f(x)=\frac{\partial}{\partial x}F(x)=\frac{\partial}{\partial x}[1-exp(-\alpha x^{\beta})]=\alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}$$

(3) $F(x)$爲連續函數

40(a).

$$f(x)=cx^2$$

$$\int_0^1 cx^2dx=1 => \frac{cx^3}{3}|_0^1=1 => c=3$$

40(b).

$$f(x)=3x^2$$

$$\int_0^x f(y)dy=\int_0^x 3y^3dy=x^3, 0\leq x\leq 1$$

40(c).

$$P(0.1\leq x\leq 0.5)=\int_{0.1}^{0.5}3x^2dx=x^3|_{0.1}^{0.5}=0.124$$

CH3

1.(a)

The last row is marginal of x. The last column is marginal of y.

	x				
y	1	2	3	4	
1	.10	.05	.02	.02	.19
2	.05	.20	.05	.02	.32
3	.02	.05	.20	.04	.31
4	.02	.02	.04	.10	.18
	.19	.32	.31	.18	

7.

$$\because \frac{\partial F(x,y)}{\partial x\partial y}=f(x,y)$$

so we can get the density of (x,y) and marginal density of x and y.

$$\Longrightarrow f(x,y)=\alpha\beta e^{-\alpha x}e^{-\beta y}, x,y\geq 0, \alpha,\beta>0$$

$$f(y)=\int_0^{\infty}f(x,y)dx=\beta e^{-\beta y}, y\geq 0, f(x)=\int_0^{\infty}f(x,y)dy=\alpha e^{-\alpha x}, x\geq 0$$

8.(a)

(i)

$$P(X>Y)=\int_0^1\int_y^1 f(x,y)dxdy=\int_0^1\int_y^1 \frac{6}{7}(x+y)^2dxdy=\frac{6}{7}\int_0^1 \frac{1}{3}x^3+x^2y+xy^2|_y^1dy$$

$$=\frac{6}{7}\int_0^1 -\frac{7}{3}y^3+y^2+y+\frac{1}{3}=\frac{6}{7}(-\frac{7}{12}y^4+\frac{1}{3}y^3+\frac{1}{2}y^2+\frac{1}{3}y|_0^1)=\frac{1}{2}$$

(ii)Let $x+y=t$

$$f(x,t)=f(x(x,t),y(x,t))|J|,|J|=\frac{d(x,y)}{d(x,t)},f(x,t)=\frac{6}{7}t^2,0\le t\le 2$$

$$P(X+Y\le 1)=\int_0^1\int_0^t \frac{6}{7}t^2dxdt=\int_0^1 \frac{6}{7}t^2x|_0^tdt$$

$$=\int_0^1 \frac{6}{7}t^3dt=\frac{3}{14}$$

(iii)

$$f(x)=\int_0^1 \frac{6}{7}(x^2+2xy+y^2)dy=\frac{6}{7}(x^2+x+\frac{1}{3}),0\le x\le 1$$

$$P(X\le \frac{1}{2})=\int_0^{\frac{1}{2}} f(x)dx=\frac{6}{7}(\frac{x^3}{3}+\frac{x^2}{2}+\frac{x}{3})=\frac{2}{7}$$

12.(a)

$$\because \int f(x,y)dxdy=1$$

$$\int_0^\infty\int_{-x}^x f(u,v)dudv=c\int_0^\infty (u^2v-\frac{1}{3}v^3)e^{-u}|_{-x}^x dv$$

$$c\int_0^\infty \frac{4}{3}x^3e^{-x}dx=\frac{4}{3}c\times\Gamma(4)=1$$

$$=>c=\frac{1}{8}$$

12.(b)
$f(x,y)=\frac{1}{8}(x^2-y^2)e^{-x},-x<y<x,0<x<\infty$
The marginal density is f(x) and f(y).

$$f(x)=\int_{-x}^x \frac{1}{8}(x^2-y^2)e^{-x}dy=\frac{1}{6}x^3e^{-x},0\le x<\infty$$

$$f(y)=\int_y^\infty \frac{1}{8}(x^2-y^2)e^{-x}dx=\frac{e^{-y}(y+1)}{4},y>0$$

$$=\int_{-y}^\infty \frac{1}{8}(x^2-y^2)e^{-x}dx=\frac{1}{4}e^y(1-y),y<0$$