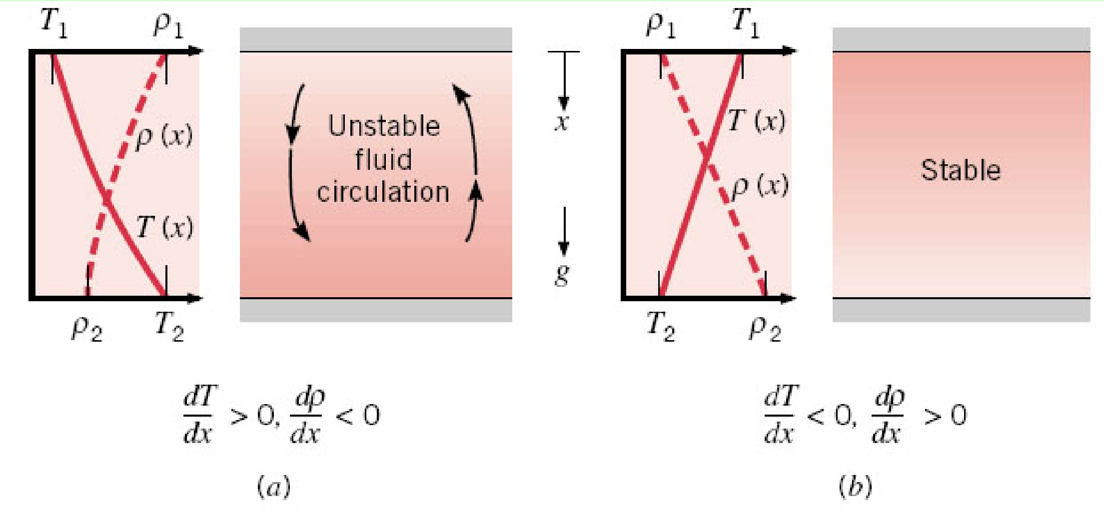
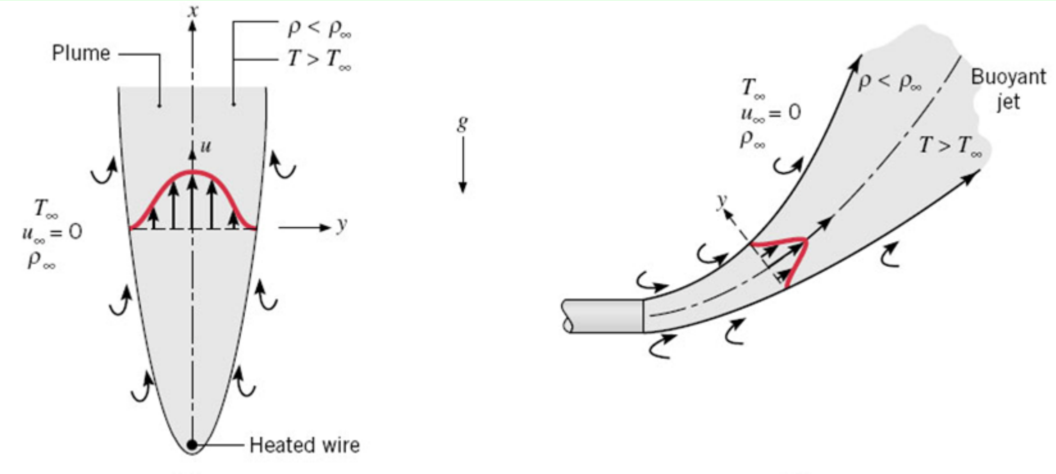
Chapter 9

Natural Convection

- **Forced convection**--with external forcing condition
- **Natural (or free) convection**--driven by buoyancy force, which is induced by body force with the presence of density gradient

9.1 Physical Considerations





9.2 The Governing Equations

x-mom. eq.:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-g+\nu\frac{\partial^2 u}{\partial y^2} \qquad (9.1)$$

With $\partial p/\partial y=0$ from the *y*-mom. eq., the *x*-pressure gradient *in* the b.l. must equal to that in the quiescent region *outside* the b.l.,

i.e.,

$$\frac{\partial p}{\partial x}=-\rho_\infty g \qquad (9.2)$$

So,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{g}{\rho}(\rho_\infty-\rho)+\nu\frac{\partial^2 u}{\partial y^2} \qquad (9.3)$$

Introducing the *volumetric thermal expansion coefficient*

$$\beta=-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p\approx-\frac{1}{\rho}\frac{\rho_\infty-\rho}{T_\infty-T} \qquad (9.4)$$

(Note: For ideal gases, $\beta=-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p=\frac{1}{\rho}\frac{p}{RT^2}=\frac{1}{T}$) (9.9)

$$\rightarrow u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\nu\frac{\partial^2 u}{\partial y^2} \qquad (9.5)$$

The set of governing equations for laminar free convection associated with a *vertical heated plate* are:

continuity eq.:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (9.6)$$

x-mom. eq.:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \qquad (9.7)$$

energy eq.:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (9.8)$$

- Note the dissipation is neglected in (9.8) and Eqs. (9.6)-(9.8) are strongly coupled and must be solved simultaneously.

9.3 Similarity Considerations

Defining $x^* \equiv \frac{x}{L}$ and $y^* \equiv \frac{y}{L}$, L is the characteristic length

$u^* \equiv \frac{u}{u_0}$ and $v^* \equiv \frac{v}{u_0}$, u_0 is an arbitrary reference velocity

$$T^* \equiv \frac{T - T_\infty}{T_s - T_\infty}$$

Eqs. (9.7) and (9.8) reduce to

$1 \rightarrow u_0 = [g\beta(T_s - T_\infty)L]^{1/2}$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{g\beta(T_s - T_\infty)L}{u_0^2} T^* + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (9.10)$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}} \quad (9.11)$$

(9.10) $\rightarrow$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = T^* + \frac{1}{(Gr_L)^{1/2}} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (9.10a)$$

where

$$Gr_L \equiv \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \quad (9.12)$$

- Gr_L plays the same role in free convection that Re_L plays in forced convection.

If there is a non-zero free stream velocity, u_∞, we may use $u_0 = u_\infty$.

Then

$$\frac{g\beta(T_s - T_\infty)L}{u_\infty^2} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \Bigg/ \left(\frac{u_\infty L}{\nu}\right)^2 = \frac{Gr_L}{Re_L^2}$$

$\rightarrow$ Eq. 9.10 becomes

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \boxed{\frac{Gr_L}{Re_L^2}} T^* + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}} \qquad (9.10b)$$

Generally,

$(Gr_L / Re_L^2) \approx 1$ $\rightarrow$ both free & forced convection to be considered

$\rightarrow Nu_L = f(Re_L, Gr_L, Pr)$

$(Gr_L / Re_L^2) << 1$ $\rightarrow$ forced convection

$\rightarrow Nu_L = f(Re_L, Pr)$

$(Gr_L / Re_L^2) >> 1$ $\rightarrow$ free convection

$\rightarrow Nu_L = f(Gr_L, Pr)$

Alternative derivation of *Gr* under purely natural convection

Eqs. (9.10) can be also written as

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{g\beta(T_s-T_\infty)L}{u_0^2}T^*+\frac{\nu}{u_0L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

If u_0 is set to make $u_0L/\nu\equiv 1$, or $u_0=\nu/L$

$$\rightarrow u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{g\beta(T_s-T_\infty)L^3}{\nu^2}T^*+\frac{\partial^2 u^*}{\partial y^{*2}} \qquad (9.10')$$

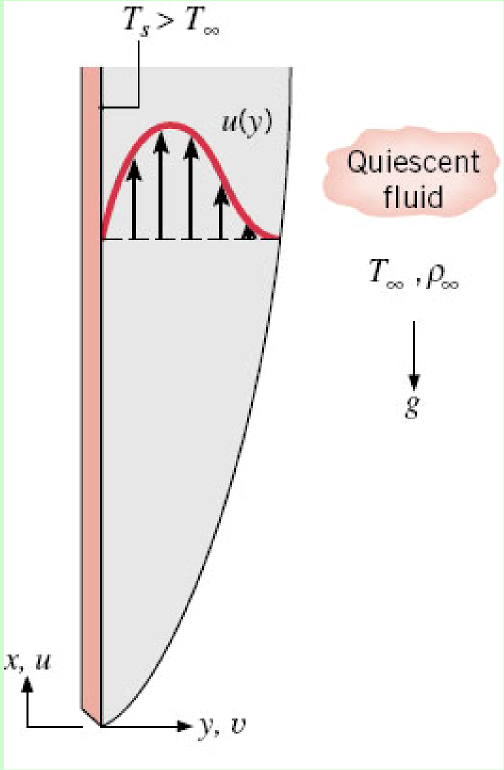
$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{Pr}\frac{\partial^2 T^*}{\partial y^{*2}} \qquad (9.11')$$

where $Gr_L\equiv\frac{g\beta(T_s-T_\infty)L^3}{\nu^2}$ (9.12)

9.4 Laminar Free Convection on a Vertical Surface

Introducing the *similarity* parameter

$$\eta \equiv \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4} \quad \text{and} \quad \psi(x,y) \equiv f(\eta)\left[4\nu\left(\frac{Gr_x}{4}\right)^{1/4}\right]$$



Eqs. (9.6 to 9.8) can be reduced to

$$f''' + 3ff'' - 2(f')^2 + T^* = 0 \qquad (9.17)$$

$$T^{*''} + 3Pr\, f\, T^{*\prime} = 0 \qquad (9.18)$$

The numerical results are shown in Fig. 9.4.

$$Nu_x = \frac{hx}{k} = -\left(\frac{Gr_x}{4}\right)^{1/4} \left.\frac{dT^*}{d\eta}\right|_{\eta=0} = \left(\frac{Gr_x}{4}\right)^{1/4} g(Pr) \qquad (9.19)$$

where $g(Pr)$ is determined numerically determined as (9.20).

And

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \frac{4}{3}\left(\frac{Gr_x}{4}\right)^{1/4} g(Pr) \qquad (9.21)$$

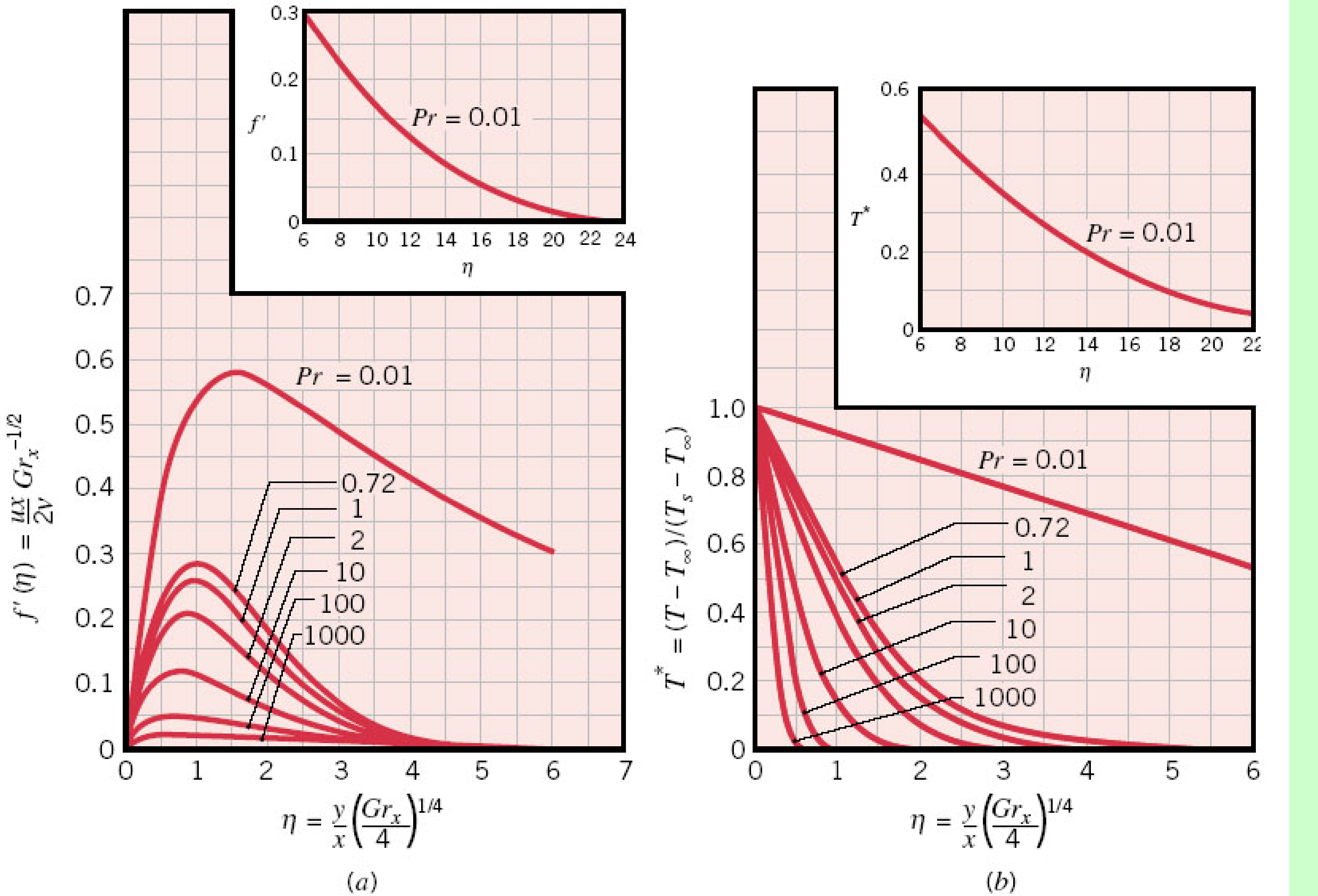
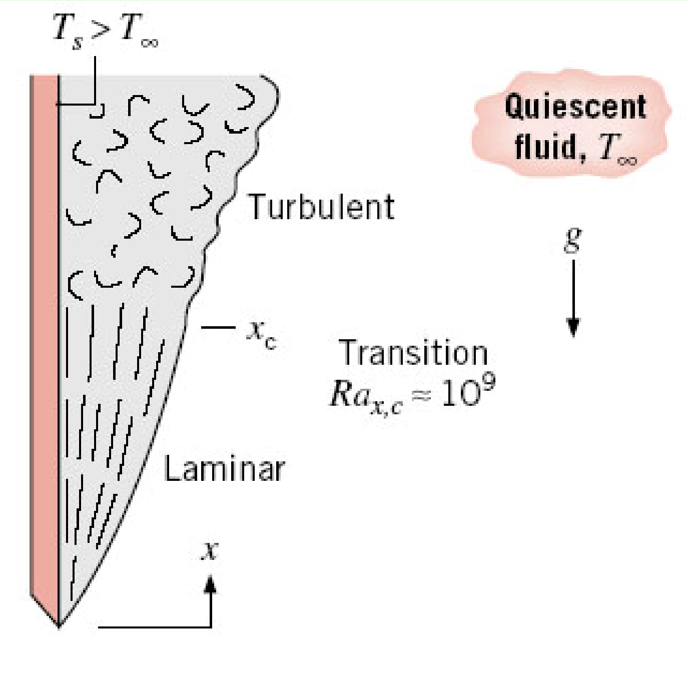


FIGURE 9.4 Laminar, free convection boundary layer conditions on an isothermal, vertical surface. (*a*) Velocity profiles. (*b*) Temperature profiles [3].

9.5 The Effects of Turbulence

For vertical plates the transition occurs at

$$Ra_{x,c} = Gr_{x,c} Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha} \approx 10^9 \qquad (9.23)$$



EX 9.1

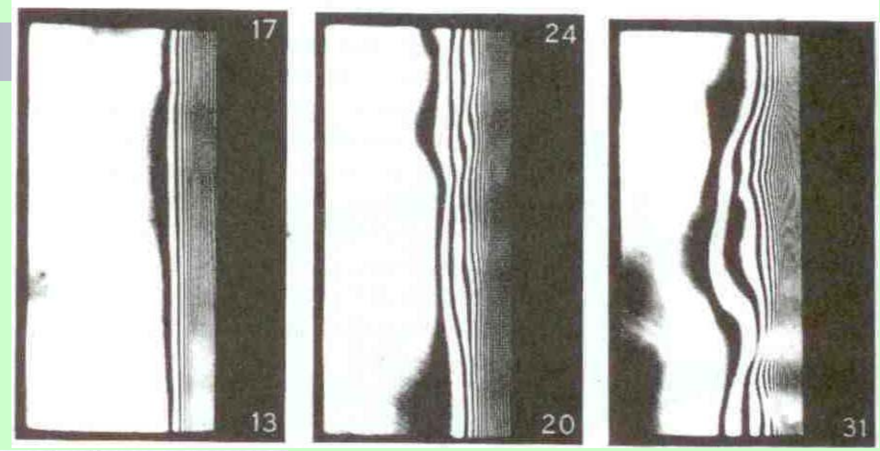
9.6 Empirical Correlations: External Free Convection Flows

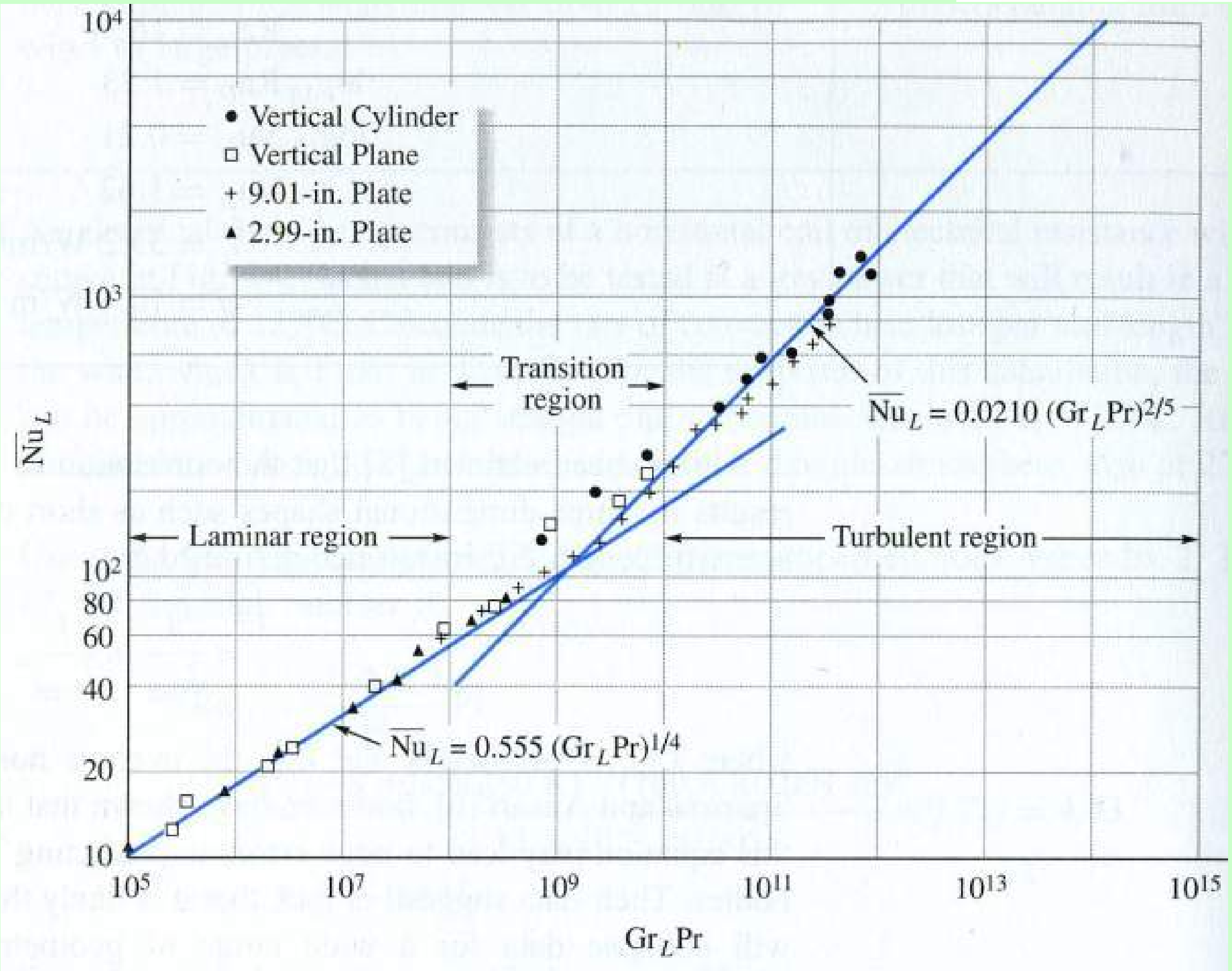
Generally,

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = C Ra_L^n$$, n=1/4 for laminar, n=1/3 for turbulent flow

Table 9.2 (p. 583) summarizes the empirical correlations for different immersed geometries.

EX 9.2





F Kreith & MS Bohn, Principles of Heat Transfer, 2001

Some other flow conditions in 9.6

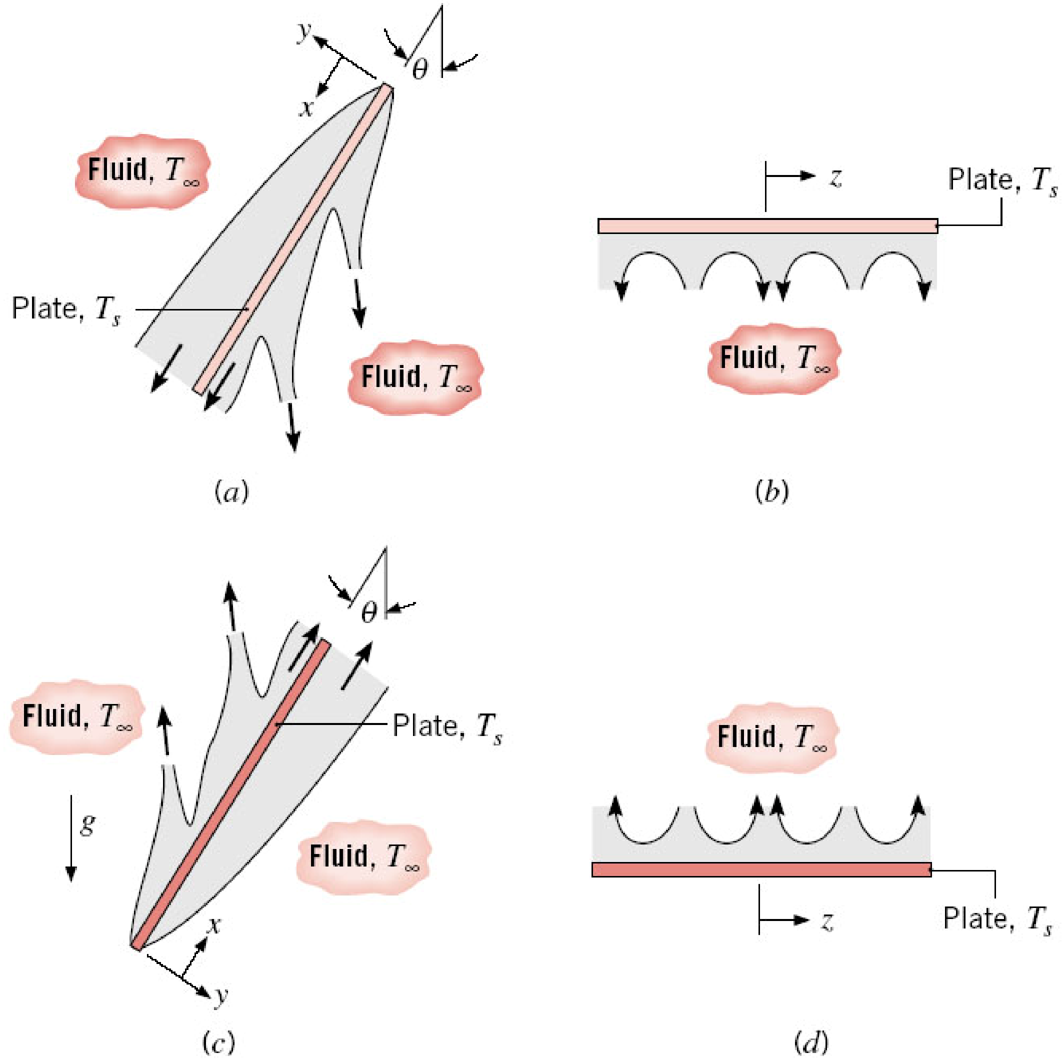


FIGURE 9.6 Buoyancy-driven flows on an inclined plate: (*a*) side view of flows at top and bottom surfaces of a cold plate ($T_s < T_\infty$), (*b*) end view of flow at bottom surface of cold plate, (*c*) side view of flows at top and bottom surfaces of a hot plate ($T_s > T_\infty$), and (*d*) end view of flow at top surface of hot plate.

Flow Pattern

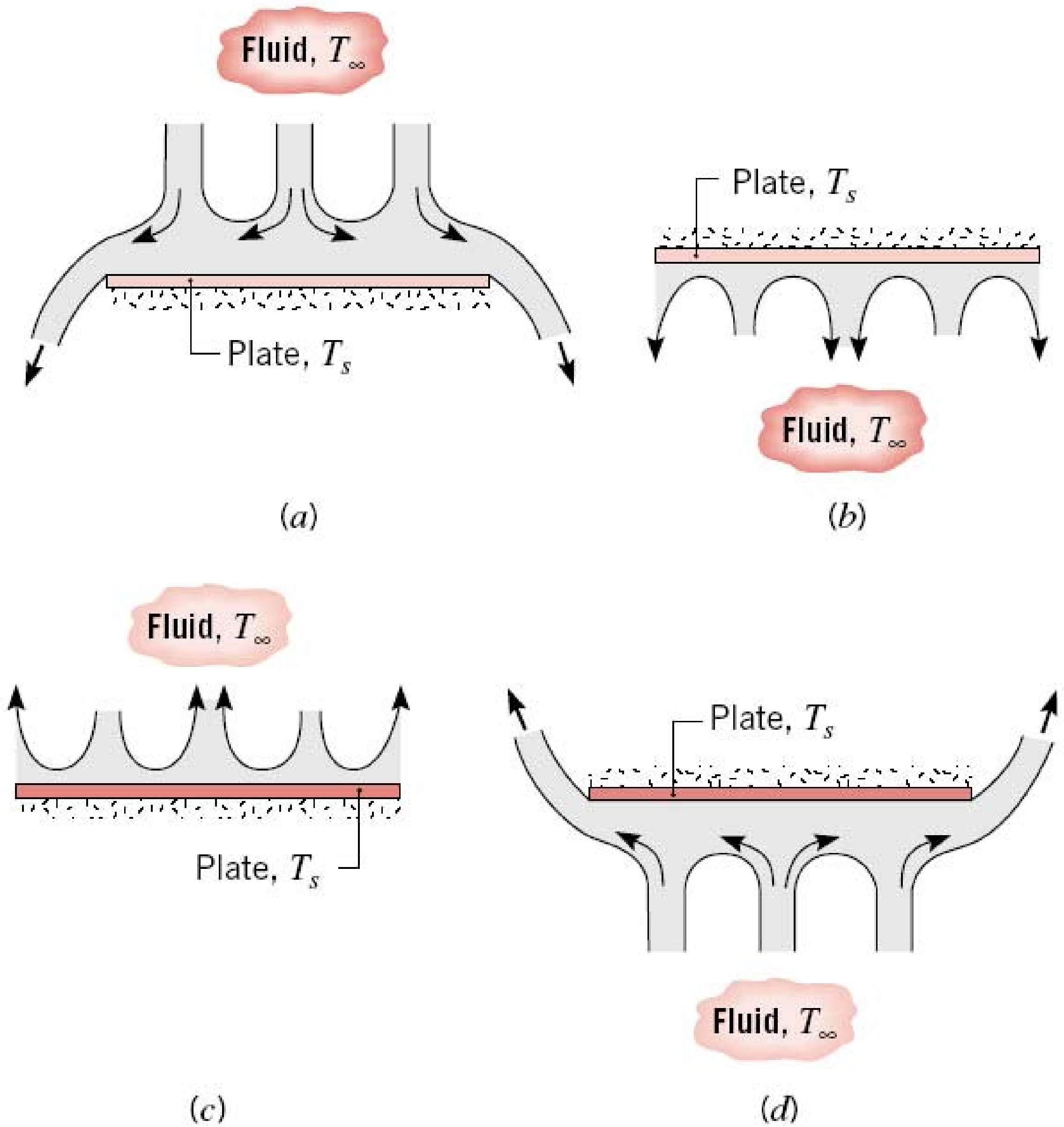


FIGURE 9.7 Buoyancy-driven flows on horizontal cold ($T_s < T_\infty$) and hot ($T_s > T_\infty$) plates: (*a*) top surface of cold plate, (*b*) bottom surface of cold plate, (*c*) top surface of hot plate, and (*d*) bottom surface of hot plate.

TABLE 9.2 Summary of free convection empirical correlations for immersed geometries

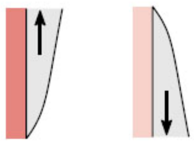
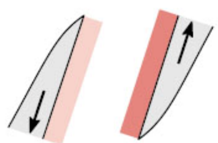
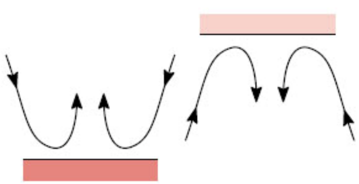
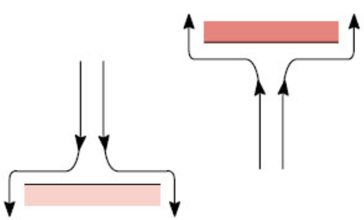
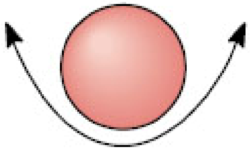
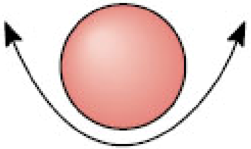
Geometry	Recommended Correlation	Restrictions
1. Vertical plates[a] 	Equation 9.26	None
2. Inclined plates Cold surface up or hot surface down 	Equation 9.26 $g \longrightarrow g \cos\theta$	$0 \leq \theta \lesssim 60°$
3. Horizontal plates (a) Hot surface up or cold surface down 	Equation 9.30 Equation 9.31	$10^4 \lesssim Ra_L \lesssim 10^7$ $10^7 \lesssim Ra_L \lesssim 10^{11}$
(b) Cold surface up or hot surface down 	Equation 9.32	$10^5 \lesssim Ra_L \lesssim 10^{10}$

Table 9.2 *Continued*

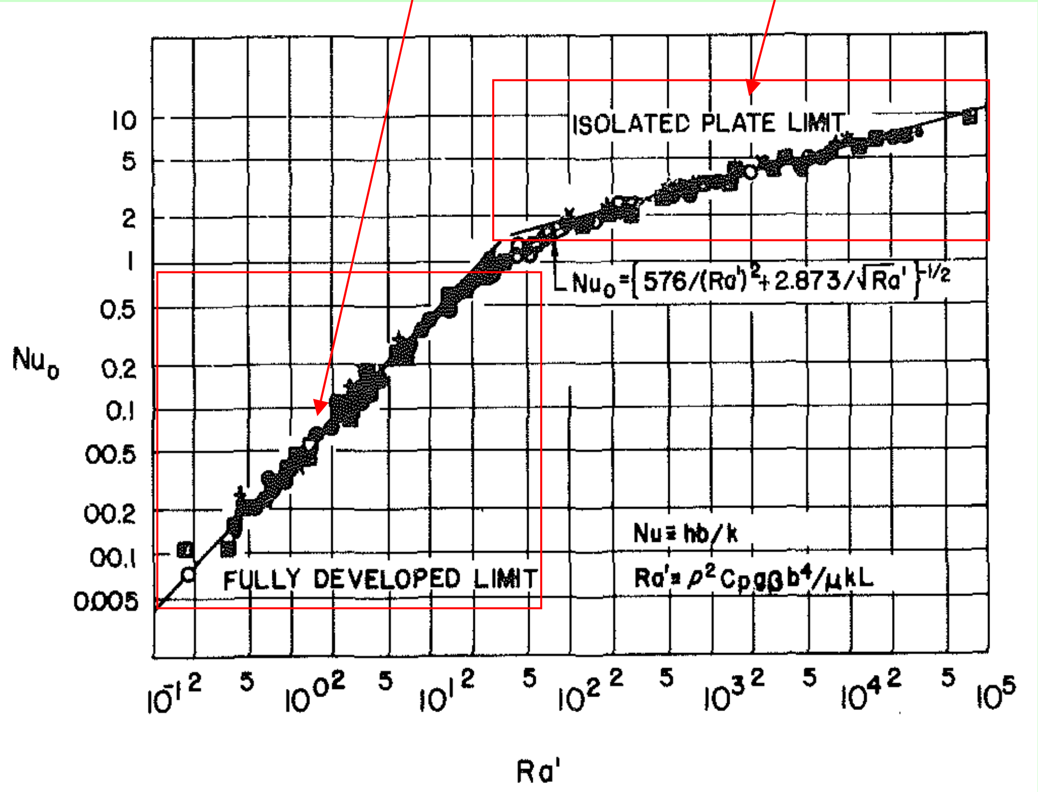
Geometry	Recommended Correlation	Restrictions
4. Horizontal cylinder 	Equation 9.34	$Ra_D \lesssim 10^{12}$
5. Sphere 	Equation 9.35	$Ra_D \lesssim 10^{11}$ $Pr \gtrsim 0.7$

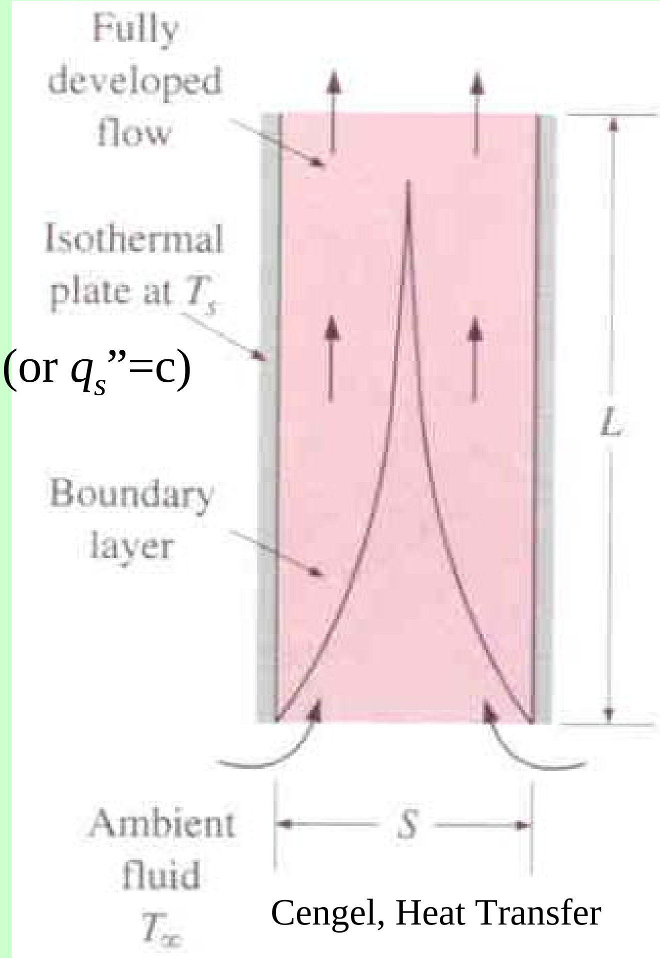
9.7 Natural Heat Transfer Between Parallel Plates

Vertical Parallel Plates:

$$\overline{Nu}_S = \left[\frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{\sqrt{Ra_S S/L}} \right]^{-1/2},$$

where $Ra_S \equiv g\beta(T_s - T_\infty)S^3/\alpha\nu$ (9.45) (or q_s"=c)





Cengel, Heat Transfer

Reference: A. Bar-Cohen and W.M. Rohsenow, Thermally optimum spacing of vertical, natural convection cooled, parallel plates, ASME J. Heat Transfer, 106 (1984) 116-123.

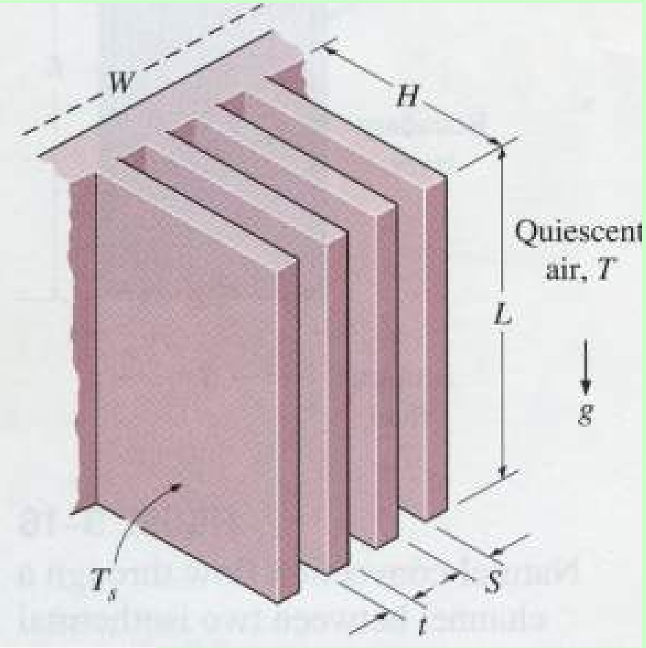
- Eq. (9.45) is suitable for different thermal conditions of the plates, isothermal or isoflux plates, symmetric or with one plate adiabatic. The different values of C_1 and C_2 for each condition are given in Table 9.3.

TABLE 9.3 Heat transfer parameters for free convection between vertical parallel plates

Surface Condition	C_1	C_2	S_{opt}	S_{max}/S_{opt}
Symmetric isothermal plates ($T_{s,1} = T_{s,2}$)	576	2.87	$2.71(Ra_S/S^3L)^{-1/4}$	1.71
Symmetric isoflux plates ($q''_{s,1} = q''_{s,2}$)	48	2.51	$2.12(Ra^*_S/S^4L)^{-1/5}$	4.77
Isothermal/adiabatic plates ($T_{s,1}, q''_{s,2} = 0$)	144	2.87	$2.15(Ra_S/S^3L)^{-1/4}$	1.71
Isoflux/adiabatic plates ($q''_{s,1}, q''_{s,2} = 0$)	24	2.51	$1.69(Ra^*_S/S^4L)^{-1/5}$	4.77

- Eq. (9.45) is commonly used for vertical plate heat sinks, although this can be inaccurate for short fins (H/S<5) due to additional boundary layers near the base plate corners.
- For inclined parallel plates, for $0 \leqq \theta \leqq 45^\circ$ and within the isolate plate limit, $Ra_S(S/L)$>200,

$$\overline{Nu}_S = 0.645(Ra_S S/L)^{1/4} \qquad (9.47)$$



Cengel, Heat Transfer

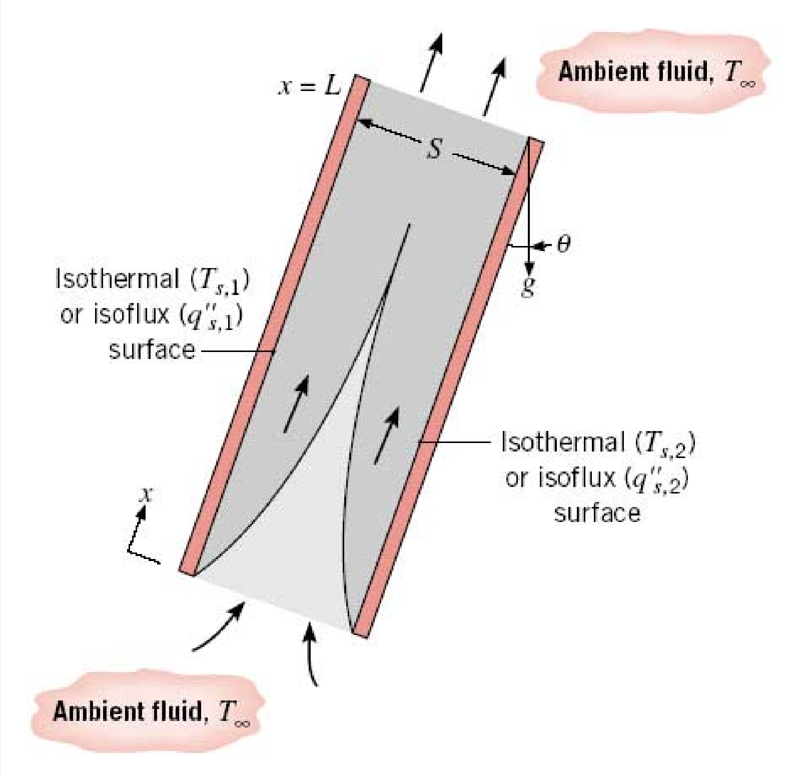


FIGURE 9.9
Free convection flow between heated parallel plates with opposite ends exposed to a quiescent fluid.