Ch 4 . 1

Let $X_i = \begin{cases} 1 & \text{, if the head is upward} \\ 0 & \text{, o.w} \end{cases}$

$\therefore X_i \sim Ber(p)$, p is the prob of the head is upward , $i=1,$

thus , $X = \sum_{i=1}^{10} X_i \sim Bin(10, p)$

$H_0: p = 0.5$, vs $H_1: p \neq 0.5$.

test statistic $= X$.

rejection region $\{ X=0 \text{ or } X=10 \}$.

(a)

$\alpha = P(\text{reject } H_0 \mid H_0 \text{ is true}) = P(X=0 \text{ or } X=10 \mid p=0.5)$

$= 2 \times (0.5)^{10} = 0.002$

b)

Power $= 1-\beta = P(\text{reject } H_0 \mid H_0 \text{ is false})$

$= P(X=0 \text{ or } X=10 \mid p=0.1)$

$= \binom{10}{0}(0.1)^0(0.9)^{10} + \binom{10}{10}(0.1)^{10}(0.9)^0$

$= 0.349$.

9-2

(a) simple, the hypothesis can be presented as $X \sim U(0,1)$, which contains only a distribution.

(b)
simple,
assume X is the number rolled on the dice, and
$P_i = P(X=i)$, $i=1,2,\cdots,6$
the hypothesis can be presented as X has the uniform distribution
i.e $P_1 = P_2 = \cdots = P_6 = \frac{1}{6}$

(c) composite, the hypothesis can be presented as
$X \sim N(0, \sigma^2)$, $\sigma^2 > 10$, which contains infinitely many distributions

(d)
Composite, the hypothesis can be presented as
$X \sim N(0, \sigma^2)$, $\sigma^2 > 0$, which contains infinitely many distributions

ch 9

3.

$X \sim Bin(100, p)$

$E[X] = 100p$, $Var[X] = 100p(1-p)$

By CLT , $X \xrightarrow{d} N(100p, 100p(1-p))$

a.

$\alpha = P_r[\,|X-50| > 10 \mid H_0 \text{ is true}]$

$= P_r[\,|\frac{X-50}{5}| > 2 \mid p = 0.5]$

$\approx P_r[\,|Z| > 2] = 0.046$ $\left(\text{When } p = 0.5 \quad Z = \frac{X-50}{5} \xrightarrow{d} N(0,1)\right)$

b.

power $= 1-\beta = P_r[\,|X-50| > 10 \mid H_A \text{ is true}]$

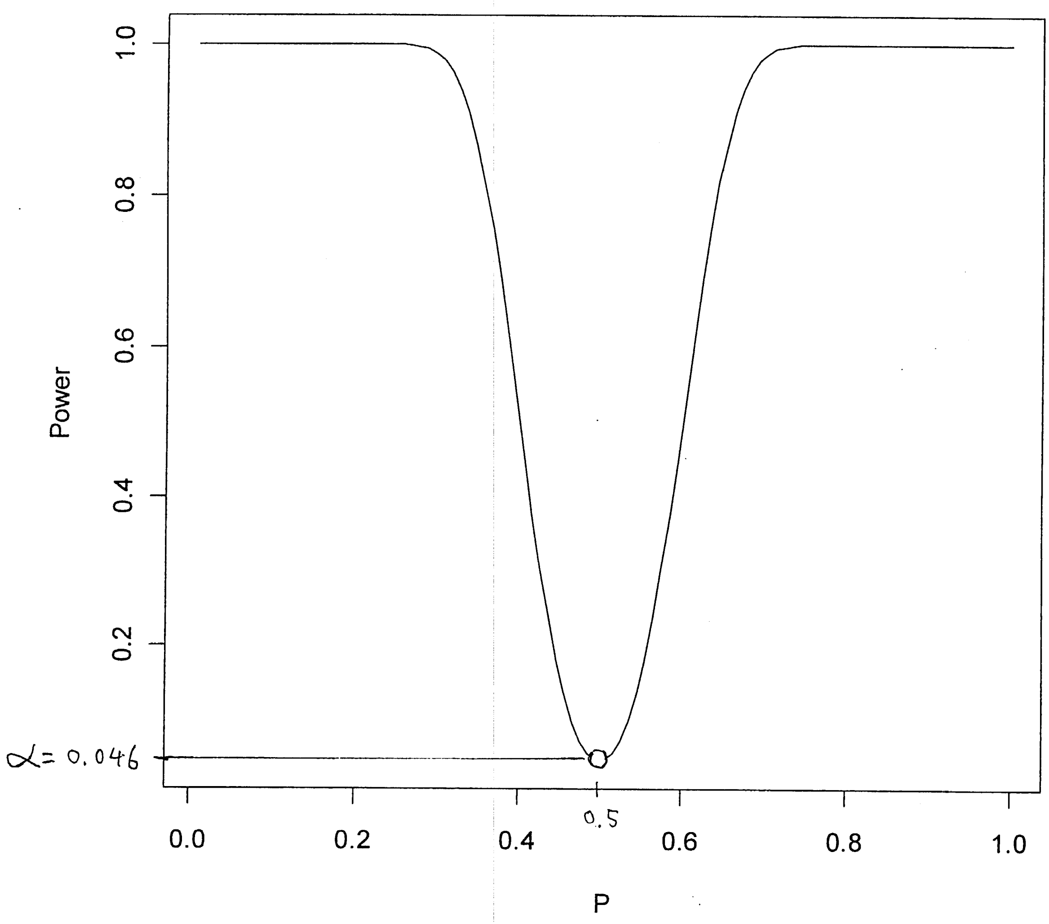
$= 1 - P_r[40 < X < 60 \mid p]$, $p \neq 0.5$

$\approx 1 - P_r\left[\frac{40-100p}{\sqrt{100p(1-p)}} < Z < \frac{60-100p}{\sqrt{100p(1-p)}} \,\middle|\, p\right]$, $p \neq 0.5$

$= 1 - \left(\Phi\left(\frac{60-100p}{\sqrt{100 \cdot p(1-p)}}\right) - \Phi\left(\frac{40-100p}{\sqrt{100 \cdot p(1-p)}}\right)\right)$, $p \neq 0.5$

$\left(Z = \frac{X-100p}{\sqrt{100p(1-p)}} \xrightarrow{d} N(0,1) \quad \text{When } X \sim Bin(100, p)\right)$

$\Downarrow$


1.0
0.8
0.6
0.4
0.2
Power
α = 0.046
0.0
0.2
0.4
0.5
0.6
0.8
1.0
P

HW 11 CH9.

5.
(a) F. the significance level $\alpha = p$ (Rejection region | H_0 is true)

In frequentist approach $p(H_0$ is true$) = \{ \begin{smallmatrix} 1 \\ 0 \end{smallmatrix}$, 亦可視為.
$\{H_0$ is true$\}$ 只有對與錯. 並沒有所謂 $\{H_0$ is true$\}$ 發生
的机率值.

(b) F. ∵ power $= 1-\beta = p$ (Rejection region | H_A is true)
$\alpha \downarrow$ Rejection region 變小. power $\downarrow$.

(c) F. α 為 type I error 發生的機率, 並沒有所謂 $\{H_0$ is true$\}$
發生的機率.

(d) F. the probability is $\alpha = p$ (Rejection region | H_0 is true),
not the power $= p$ (Rejection region | H_A is true)

(e) F. 當 H_0 為真, test statistic $\in$ Rejection region 才是 type I error
若 H_A 為真時, test statistic $\in$ Rejection region, 会做出正確決定
則不是 type I error.

(f) F. see textbook p.336. 第二頁
Ex: H_0: 新藥無效 vs H_1: 新藥有效
Type I error: 新藥無效卻宣稱有效.
Type II error: 新藥有效卻宣稱無效.
故這兩種 error 後果較嚴重者, 应使其為 Type I error.

(g). F. the power is the probability of rejection region determined by the alternative distribution of the test statistics.

(h) T.