Ch9.7

$$\phi(x) = \begin{cases} 1, & \text{if } f(\underline{x};\lambda_1) > kf(\underline{x};\lambda_0) \\ r, & \text{if } f(\underline{x};\lambda_1) = kf(\underline{x};\lambda_0) \\ 0, & \text{if } f(\underline{x};\lambda_1) < kf(\underline{x};\lambda_0) \end{cases}$$

$$\frac{f_0(x)}{f_1(x)} = \frac{\lambda_0^{\Sigma X_i} e^{-n\lambda_0}}{\lambda_1^{\Sigma X_i} e^{-n\lambda_1}} = e^{n(\lambda_1-\lambda_0)} \left(\frac{\lambda_0}{\lambda_1}\right)^{\Sigma X_i} \propto \left(\frac{\lambda_0}{\lambda_1}\right)^{\Sigma X_i}$$

$\because \lambda_1 > \lambda_0$

$\therefore \left(\frac{\lambda_0}{\lambda_1}\right)^{\Sigma X_i} \in \downarrow \Rightarrow \frac{f_0(x)}{f_1(x)} < C \Leftrightarrow \Sigma X_i > C^*$

So we reject H_0 when $\Sigma X_i > C^*$

且其中 r 滿足 $E(\phi(x)) = \alpha$. 又 $\because \Sigma X_i \sim Poi(n\lambda_0)$ under H_0

C^*滿足 $C^* = \min_c \left\{ c \mid c \in Z, \sum_{k=c+2}^{\infty} \frac{(n\lambda_0)^k e^{-n\lambda_0}}{k!} + r \cdot \frac{(n\lambda_0)^{(c+1)} e^{-n\lambda_0}}{(c+1)!} \le \alpha \right\}$.

Ch9.8

from 9.7, $\forall \lambda_1 > \lambda_0$, reject

By Neyman-Pearson Lemma, $\forall \lambda_1, \lambda_0$ s.t $\lambda_1 > \lambda_0$

the test $H_0: \lambda = \lambda_0$ vs $H_1: \lambda = \lambda_1$; 且 rejection region 和 λ_1 無關

$R = \{ (x_1, \sim, x_n) \mid \Sigma X_i > C^* \}$

$\Rightarrow$ is uniformly most Powerful for testing

$$\begin{cases} H_0: \lambda = \lambda_0 \\ H_1: \lambda > \lambda_0 \end{cases}$$

Jointly made by 趙致平, 許家綸, 黃彥霖 助教

Ch9.29

$\because$ g is a monotone-increasing function.

$\therefore$ $T > t_0 \Leftrightarrow g(T) > g(t_0)$

$\Rightarrow P(S > g(t_0) | H_0) = P(T > t_0 | H_0) = \alpha$

5. $X_i \overset{iid}{\sim} f$, $i=1,2,\cdots,n$

Test: $H_0: f=f_0$ V.S. $H_A: f=f_1$, $f_0 \sim Poi(1)$, $f_1 \sim Geo(\frac{1}{2})$

Let $\lambda(\underset{\sim}{x}) = \dfrac{f_0(\underset{\sim}{x})}{f_1(\underset{\sim}{x})} \propto \dfrac{e^{-n}}{\left(\prod_{i=1}^{n} x_i!\right)\left(\frac{1}{2}\right)^{\sum_{i=1}^{n} x_i}} < c$

The MP test level $\alpha=0.05$ test for $H_0: f=f_0$ V.S. $H_A: f=f_1$ is given by

$$\phi(\underset{\sim}{x}) = \begin{cases} 1, & \text{if } \lambda(\underset{\sim}{x}) < c \\ r, & \text{if } \lambda(\underset{\sim}{x}) = c \\ 0, & \text{if } \lambda(\underset{\sim}{x}) > c \end{cases},$$

where c and r is determined by

$$E_{H_0}(\phi) = P\left[\lambda(\underset{\sim}{x}) < c \mid H_0\right] + r\,P\left[\lambda(\underset{\sim}{x}) = c \mid H_0\right] = \alpha = 0.05.$$

Ch 9

30.

(a) P-value: $V(t_{obs}) = P_r[T > t_{obs} \mid H_0 \text{ is true}]$

$= 1 - F(t_{obs})$

* That t_{obs} is observed value from T, so $V = 1 - F(T)$

※

(b)

$\because$ under H_0, $F(T)$ is the cdf of $\underbrace{T}_{\text{the continuous r.v.}}$, so $F(T) \sim U(0,1)$

$\underbrace{\text{and}}_{\text{also}}$ $V = 1 - F(T) \sim U(0,1)$, under H_0.

(c)

$P[V > 0.1 \mid H_0 \text{ is true}]$

$= P[U(0,1) > 0.1 \mid H_0]$ ⟵ (by (b) $V \sim U(0,1)$)

$= 0.9$

(d)

$P[\text{rejected } H_0 \mid H_0 \text{ is true}]$

$= P[V < \alpha \mid H_0] = P[U(0,1) < \alpha \mid H_0] = \alpha$

$\therefore$ the test $V < \alpha$ has significant level α.

※

HW12.

6. $\because f(\underset{\sim}{x}|\theta) = \exp(-\frac{1}{\theta}\Sigma x_i - n\log\theta) \in$ one-parameter exponential family

$T(\underset{\sim}{X}) = \Sigma X_i$. $c(\theta) = -\frac{1}{\theta}$ is strictly monotone increasing func. of θ.

By L.N. CH9 P.22 Thm. 9.3.

The α-level UMP-test $H_0: \theta \geq \theta_0$ vs $H_1: \theta < \theta_0$

is given by $\phi(\underset{\sim}{x}) = \begin{cases} 1, & T(\underset{\sim}{X}) = \Sigma X_i < c \\ 0, & \Sigma X_i > c \end{cases}$

α can be determined by.

$E_{\theta_0}(\phi(\underset{\sim}{X})) = P_{\theta_0}(\Sigma X_i < c) = \alpha$. $\because \Sigma X_i \sim \Gamma(n, \frac{1}{\theta}) \Rightarrow \frac{2}{\theta}\Sigma X_i \sim \Gamma(\frac{2n}{2}, \frac{1}{2}) \equiv \chi^2(2n)$

$\therefore P_{\theta_0}(\frac{2}{\theta_0}\Sigma X_i < \frac{2}{\theta_0}c) = \alpha \Rightarrow c = \frac{\theta_0}{2}\chi^2(2n)$

7.

令 $Y = X_1 + X_2 + X_3$.

$P(y) = \binom{3}{y}\exp\left(y\ln\frac{p}{1-p} + 3\ln(1-p)\right)$

$T(\underset{\sim}{X}) = Y$. $c(\theta) = \ln\frac{p}{1-p}$ is strictly monotone increasing function of p.

The α-level UMPU test for

$$H_0: p = 0.25 \quad vs \quad p \neq 0.25$$

TS given by

$$\phi(\underset{\sim}{X}) = \begin{cases} 1 & , T(\underset{\sim}{X}) < c_1 \quad T(\underset{\sim}{X}) > c_2 \\ \gamma_i & T(\underset{\sim}{X}) = c_i, \quad i = 1, 2 \\ 0 & c_1 < T(\underset{\sim}{X}) < c_2. \end{cases}$$

$\Rightarrow$ We take $c_1 = 0$, $c_2 = 3$. γ_1, γ_2 can be determined by

$$\begin{cases} E_{p=0.25}(\phi(\underset{\sim}{X})) = \gamma_1 p(Y=0) + \gamma_2 p(Y=3) = 0.05 \\ E_{p=0.25}(\phi(T) \times T) = \gamma_1 \times 0 \times p(Y=0) + \gamma_2 \times 3 \times p(Y=3) = 0.05 \times \frac{3}{4} \end{cases}$$

$$\Rightarrow \quad \gamma_2 = \frac{5}{8} = 0.625. \quad \gamma_1 = \frac{0.0475}{0.42} \approx 0.1131.$$