Chapter 6

Fundamental Concepts of Convection

6.1 The Convection Boundary Layers

Velocity boundary layer:

surface shear stress: $\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$ (6.2)

local friction coeff.: $C_f \equiv \frac{\tau_s}{\rho u_\infty^2 / 2}$ (6.1)

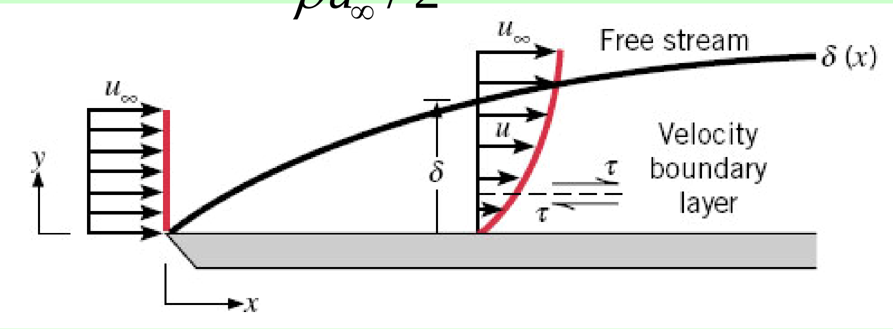


FIGURE 6.1
Velocity boundary layer development on a flat plate.

Thermal boundary layer:

local heat flux: $q_s'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}$ (6.3)

local heat transfer coeff.: $h = \frac{q_s''}{T_s - T_\infty} = \frac{-k_f \,\partial T / \partial y|_{y=0}}{T_s - T_\infty}$ (6.5)

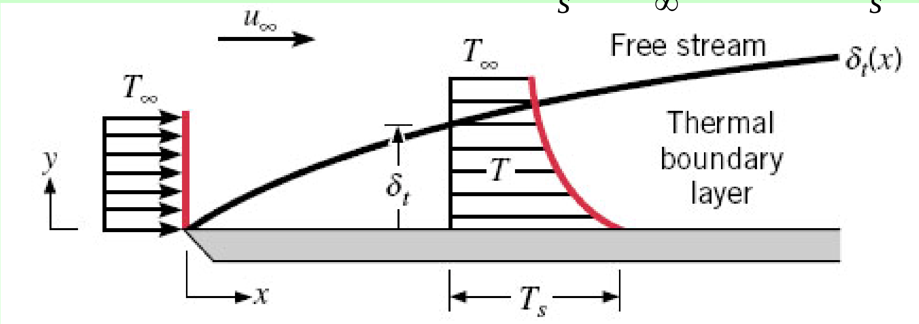


FIGURE 6.2
Thermal boundary layer development on an isothermal flat plate.

Concentration boundary layer:

local species flux: $$N''_A = -D_{AB} \left.\frac{\partial C_A}{\partial y}\right|_{y=0} \quad [\text{kmol/s} \cdot \text{m}^2] \qquad (6.6)$$

or (mass basis): $$n''_A = -D_{AB} \left.\frac{\partial \rho_A}{\partial y}\right|_{y=0} \quad [\text{kg/s} \cdot \text{m}^2]$$

local mass transfer coeff.: $$h_m = \frac{N''_{A,s}}{C_{A,s} - C_{A,\infty}} = \frac{-D_{AB}\, \partial C_A / \partial y|_{y=0}}{C_{A,s} - C_{A,\infty}} \qquad (6.9)$$

or (mass basis): $$h_m = \frac{-D_{AB}\, \partial \rho_A / \partial y|_{y=0}}{\rho_{A,s} - \rho_{A,\infty}}$$

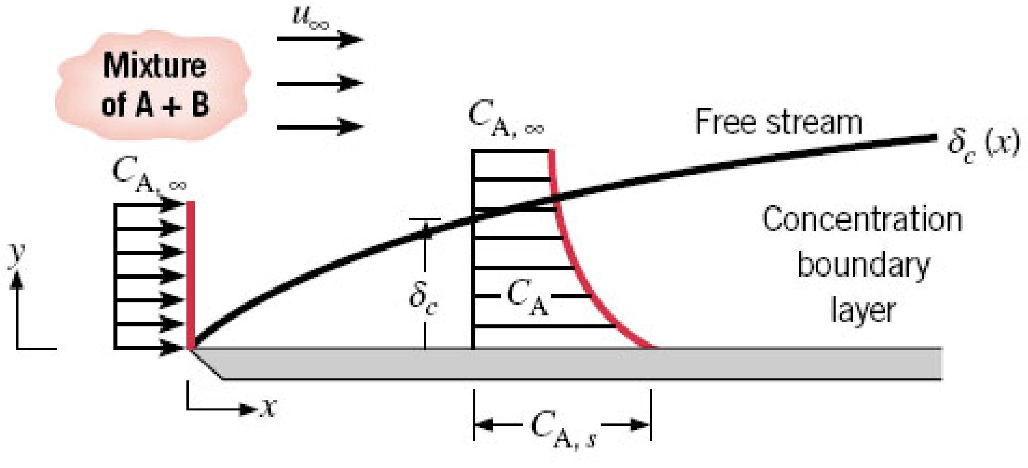


FIGURE 6.3
Species concentration boundary layer development on a flat plate.

6.1.4 Significance of the Boundary Layers

- **Velocity boundary layer:** always exists for flow over any surface
- **Thermal boundary layer:** exists if the surface and free stream temperature differ
- **Concentration boundary layer:** exists if the surface concentration of a species differs from the free stream value

The principal manifestations and boundary layer parameters are

- **Velocity boundary layer**: surface friction and friction coefficient C_f
- **Thermal boundary layer**: convection heat transfer and heat transfer convection coefficient h
- **Concentration boundary layer**: convection mass transfer and mass transfer convection coefficient h_m

6.2 Local and Average Convection Coefficients

6.2.1 Heat Transfer

The *local* heat flux q'' may be expressed as (Newton's law of cooling)

$q'' = h(T_s - T_\infty)$, h = heat transfer convection coeff.

= f (fluid properties, surface geometry, flow conditions)

The *total* heat transfer rate q may be obtained by integration

$$q = \int_{A_s} q'' dA_s = (T_s - T_\infty)\int_{A_s} h dA_s \text{, if } T_s \text{ is uniform} \quad (6.10\text{-}12)$$

$$= \bar{h} A_s (T_s - T_\infty),$$

where $$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s \ . \quad (6.13)$$

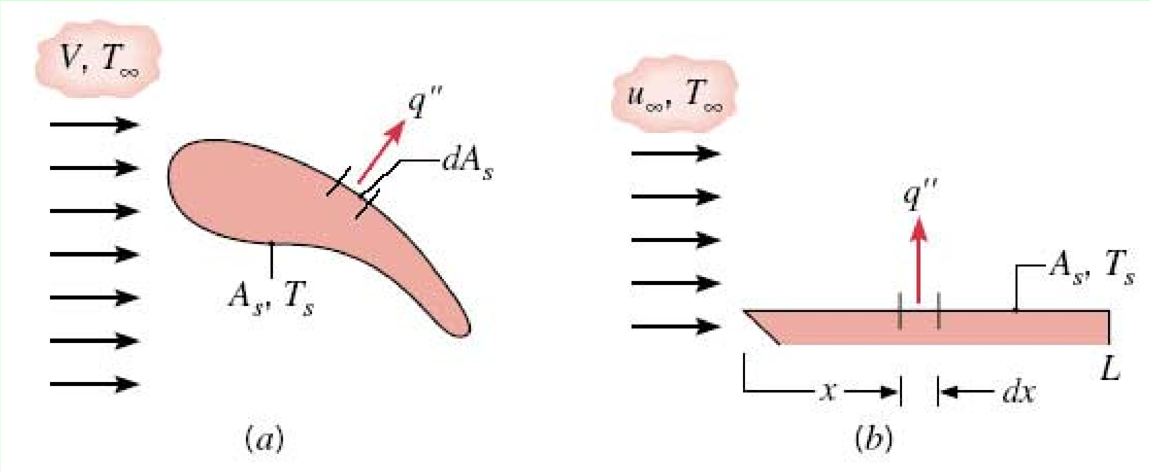


FIGURE 6.4 Local and total convection heat transfer. (a) Surface of arbitrary shape. (b) Flat plate.

6.2.2 Mass Transfer

Similar for mass transfer of species A, we have

$$N''_{\mathrm{A}} = h_m (C_{\mathrm{A},s} - C_{\mathrm{A},\infty}) \ \ [\mathrm{kmol/s \cdot m^2}]$$ --local

--Molar transfer rate

$$N_{\mathrm{A}} = \bar{h}_m A_s (C_{\mathrm{A},s} - C_{\mathrm{A},\infty}) \ \ [\mathrm{kmol/s}]$$ --average (6.15)

where $\bar{h}_m = \int_{A_s} h_m dA_s$ (6.16)

or $n''_{\mathrm{A}} = h_m (\rho_{\mathrm{A},s} - \rho_{\mathrm{A},\infty}) \ \ [\mathrm{kg/s \cdot m^2}]$ (6.18)

--Mass transfer rate

$$n_{\mathrm{A}} = \bar{h}_m A_s (\rho_{\mathrm{A},s} - \rho_{\mathrm{A},\infty}) \ \ [\mathrm{kg/s}]$$ (6.19)

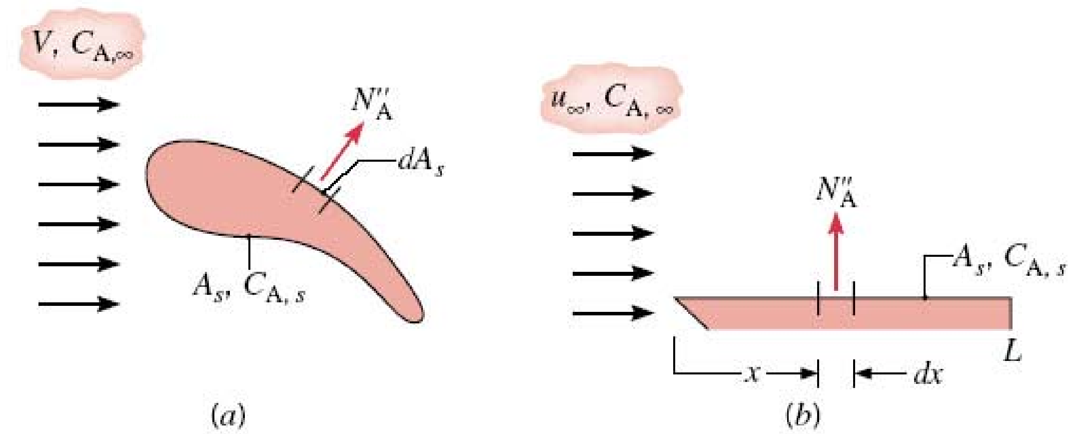


FIGURE 6.5 Local and total convection species transfer. (a) Surface of arbitrary shape. (b) Flat plate.

We can also write Fick's law on a mass basis by multiplying Eq. 6.7 by $\mathcal{M}_A$ to yield

$$n''_A = -D_{AB} \left.\frac{\partial \rho_A}{\partial y}\right|_{y=0} = h_m(\rho_{A,s} - \rho_{A,\infty}) \tag{6.20}$$

$$h_m = \frac{-D_{AB}\,\partial \rho_A / \partial y\big|_{y=0}}{\rho_{A,s} - \rho_{A,\infty}} \tag{6.21}$$

The value of $C_{A,s}$ or $\rho_{A,s}$ can be determined by assuming thermodynamic equilibrium at the interface between the gas and the liquid or solid surface. Thus,

$$C_{A,s} = \frac{p_{sat}(T_s)}{\mathcal{R}T} \tag{6.22}$$

6.2.3 The Problem of Convection

The *local* flux and/or the total transfer rate are of paramount importance in any convection problem. So, determination of these coefficients (local h or h_{m} and average $\bar{h}$ or $\bar{h}_m$) is viewed as *the problem of convection*.

However, the problem is not a simple one, as

$$\bar{h} \text{ or } \bar{h}_m = f(\text{fluid properties, surface geometry, flow conditions})$$

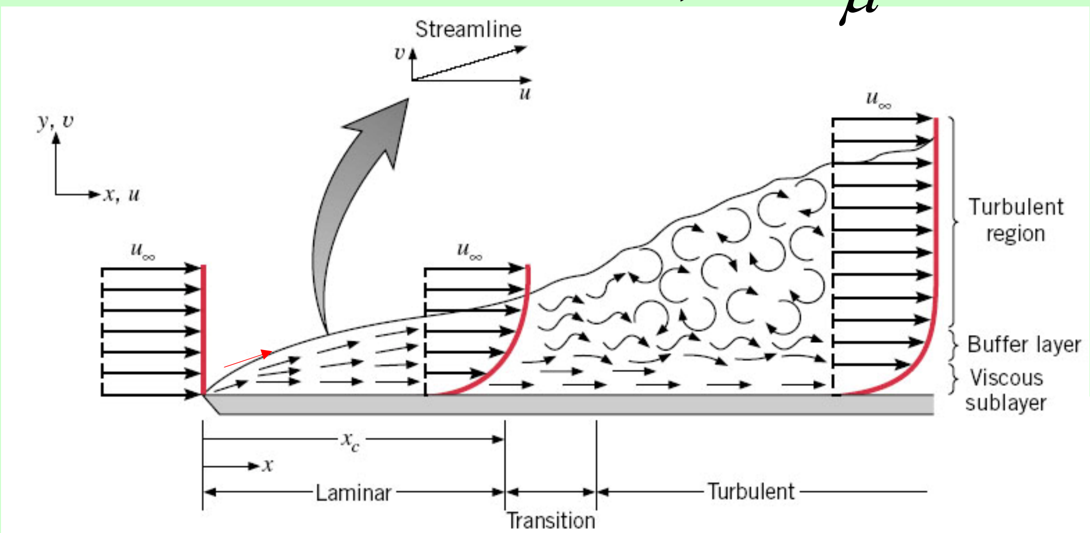
$$\uparrow \text{i.e., } \rho, \mu, k_f, c_{p,f}$$

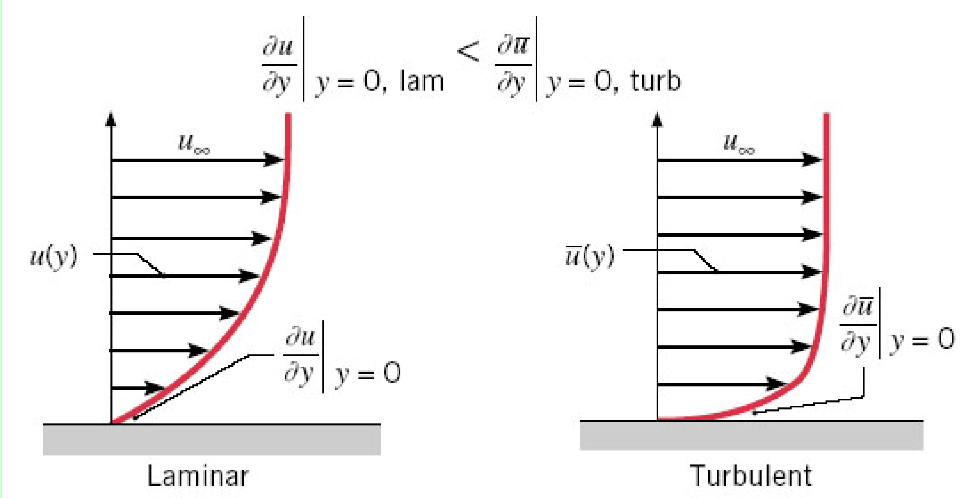
EXs 6.1-6.3

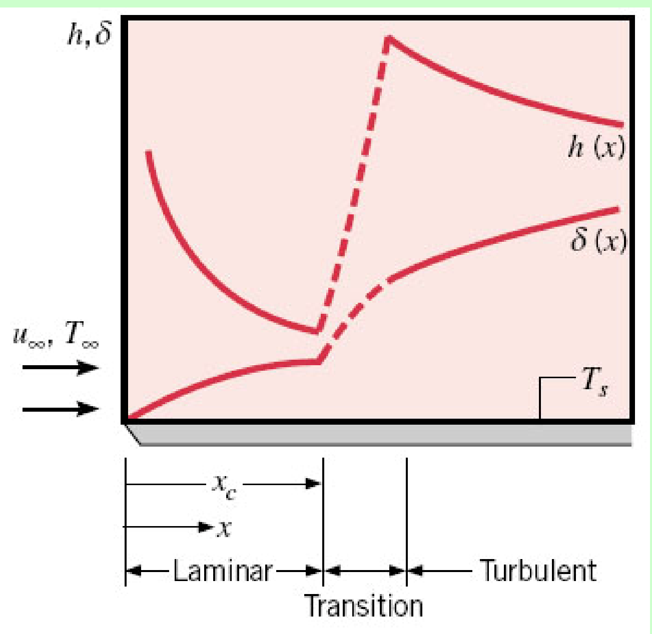
6.3 Laminar and Turbulent Flow

Critical Reynolds no. where transition from laminar boundary layer to turbulent occurs:

$$Re_{x,c} = \frac{\rho u_\infty x_c}{\mu} = 5 \times 10^5$$







EX 6.4

6.4 The Boundary Layer Equations (2-D, Steady)

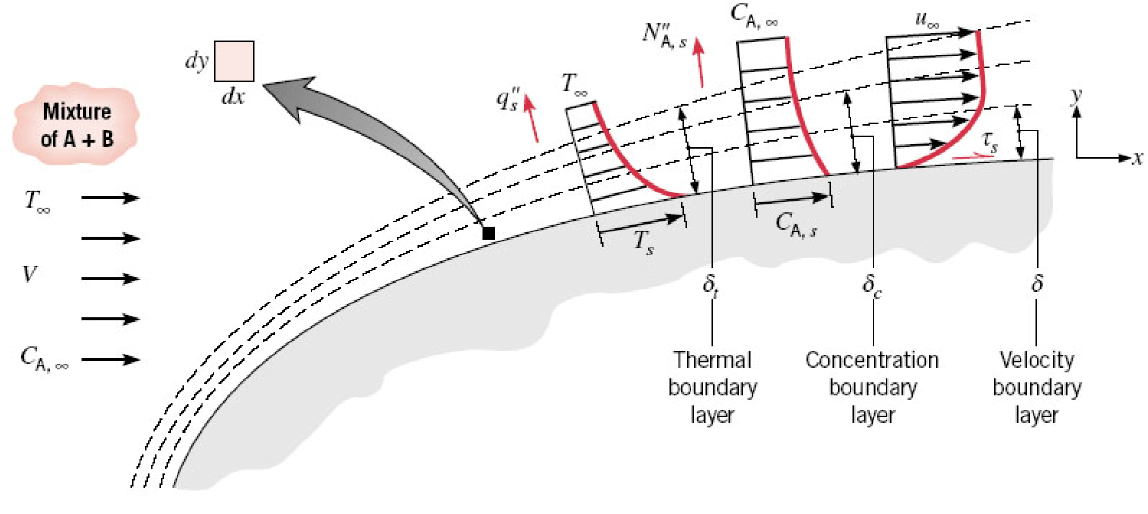


FIGURE 6.9 Development of the velocity, thermal, and concentration boundary layers for an arbitrary surface.

6.4.1 Boundary Layer Equations for Laminar Flow

For the *steady, two-dimensional flow* of an *incompressible fluid* with *constant properties*, the following equations are involved. (Read 6S.1 for detailed derivation.)

- continuity eq.: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{(D.1)}$$

- momentum eq. (incompressible):

x-dir. $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+X \qquad \text{(D.2)}$$

y-dir. $$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+Y \qquad \text{(D.3)}$$

- energy equation:

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\Phi+\dot{q} \qquad \text{(D.4)}$$

energy transport thru convection — heat transport thru conduction — heat generation

where $$\mu\Phi=\mu\left\{\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]\right\} \qquad \text{(D.5)}$$

viscous dissipation

- species equations:

$$u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y}=D_{\mathrm{AB}}\left(\frac{\partial^2 C_A}{\partial x^2}+\frac{\partial^2 C_A}{\partial y^2}\right)+\dot{N}_{\mathrm{A}} \qquad \text{(D.6)}$$

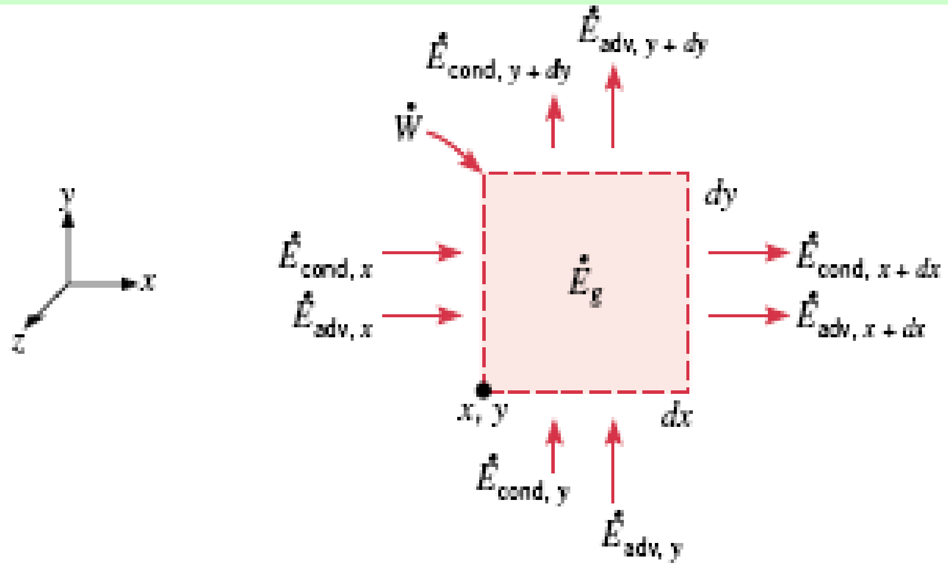


FIGURE 6S.5 Differential control volume $(dx \cdot dy \cdot 1)$ for energy conservation in two-dimensional flow of a viscous fluid with heat transfer.

$$\dot{E}_{\text{adv},x} - \dot{E}_{\text{adv},x+dx} \equiv \rho u\left(e + \frac{V^2}{2}\right)dy - \left\{\rho u\left(e + \frac{V^2}{2}\right) + \frac{\partial}{\partial x}\left[\rho u\left(e + \frac{V^2}{2}\right)\right]dx\right\}dy$$

$$= -\frac{\partial}{\partial x}\left[\rho u\left(e + \frac{V^2}{2}\right)\right]dx\,dy \qquad (6S.15)$$

$$\dot{E}_{\text{cond},x} - \dot{E}_{\text{cond},x+dx} = -\left(k\frac{\partial T}{\partial x}\right)dy - \left[-k\frac{\partial T}{\partial x} - \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]dy$$

$$= \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\,dy \qquad (6S.16)$$

Boundary layer approximations:

$$u >> v$$

$$\frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x}, \quad \frac{\partial v}{\partial y}, \quad \frac{\partial v}{\partial x}$$ Velocity boundary layer

$$\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x}$$ Thermal boundary layer

$$\frac{\partial C_A}{\partial y} >> \frac{\partial C_A}{\partial x}$$ Concentration boundary layer

- Usual additional simplifications: incompressible, constant properties, negligible body forces, nonreacting, no energy generation

Eqs. 6.27 is unchanged and the x-mom equation reduces to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp_\infty}{dx}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (6.28)$$

The energy equation reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (6.29)$$

usually negligible

energy transport thru convection

heat transport thru conduction

and the species equation becomes

$$u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y}=D_{\mathrm{AB}}\frac{\partial^2 C_A}{\partial y^2} \qquad (6.30)$$

- For incompressible, constant property flow, Eq. (6.28) is *uncoupled* from (6.29) & (6.30). Eqs. (6.27) & (6.28) need to be solved first, for the velocity field $u(x, y)$ and $v(x, y)$, before (6.29) & (6.30) can be solved, i.e., the temperature and species fields are *coupled* to the velocity field.

6.5 Boundary Layer Similarity: The Normalized Convection Transfer Equations

6.5.1 Boundary Layer Similarity Parameters

Defining $x^* \equiv \frac{x}{L}$ and $y^* \equiv \frac{y}{L}$ L is the characteristic length

$u^* \equiv \frac{u}{V}$ and $v^* \equiv \frac{v}{V}$ V is the free stream velocity

$$T^* \equiv \frac{T - T_s}{T_\infty - T_s} \quad \text{and} \quad C_A^* \equiv \frac{C_A - C_{A,s}}{C_{A,\infty} - C_{A,s}}$$

we arrive at the convection transfer equations and their boundary conditions in nondimensional form, as shown in Table 6.1.

Dimensionless groups:

- Reynolds no.: $Re_L \equiv \frac{VL}{\nu}$
- Prandtl no.: $Pr \equiv \frac{\nu}{\alpha}$
- Schmidt no.: $Sc \equiv \frac{\nu}{D_{AB}}$

The final dimensionless governing eqs.: (6.35)-(6.37)--*similar* in form

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} \qquad (6.35)$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{\mathrm{Re}_L\,\mathrm{Pr}}\frac{\partial^2 T^*}{\partial y^{*2}} \qquad (6.36)$$

$$u^*\frac{\partial C_A^*}{\partial x^*}+v^*\frac{\partial C_A^*}{\partial y^*}=\frac{1}{\mathrm{Re}_L\,\mathrm{Sc}}\frac{\partial^2 C_A^*}{\partial y^{*2}} \qquad (6.37)$$

TABLE 6.1 The boundary layer equations and their y-direction boundary conditions in nondimensional form

Boundary Layer	Conservation Equation	Boundary Conditions: Wall	Boundary Conditions: Free Stream	Similarity Parameter(s)
Velocity	$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$ (6.35)	$u^*(x^*,0)=0$ $v^*(x^*,0)=0$	$u^*(x^*,\infty)=\frac{u_\infty(x^*)}{V}$ (6.38)	$Re_L=\frac{VL}{\nu}$ (6.41)
Thermal	$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$ (6.36)	$T^*(x^*,0)=0$	$T^*(x^*,\infty)=1$ (6.39)	$Re_L,\ Pr=\frac{\nu}{\alpha}$ (6.42)
Concentration	$u^*\frac{\partial C_A^*}{\partial x^*}+v^*\frac{\partial C_A^*}{\partial y^*}=\frac{1}{Re_L Sc}\frac{\partial^2 C_A^*}{\partial y^{*2}}$ (6.37)	$C_A^*(x^*,0)=0$	$C_A^*(x^*,\infty)=1$ (6.40)	$Re_L,\ Sc=\frac{\nu}{D_{AB}}$ (6.43)

6.5.2 Functional Form of the Solutions

From (6.35), we can write

$$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*} \right) \qquad (6.44)$$

where *dp*/dx** depends on the surface geometry.

The shear stress and the friction coefficient at the surface are

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left(\frac{\mu V}{L} \right) \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

$$C_f = \frac{\tau_s}{\rho V^2 / 2} = \frac{2}{\mathrm{Re}_L} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \frac{2}{Re_L} f(x^*, Re_L) \qquad (6.45, 46)$$

For a prescribed geometry, (6.45) is *universally* applicable.

From (6.36)

$$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*} \right) \quad (6.47)$$

$$h = -\frac{k_f}{L}\frac{(T_\infty - T_s)}{(T_s - T_\infty)}\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} = +\frac{k_f}{L}\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

Nusselt number can be defined as

$$Nu \equiv \frac{hL}{k_f} = +\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \quad (6.48)$$

For a prescribed geometry, from (6.47)

$$Nu = f\left(x^*, Re_L, Pr\right) \quad (6.49)$$

The spatially average Nusselt number is then

$$\overline{Nu} = \frac{\bar{h}L}{k_f} = f\left(Re_L, Pr\right) \quad (6.50)$$

Similarly for mass transfer, $\overline{Sh} = \frac{\bar{h}_m L}{D_{AB}} = f\left(Re_L, Sc\right)$ (6.54)

EX 6.5

6.6 Physical Significance of the Dimensionless Parameters

See Table 6.2.

- For heat transfer, the important dimensionless parameters are:

  Re, Nu, Pr, Bi, Fo, Pe (= $RePr$), Gr

- For mass transfer,

  Re, Sh, Sc, Bi_m, Fo_m, Le (= α/D_{AB})

For boundary layer thicknesses,

$$\frac{\delta}{\delta_t} \approx Pr^n, \quad \frac{\delta}{\delta_c} \approx Sc^n, \quad \frac{\delta_t}{\delta_c} \approx Le^n \qquad n = \frac{1}{3}$$ for laminar boundary layer (6.55, 56, 58)

$$\delta \approx \delta_t \approx \delta_c$$ for turbulent boundary layer (why?)

6.7 Boundary Layer Analogies

6.7.1 The Heat and Mass Transfer Analogy

TABLE 6.3 Functional relations pertinent to the boundary layer analogies

Fluid Flow		Heat Transfer		Mass Transfer	
$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right)$	(6.44)	$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$	(6.47)	$C_A^* = f\left(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*}\right)$	(6.51)
$C_f = \frac{2}{Re_L} \frac{\partial u^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.45)	$Nu = \frac{hL}{k} = +\frac{\partial T^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.48)	$Sh = \frac{h_m L}{D_{AB}} = +\frac{\partial C_A^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.52)
$C_f = \frac{2}{Re_L} f(x^*, Re_L)$	(6.46)	$Nu = f(x^*, Re_L, Pr)$	(6.49)	$Sh = f(x^*, Re_L, Sc)$	(6.53)
		$\overline{Nu} = f(Re_L, Pr)$	(6.50)	$\overline{Sh} = f(Re_L, Sc)$	(6.54)

- Since the dimensionless relations that govern the thermal and the concentration boundary layers are the same, *the heat and mass transfer relations for a particular geometry are interchangeable*.

With

$$Nu = f(x^*, Re_L)Pr^n$$

and

$$Sh = f(x^*, Re_L)Sc^n$$

and equivalent functions, $f(x^*, Re_L)$,

Then, $$\frac{Nu}{Pr^n} = \frac{Sh}{Sc^n} \rightarrow \frac{hL/k}{Pr^n} = \frac{h_m L / D_{AB}}{Sc^n}$$

and we have $$\frac{h}{h_m} = \frac{k}{D_{AB} Le^n} = \rho c_p Le^{1-n}$$, n=1/3 for most applications

(6.60)

EX 6.6

6.7.3 The Reynolds Analogy

- For $dp^*/dx^* = 0$ and $Pr = Sc = 1$, Eqs. 6.35-6.37 are of precisely the same form. Moreover, if $dp^*/dx^*=0$, the boundary conditions, 6.38-6.40 also have the same form. From Equations 6.45, 6.48, 6.52, it follows that

$$C_f \frac{Re_L}{2} = Nu = Sh \qquad (6.66)$$

- Replacing Nu and Sh with Stanton number (St) and mass transfer Stanton number (St_m), as defined in (6.67) and (6.68), we have

$$C_f / 2 = St = St_m \qquad \text{-- Reynolds Analogy} \qquad (6.69)$$

$$St \equiv \frac{h}{\rho V c_p} = \frac{Nu}{RePr}, \quad St_m \equiv \frac{h_m}{V} = \frac{Sh}{ReSc}$$

- The modified Reynolds Analogy (or Chilton-Colburn Analogy) is

$$\frac{C_f}{2} = StPr^{2/3} \equiv j_H = \frac{Nu}{RePr^{1/3}}; \; \frac{C_f}{2} = St_m Sc^{2/3} \equiv j_m = \frac{Sh}{ReSc^{1/3}} \qquad (6.70,71)$$

Eqs. 6.70, 71 are appropriate for laminar flow when $dp^*/dx^* \sim 0$; for turbulent flow, conditions are less sensitive to pressure gradient.

6.7.2 Evaporative Cooling

For evaporative cooling shown in Fig. 6.10,

$$q''_{\text{conv}} + q''_{\text{add}} = q''_{\text{evap}}$$

where $q''_{\text{evap}} = n''_{\text{A}} h_{fg}$ (6.62)

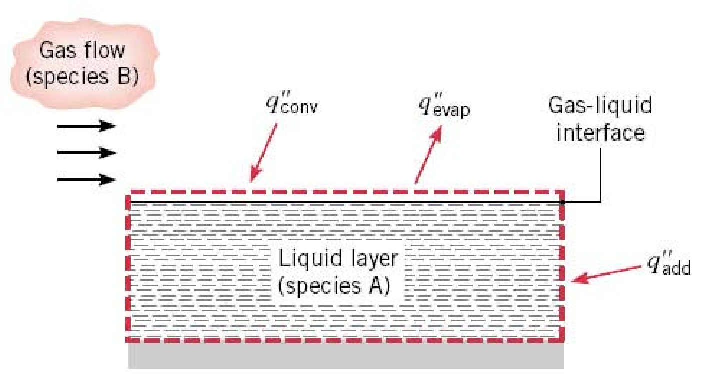


FIGURE 6.10 Latent and sensible heat exchange at a gas–liquid interface.

If q''_{add} is absent, then

$$h(T_\infty - T_s) = h_{fg} h_m [\rho_{\text{A,sat}}(T_s) - \rho_{\text{A},\infty}]$$

$$\rightarrow T_\infty - T_s = h_{fg}\left(\frac{h_m}{h}\right)[\rho_{\text{A,sat}}(T_s) - \rho_{\text{A},\infty}] \qquad (6.64)$$

Using the heat and mass transfer analogy, (6.64) becomes

$$(T_\infty - T_s) = \frac{M_A h_{fg}}{\Re \rho c_p Le^{2/3}}\left[\frac{p_{\text{A,sat}}(T_s)}{T_s} - \frac{p_{\text{A},\infty}}{T_\infty}\right] \qquad (6.65)$$

Note: Gas properties ρ, c_p, Le should be evaluated at the arithmetic mean temperature of the thermal b. l., $T_{\text{am}} = (T_s + T_\infty)/2$.

EX 6.7

Special Topic—Heat Pipes

Representative Ranges of Convection Thermal Resistance

	h (W/m^{2}K)	Areal $R_{th,\text{conv}}$ (Kcm^2/W)
Natural Convection		
Air	2~25	5,000~400
Oils	20~200	500~50
Water	100~1,000	100~10
Forced Convection		
Air	20~200	500~50
Oils	200~2,000	50~5
Water	1,000~10,000	10~1
Microchannel Cooling	40,000	0.25
Impinging Jet Cooling	2,400~49,300	4.1~0.20
Heat Pipe		0.049*

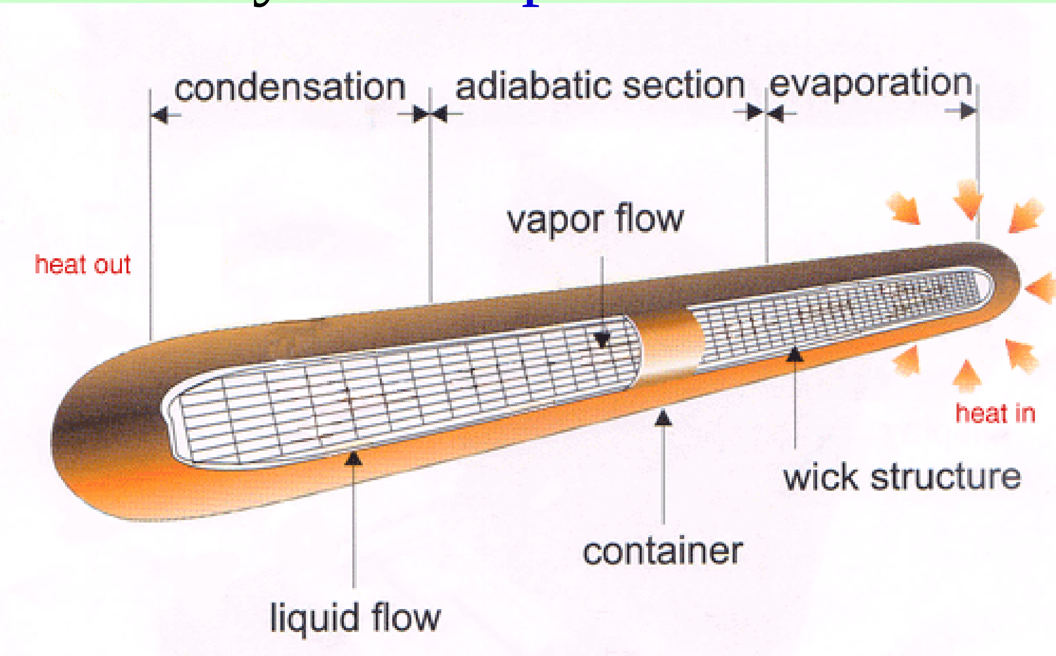
* based on THERMACORE 5 mm heat pipe having capacity of 20W with $\Delta T = 5K$.

The Working Principle of Heat Pipes

--Based on phase-change heat transfer

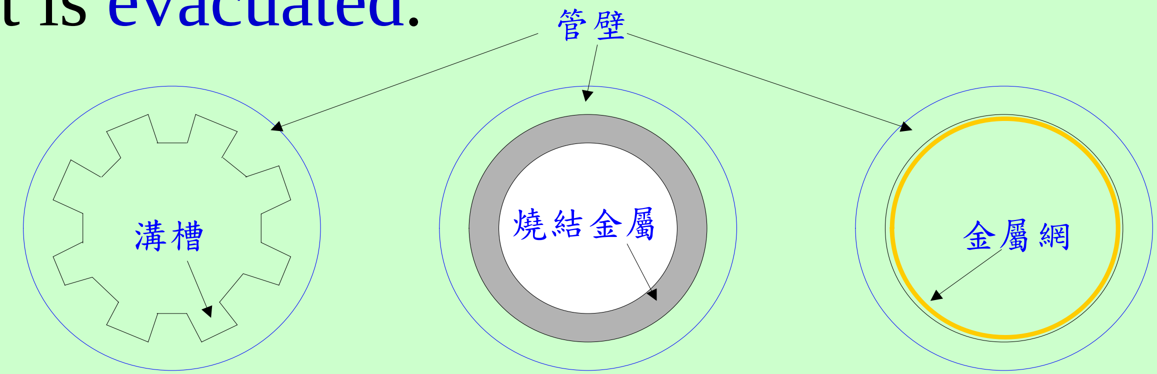
- Evaporation Section
- Adiabatic Section
- Condensation Section

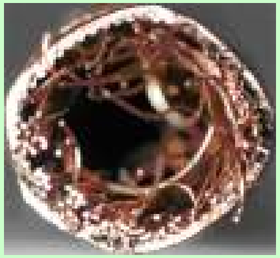
The performance of a heat pipe is dictated by its thermal resistance R_{th} and maximum heat load, Q_{max}, which are mainly determined by the evaporation characteristics.



A small amount of working fluid (mostly water) is filled in the heat pipe after it is evacuated.

- container
- Working fluid
- wick





Advantages of Heat Pipes

- superior heat spreading ability
- fast thermal response
- light weight and flexible
- low cost
- simple without active units