# Chapter 4

# Steady-State Multi-Dimensional Conduction

# 4.1 Alternative Approaches

For 2-D, steady-state conduction with no heat generation and constant *k*, the differential equation (2.16) reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

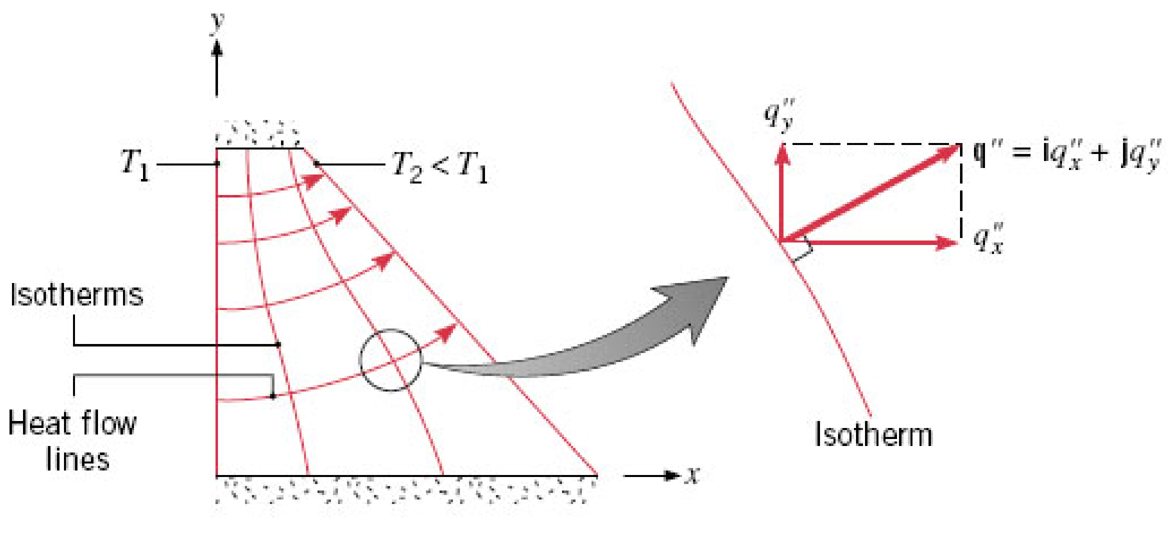


FIGURE 4.1 Two-dimensional conduction.

Three approaches to solve Eq. 4.1:

- Analytic--exact solution
- Graphical--rough
- Numerical--for complex geometries

## 4.2 The Method of Separation of Variables

**Notes**: The method of separation of variables is applicable to steady-state, 2-D problems if

i) one of the directions of the problem is expressed by a *homogeneous differential equation* subject to *homogeneous boundary conditions* (*the homogeneous direction*), while the other direction is expressed by a homogeneous differential equation subject to one homogeneous and one nonhomogeneous boundary condition (*the nonhomogeneous direction*), and

ii) the sign of $\lambda^2$ is chosen such that the boundary-value problem of the *homogeneous direction* leads to an *eigenvalue problem*.

## Steps of the Method of Separation of Variables

Read the textbook.

1. Separate the PDE into 2 interconnected ODEs.
2. Choose the homogeneous direction (homogeneous ODE with two homogeneous BCs.), solve both ODEs with one homogeneous BC for each, and combine their solutions into one group.
3. Apply the third homogeneous BC to determine the eigenvalues.
4. Make linear combinations of the combined solution set into a Fourier series to which the last nonhomogeneous BC is applied.
5. Coefficients of each term of the Fourier series can be determined for the last BC based on the orthogonality of the eigenfunction set.