Chapter 12

Fundamental Concepts of Radiation

12.1 Fundamental Concepts

Radiation heat exchange: Fig. 12.1

Volumetric emission and surface emission: Fig. 12.2

Thermal radiation: the electromagnetic wave with wavelength between 0.1 to 100 μm (Fig. 12.3), including a portion of ultraviolet (UV), visible light and infrared (IR).

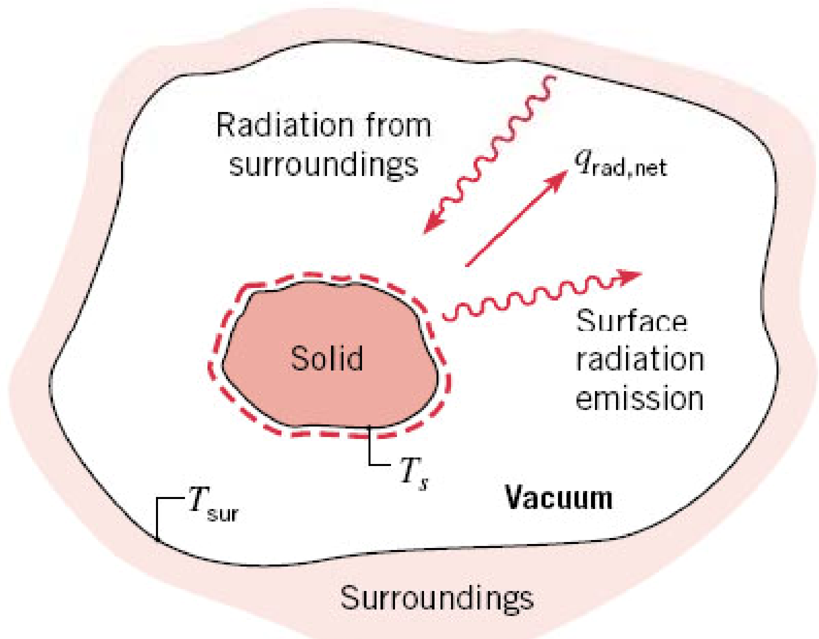


FIGURE 12.1
Radiation cooling of a heated solid.

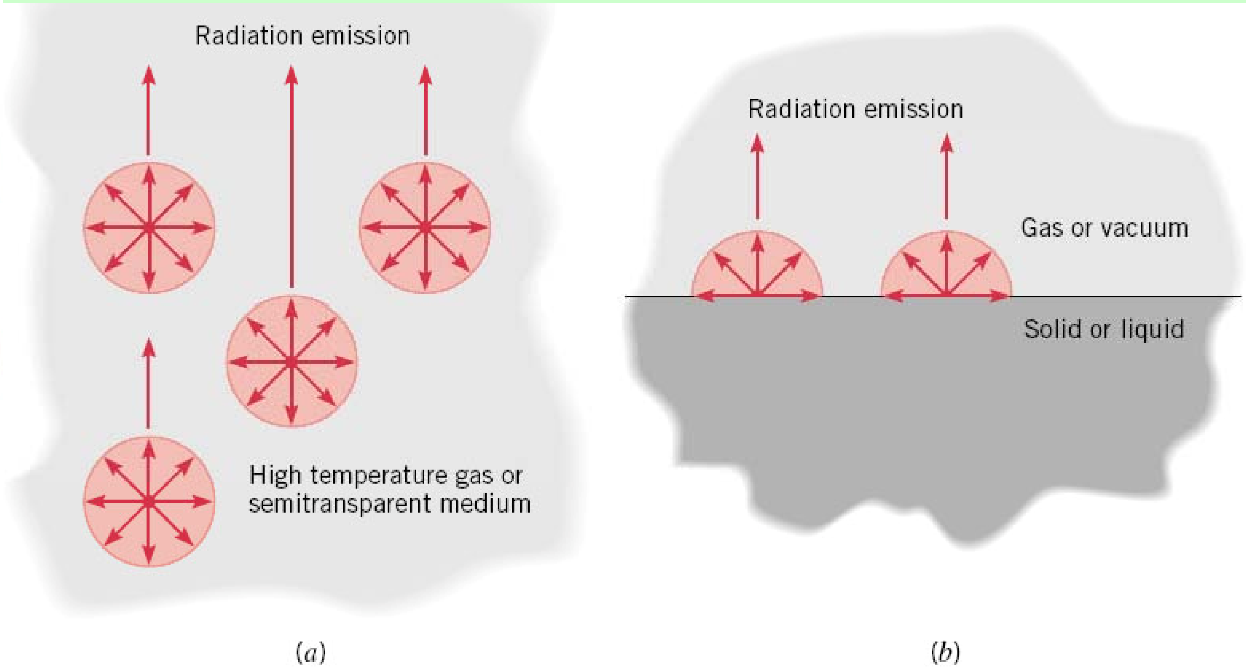


FIGURE 12.2 The emission process. (a) As a volumetric phenomenon. (b) Surface phenomenon.

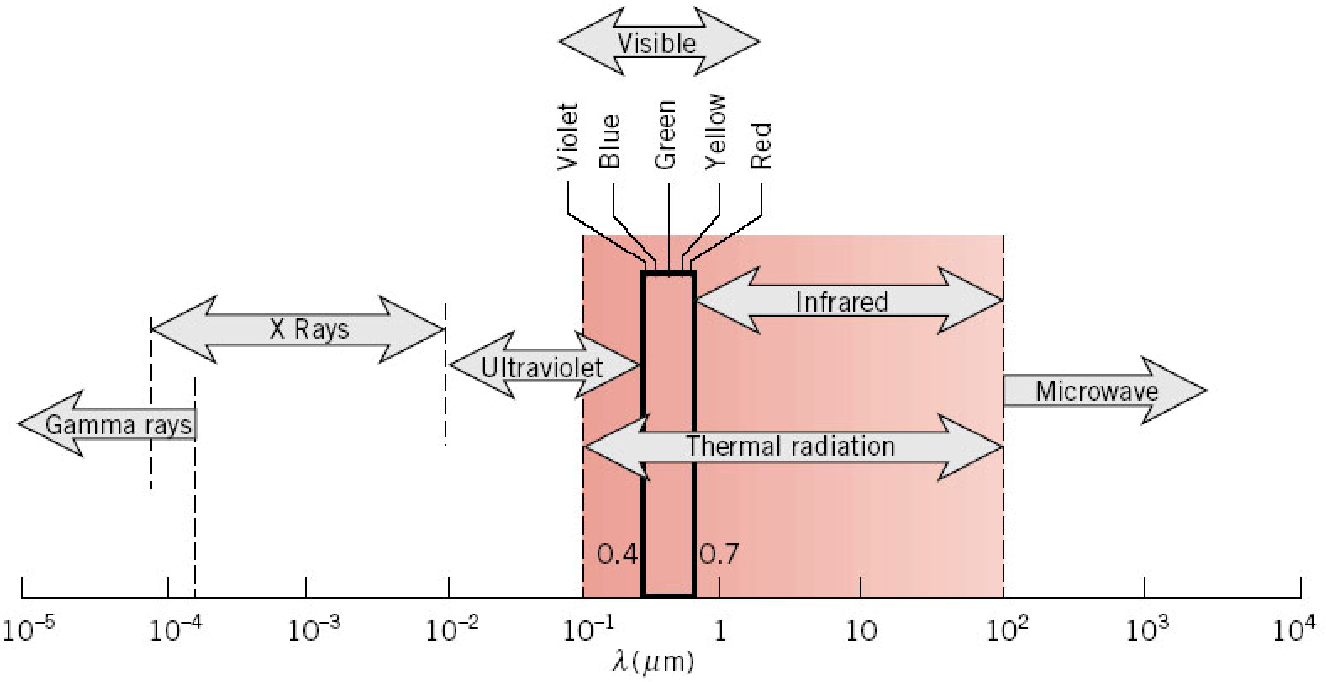


FIGURE 12.3 Spectrum of electromagnetic radiation.

Spectral and *directional* distribution: Fig. 12.4

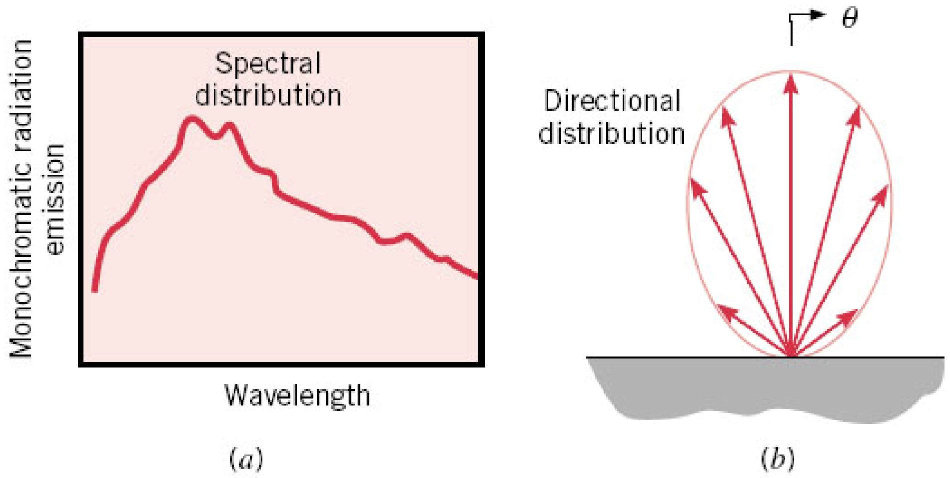
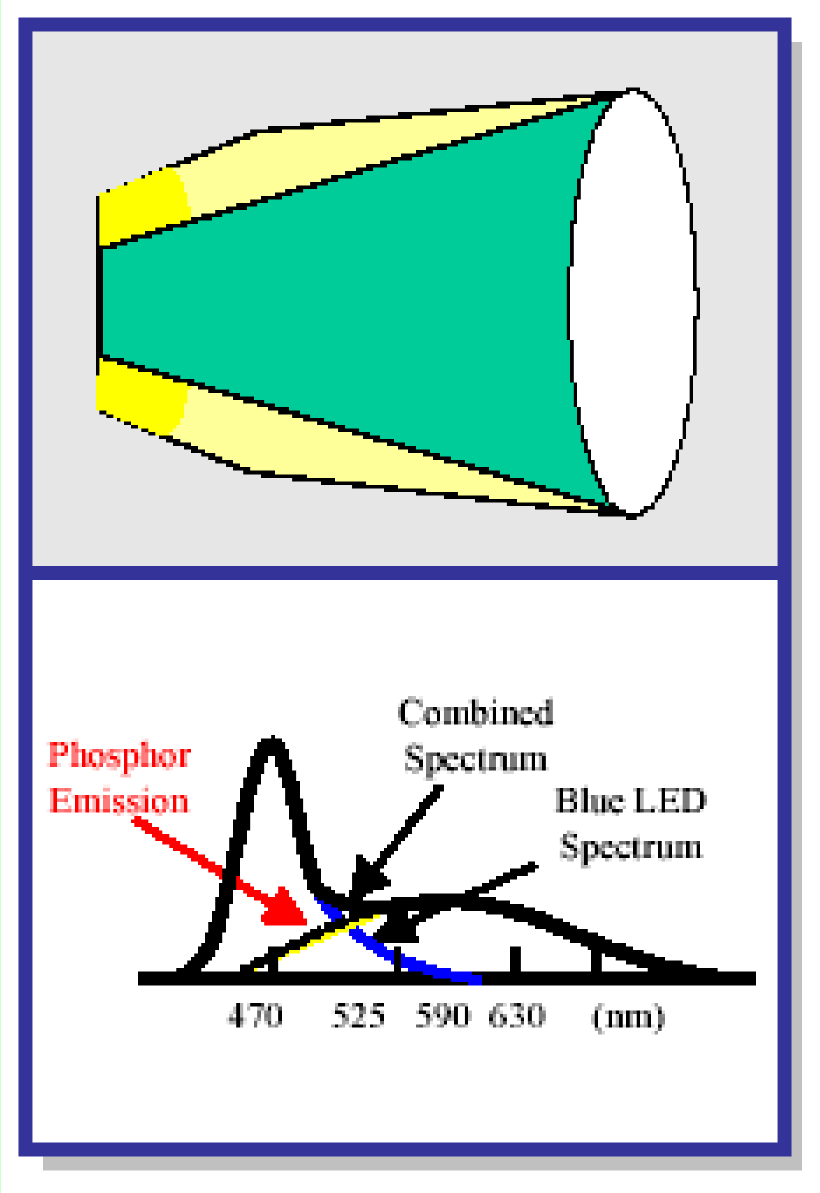


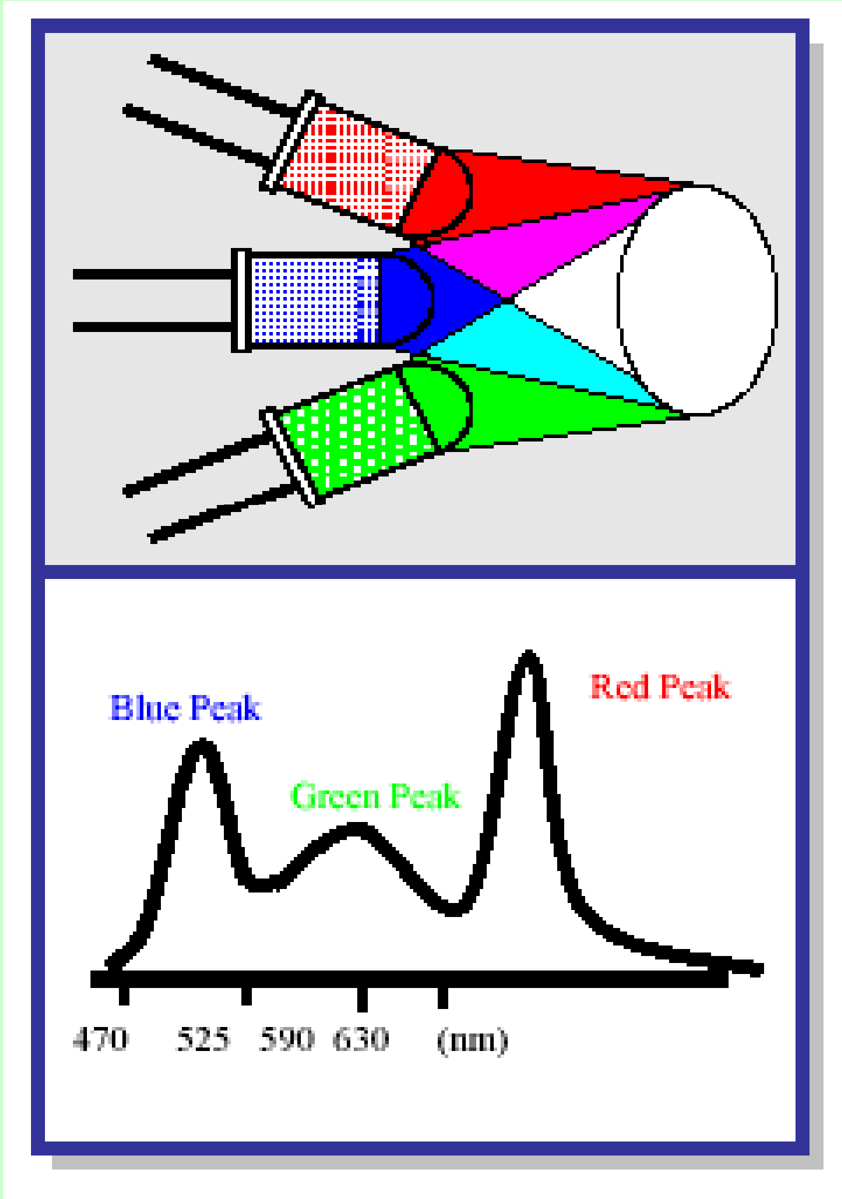
FIGURE 12.4 Radiation emitted by a surface. (*a*) Spectral distribution. (*b*) Directional distribution.

Spectrum of White-Light LEDs

Blue LED+Yellow Phosphor



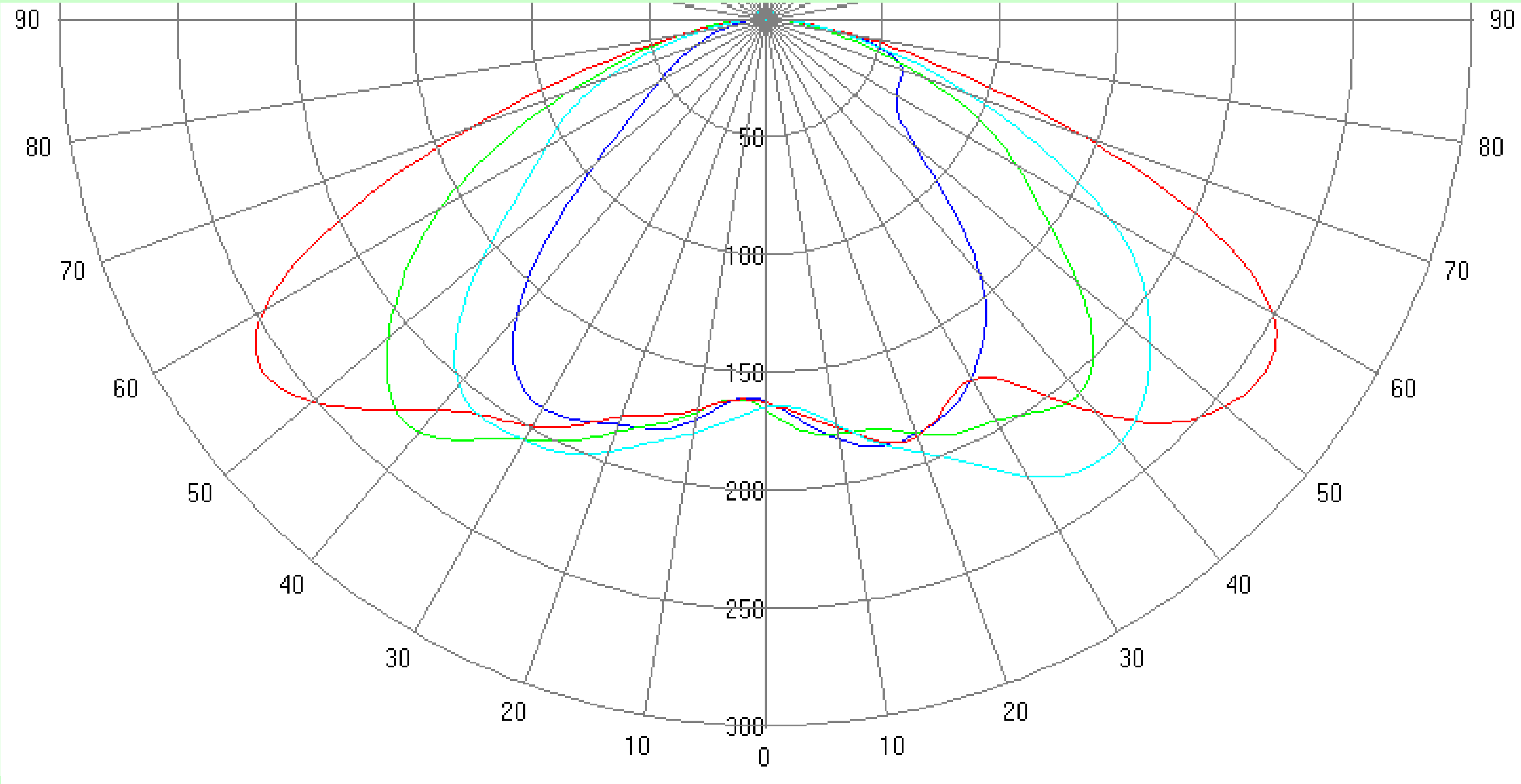
Red+Green+Blue LED



Directional Distribution of an LED

Luminous intensity (光強度)

Wavelength-weighted power emitted by a light source in a particular direction per unit solid angle, Unit: cd = lm/sr (燭光)



Candela plot / Luminous intensity distribution/ 配光曲線圖

12.2 Radiation Intensity

12.2.1 Mathematical Definitions

Solid angle: $$d\omega \equiv \frac{dA_n}{r^2} = \sin\theta d\theta d\phi \qquad (12.2, 3)$$

Unit of the solid angle: steradian (sr); for plane angle: radian (rad)

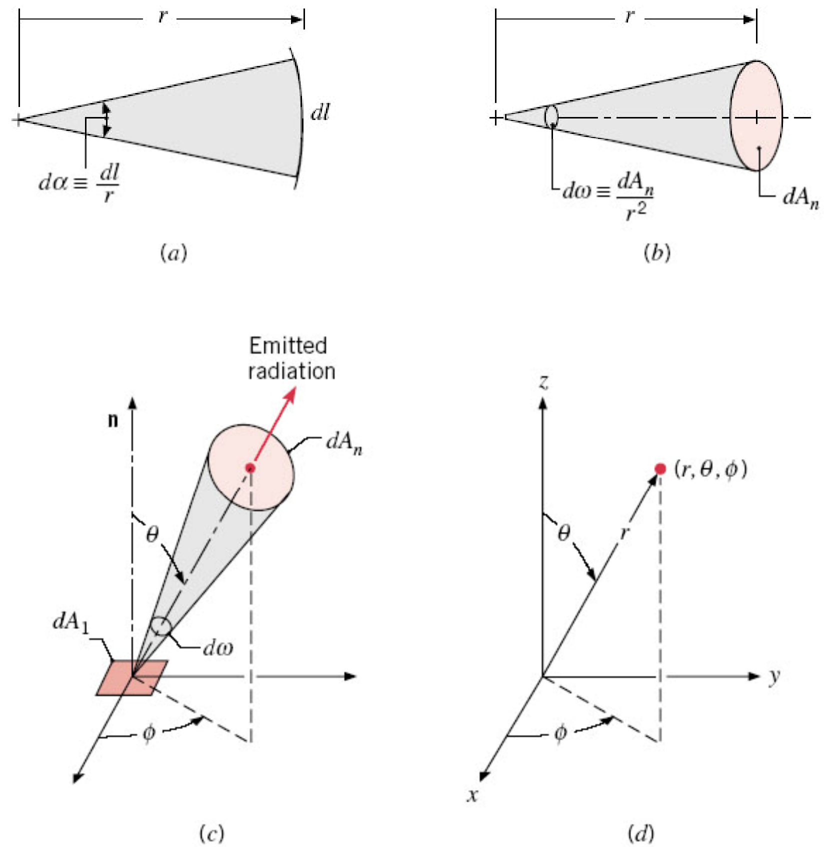


FIGURE 12.5 Mathematical definitions. (*a*) Plane angle. (*b*) Solid angle. (*c*) Emission of radiation from a differential area dA_1 into a solid angle $d\omega$ subtended by dA_n at a point on dA_1. (*d*) The spherical coordinate system.

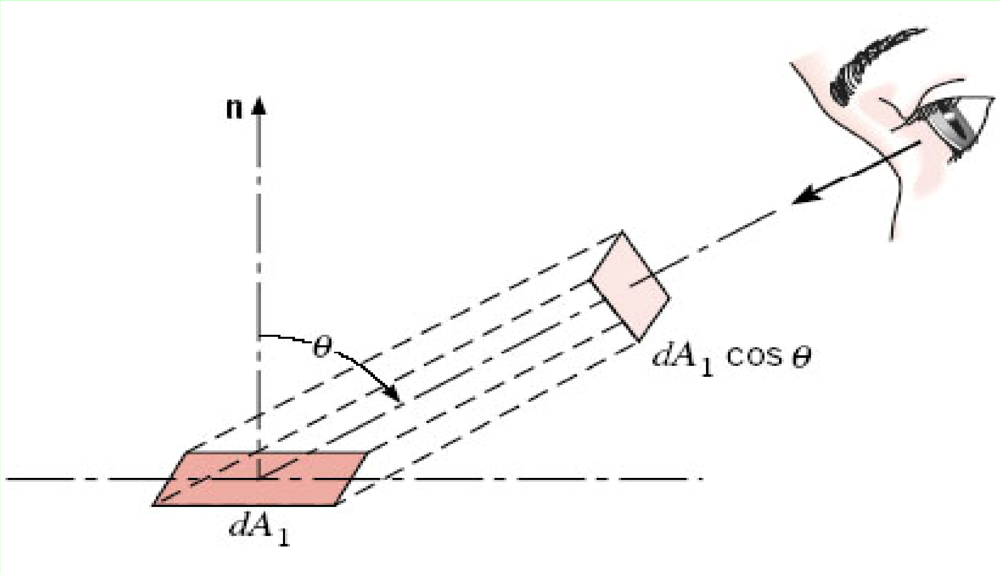


FIGURE 12.7 The projection of dA_1 normal to the direction of radiation.

Solid angle for a hemisphere: Fig. 12.6

$$\int_h d\omega = \int_0^{2\pi}\int_0^{\pi/2} \sin\theta d\theta d\phi = 2\pi\int_0^{2\pi} \sin\theta d\theta = 2\pi \text{ (sr)} \qquad (12.4)$$

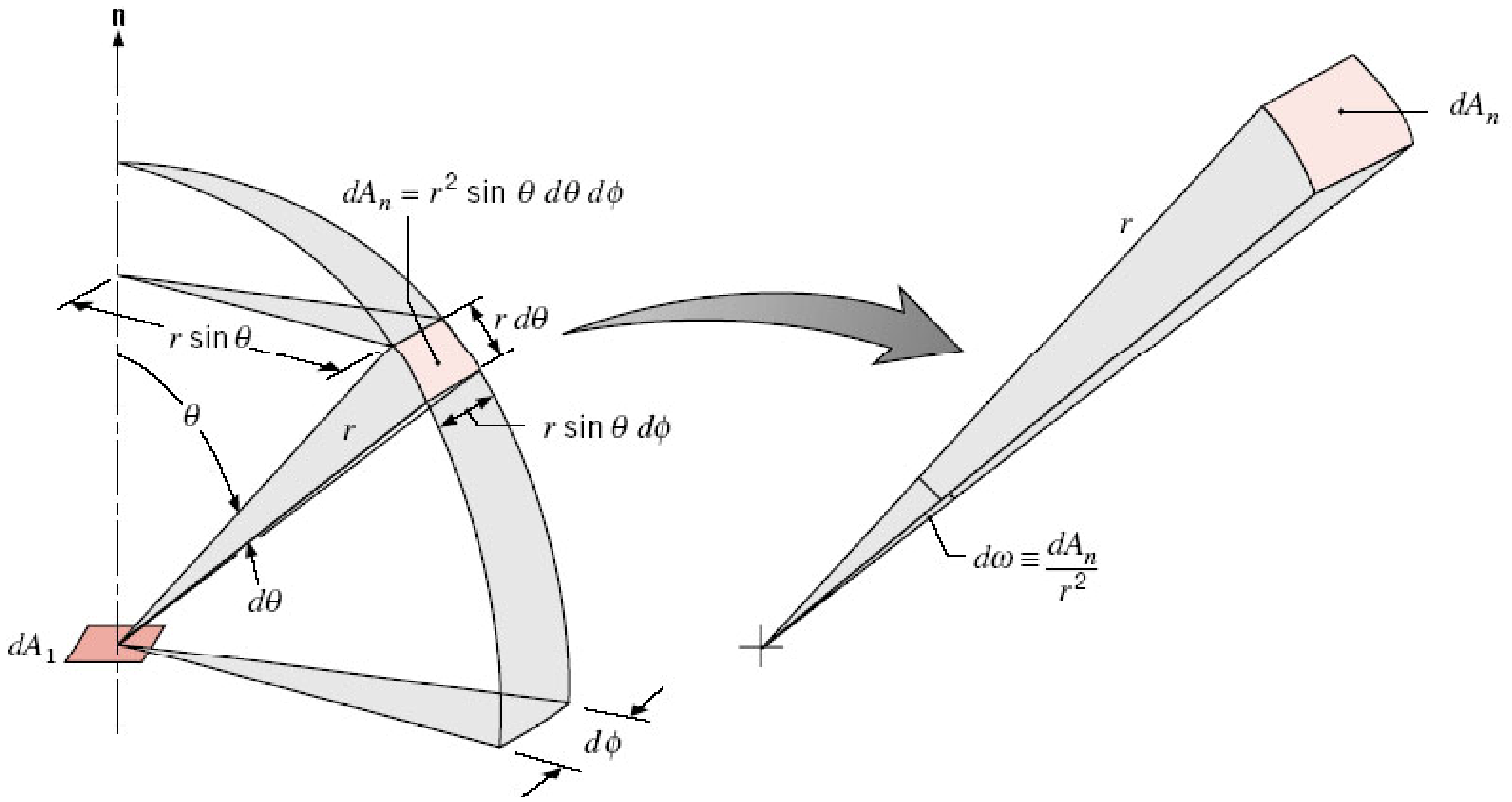


FIGURE 12.6 The solid angle subtended by dA_n at a point on dA_1 in the spherical coordinate system.

12.2.2 Radiation Intensity and Its Relation to Emission

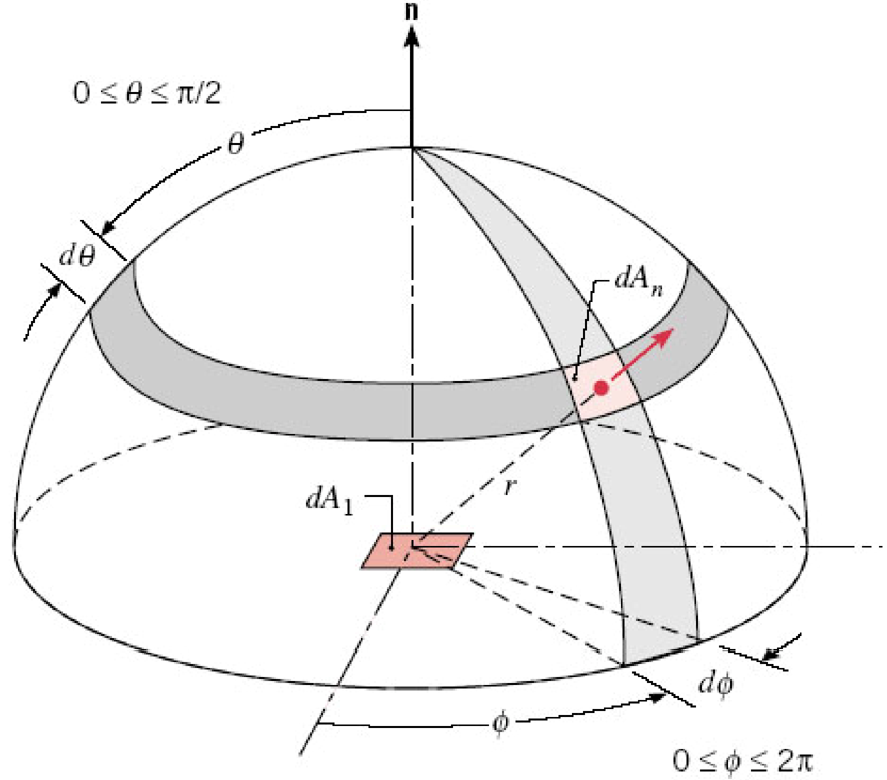


FIGURE 12.8 Emission from a differential element of area dA_1 into a hypothetical hemisphere centered at a point on dA_1.

- *Spectral intensity* of emission, $I_{\lambda,\mathrm{e}}$: the *rate* at which radiant energy is *emitted at the wavelength λ in the direction* (θ, ϕ), *per unit area* of the emitting surface normal to this direction, *per unit solid angle* about this direction, *and per unit wavelength* interval $d\lambda$ about λ.

$$I_{\lambda,e}(\lambda, \theta, \phi) \equiv \frac{dq}{dA_1 \cos\theta \cdot d\omega \cdot d\lambda} \qquad (12.5)$$

- Rate for radiation of wavelength λ leaving dA_1 and passing thru dA_n : $dq_\lambda = I_{\lambda,e}(\lambda,\theta,\phi) dA_1 \cos\theta \cdot d\omega$ (12.6)
- Spectral radiation *flux* assoc. with dA_1 (per unit area of the emitting surface), passing through dA_n:

$$dq''_\lambda = dq_\lambda / dA_1 = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi \qquad (12.7)$$

- Spectral (hemispherical) emissive power $E_{\lambda,e}$: (W/m$^2 \cdot \mu$m): spectral radiation *heat flux* assoc. with emission into a hypothetical hemisphere: (integration of $I_{\lambda,e}$ over the hemisphere)

$$E_\lambda(\lambda) = q''_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi \qquad (12.8)$$

* Note that E_λ is a flux based on the *actual* surface area, whereas $I_{\lambda,e}$ is based on the *projected* area.

- Total (hemispherical) emissive power E: (W/m^2)

$$E = \int_0^\infty E_\lambda(\lambda) d\lambda \qquad (12.9)$$

<**Special Case**>: For a *diffuse* emitter, $I_{\lambda,e}(\lambda,\theta,\phi) = I_{\lambda,e}(\lambda)$

(12.10) $\rightarrow$ $E_\lambda(\lambda) = \pi I_{\lambda,e}(\lambda), \quad E = \pi I_e$ (12.11, 12)

EX 12.1—This is an instructive example.

12.2.3 Relation to Irradiation

- Spectral intensity of incident radiation, $I_{\lambda,i}(\lambda, \theta, \phi)$: rate at which radiant energy of wavelength is incident from the direction (θ, ϕ) , per unit area of the *intercepting surface* normal to this direction, per unit solid angle about this direction, and per unit wavelength $d\lambda$ about λ.

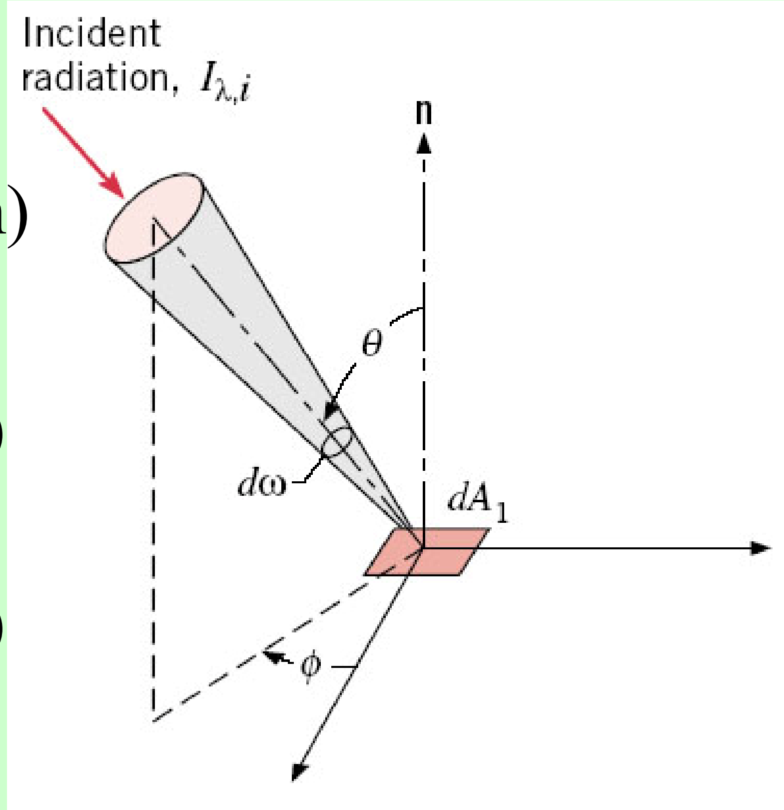


FIGURE 12.9
Directional nature of incident radiation.

- *Spectral irradiation*, $G_\lambda(\lambda)$: (W/m^2 · μm)

$$G_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda, \theta, \phi)\cos\theta \sin\theta d\theta d\phi \quad (12.13)$$

- *Total irradiation*, G:

$$G = \int_0^{\infty} G_\lambda(\lambda) d\lambda \quad (12.14)$$

* For diffuse incident radiation:

$$G_\lambda(\lambda) = \pi I_{\lambda,i}(\lambda) \quad \text{and} \quad G = \pi I_i \quad (12.16, 17)$$

EX 12.2

12.2.4 Relation to Radiosity

- *Radiosity* is all the radiant energy leaving a surface, including emitted and reflected energy. (Fig. 12.10)

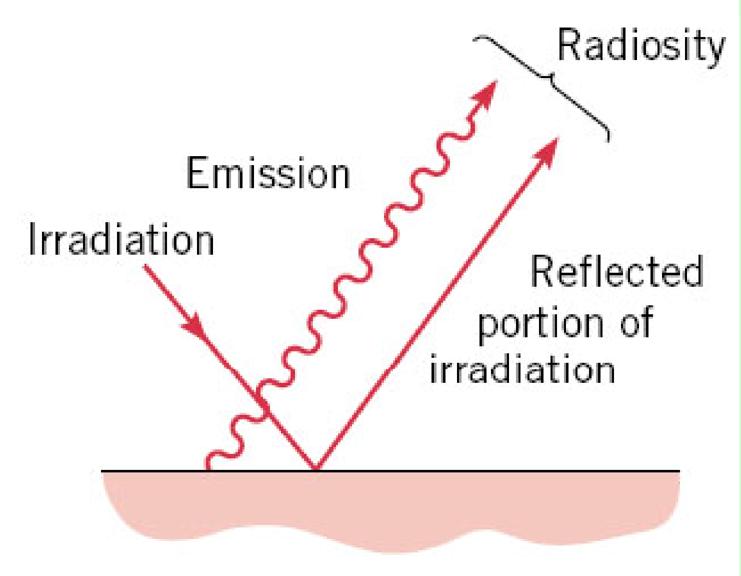


FIGURE 12.10
Surface radiosity.

- *Spectral radiosity*, $J_\lambda(\lambda)$: : ($W/m^2 \cdot \mu m$)

$$J_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi \qquad (12.18)$$

- *Total radiosity*, J:

$$J = \int_0^\infty J_\lambda(\lambda) d\lambda \qquad (12.19)$$

* For a surface being both a diffuse reflector and a diffuse emitter:

$$J_\lambda(\lambda) = \pi I_{e+r}(\lambda) \quad \text{and} \quad J = \pi I_{e+r} \qquad (12.21, 22)$$

12.3 Blackbody Radiation

A ***blackbody*** is an ideal surface having the following properties:

1. A blackbody absorbs *all* incident radiation, regardless of wavelength and direction.
2. For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody.
3. Although the radiation emitted by a blackbody is a function of wavelength and temperature, it is independent of direction. That is, the blackbody is a *diffuse* emitter.

No surface has precisely the properties of a blackbody. A cavity is a closest approximation.

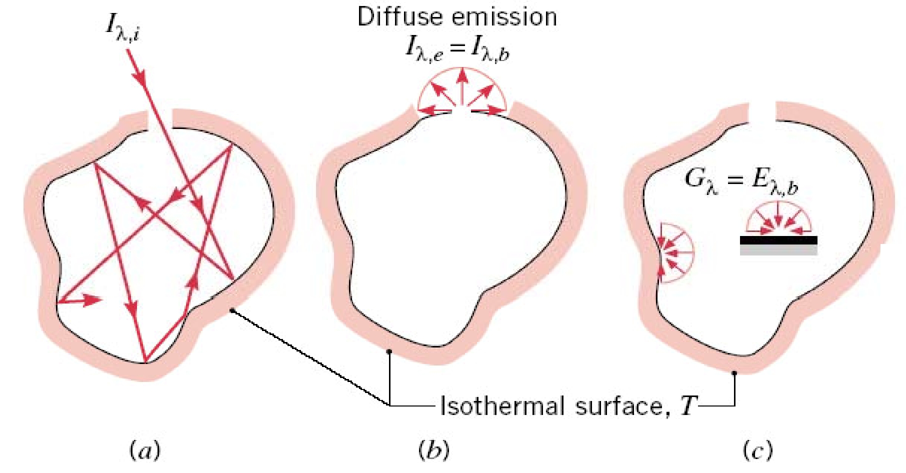


FIGURE 12.11 Characteristics of an isothermal blackbody cavity. (a) Complete absorption. (b) Diffuse emission from an aperture. (c) Diffuse irradiation of interior surfaces.

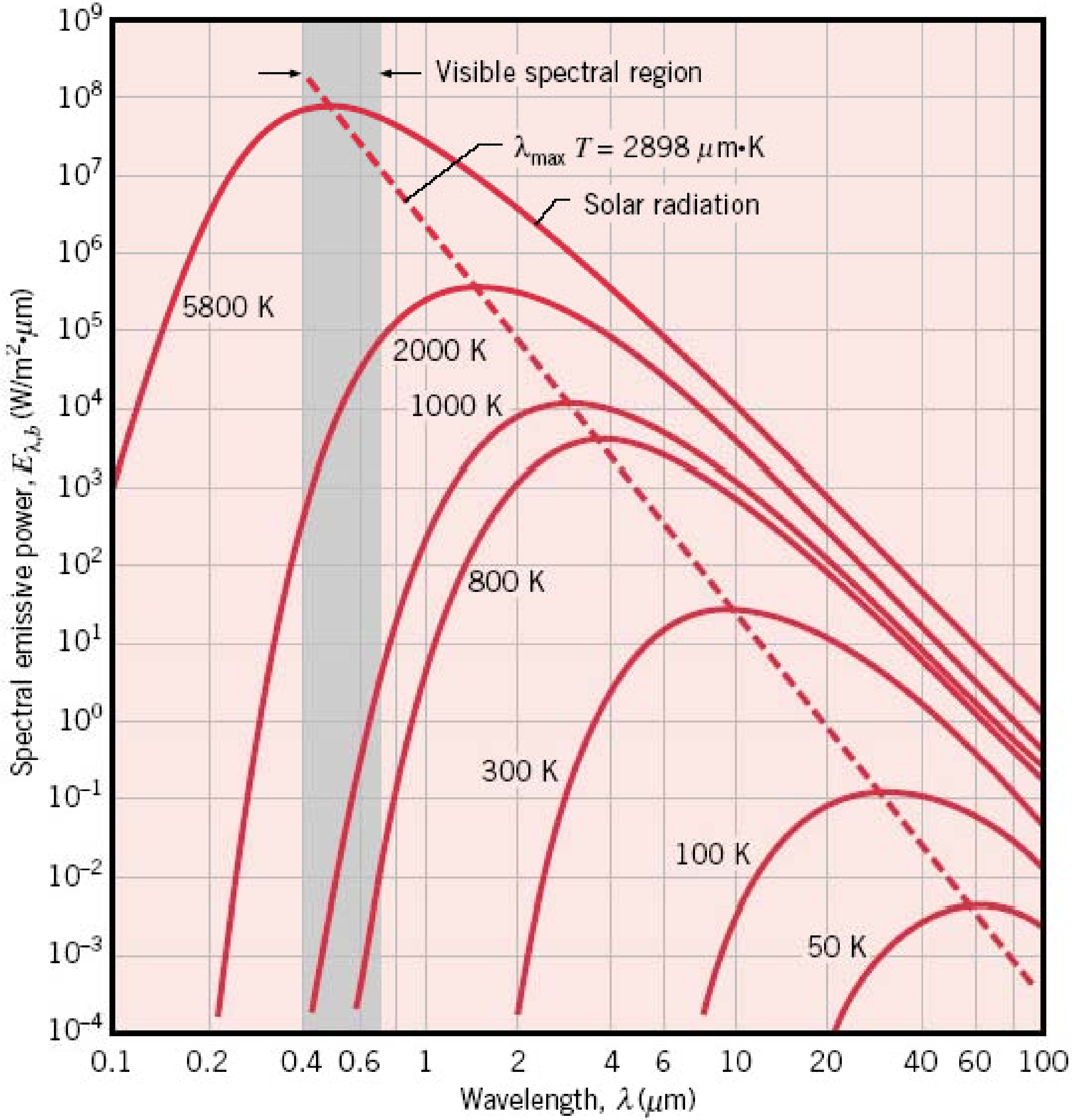


FIGURE 12.12 Spectral blackbody emissive power.

12.3.1 The Planck Distribution

$$I_{\lambda,b}(\lambda,T)=\frac{2hc_0^2}{\lambda^5[\exp(hc_0/\lambda kT)-1]} \qquad (12.23)$$

Since a blackbody is a diffuse emitter, it follows that

$$E_{\lambda,b}(\lambda,T)=\pi I_{\lambda,b}(\lambda,T)=\frac{C_1}{\lambda^5[exp(C_2/\lambda T)-1]} \qquad (12.24)$$

Read the text for the important features.

12.3.2 Wien's Displacement Law

Differentiating (12.24) w.r.t. λ to obtain

$$\lambda_{\max}T=C_3=2898\ \mu\text{m K} \qquad (12.25)$$

This relation locates $\lambda_{\max}$ of blackbody emission at a certain T.

* Note its linear appearance in Fig. 12.12.

12.3.3 The Stefan-Boltzmann Law

The total emissive power of a blackbody:

$$E_b(T) = \int_0^\infty \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} d\lambda = \sigma T^4 \tag{12.26}$$

where σ is the Stefan-Boltzmann constant.

Since a blackbody is diffuse,

$$I_b = E_b / \pi \tag{12.27}$$

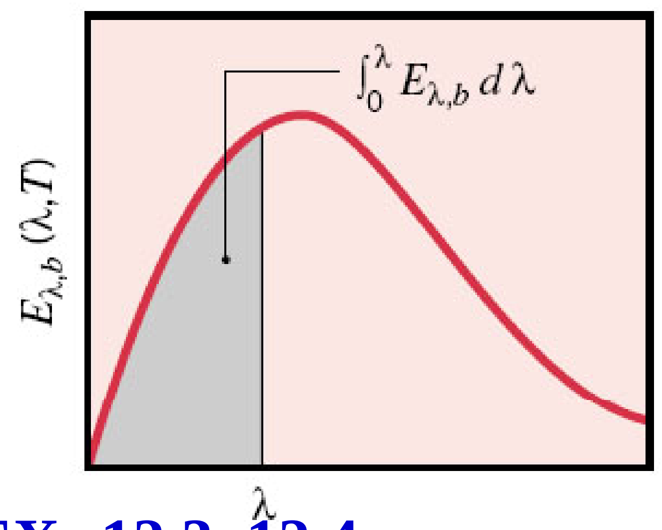
12.3.4 Band Emission

The fraction of the total emission from a blackbody that is in a certain wavelength interval of *band*:

$F_{(0\to\lambda)}$ is shown in Fig. 12.14 and Table 12.1, from which

$$F_{(0\to\lambda)} \equiv \frac{\int_0^{\lambda} E_{\lambda,b}d\lambda}{\int_0^{\infty} E_{\lambda,b}d\lambda} = \frac{\int_0^{\lambda} E_{\lambda,b}d\lambda}{\sigma T^4} = \int_0^{\lambda T} \frac{E_{\lambda,b}}{\sigma T^5} d(\lambda T) = f(\lambda T) \qquad (12.28)$$

$$F_{(\lambda_1\to\lambda_2)} = \frac{\int_0^{\lambda_2} E_{\lambda,b}d\lambda - \int_0^{\lambda_1} E_{\lambda,b}d\lambda}{\sigma T^4} = F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)} \qquad (12.29)$$



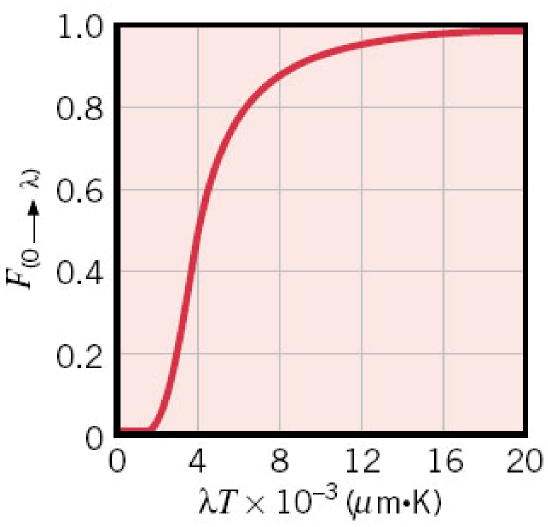


FIGURE 12.13 Radiation emission from a blackbody in the spectral band 0 to λ.

FIGURE 12.14 Fraction of the total blackbody emission in the spectral band from 0 to λ as a function of λT.

EXs 12.3, 12.4

12.4 Emission from Real Surfaces

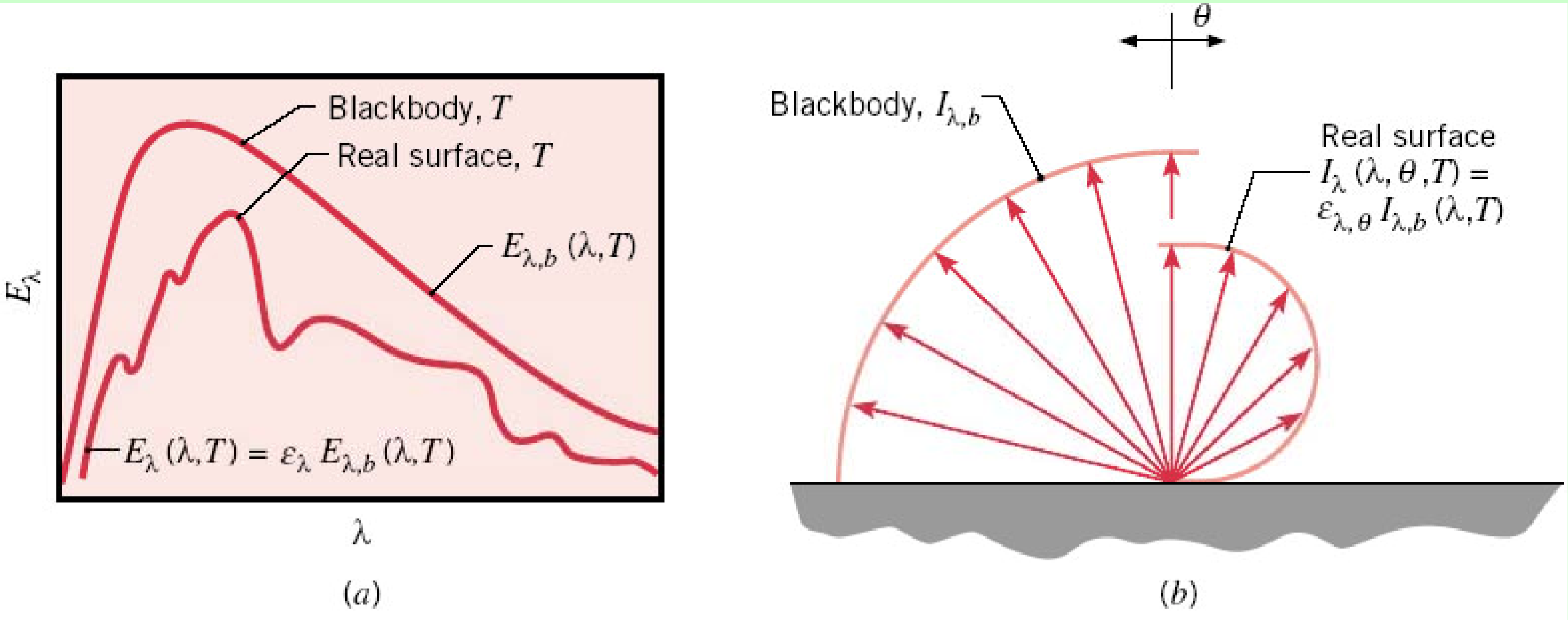


FIGURE 12.15 Comparison of blackbody and real surface emission. (a) Spectral distribution. (b) Directional distribution.

- *Spectral, directional* emissivity $\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T)$:

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) \equiv \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,b}(\lambda,T)} \tag{12.30}$$

- *Total, directional* emissivity $\varepsilon_\theta(\theta,\phi,T)$:

$$\varepsilon_\theta(\theta,\phi,T) \equiv \frac{I_e(\theta,\phi,T)}{I_b(T)} \tag{12.31}$$

- *Spectral, hemispherical* emissivity :

$$\varepsilon_\lambda(\lambda,T) \equiv \frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)} \qquad (12.32)$$

- *Total, hemispherical* emissivity :

$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda,T)E_{\lambda,b}(\lambda,T)d\lambda}{E_b(T)} \qquad (12.35, 36)$$

Usually, $\varepsilon \sim \varepsilon_n$; $\varepsilon_\lambda = \varepsilon_{\lambda,n}$; (12.37)

The values of ε_n and $\varepsilon_{\lambda,n}$ for some materials: Figs. 12.17, 18. Read the text for some generalizations.

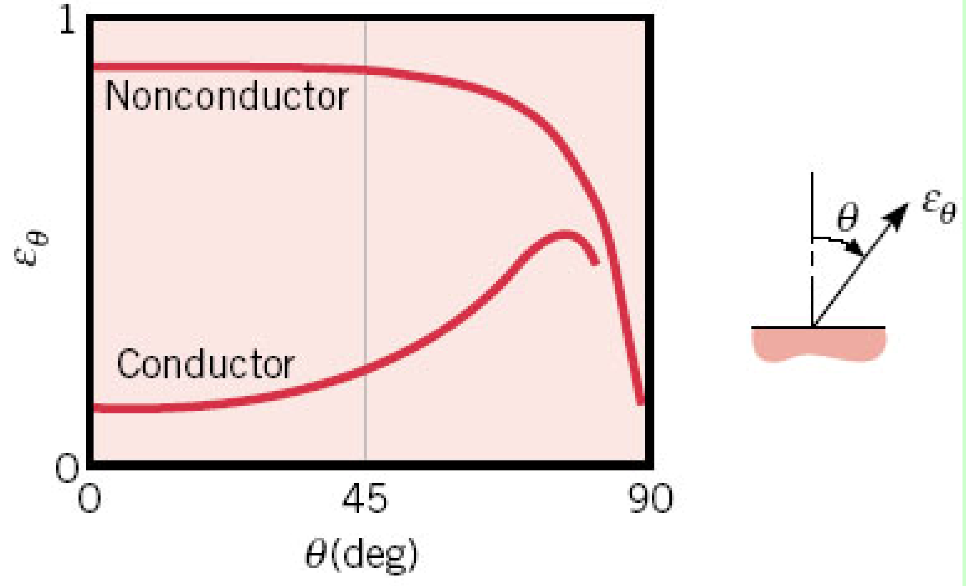


FIGURE 12.16
Representative directional distributions of the total, directional emissivity.

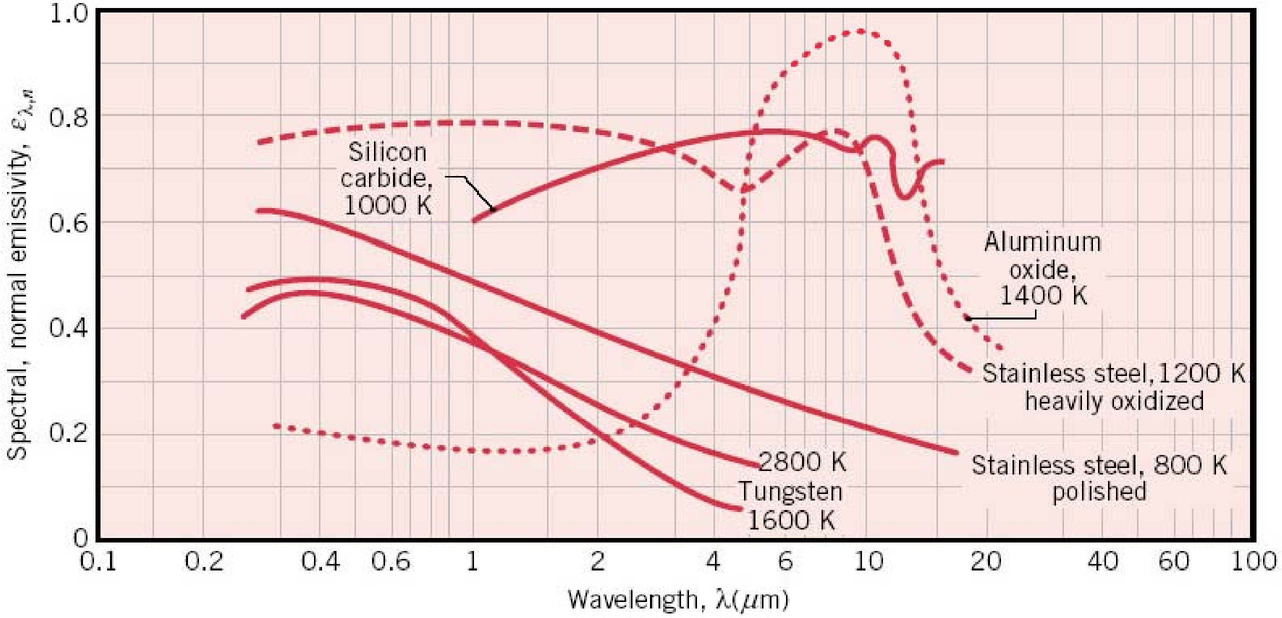


FIGURE 12.17 Spectral dependence of the spectral, normal emissivity $\varepsilon_{\lambda,n}$ of selected materials.

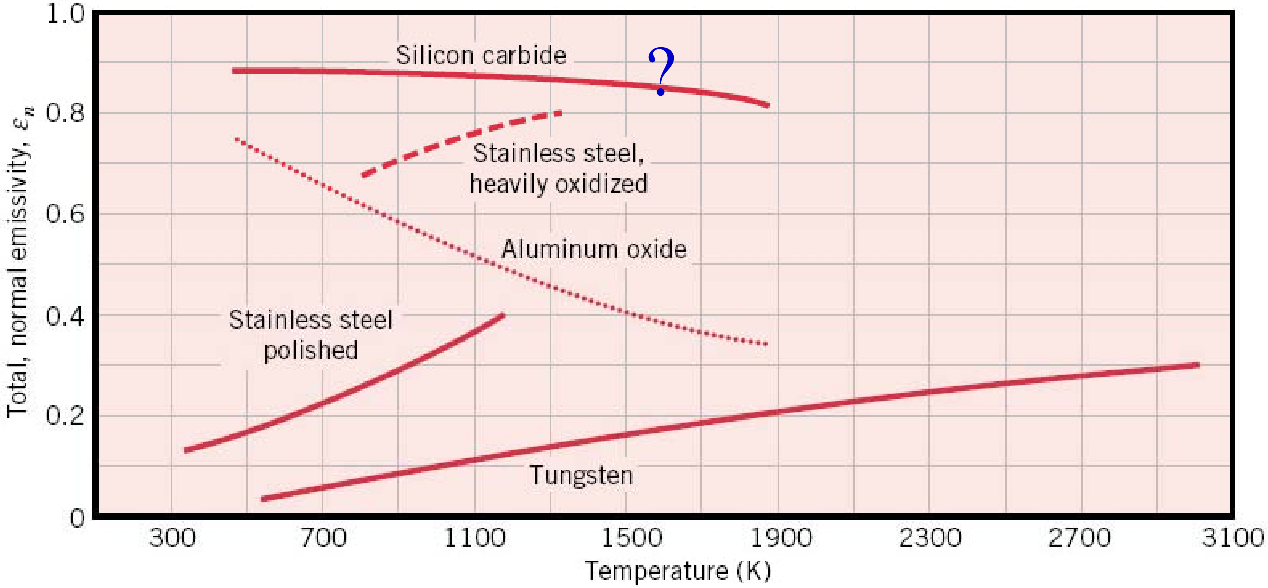


FIGURE 12.18 Temperature dependence of the total, normal emissivity ε_n of selected materials.

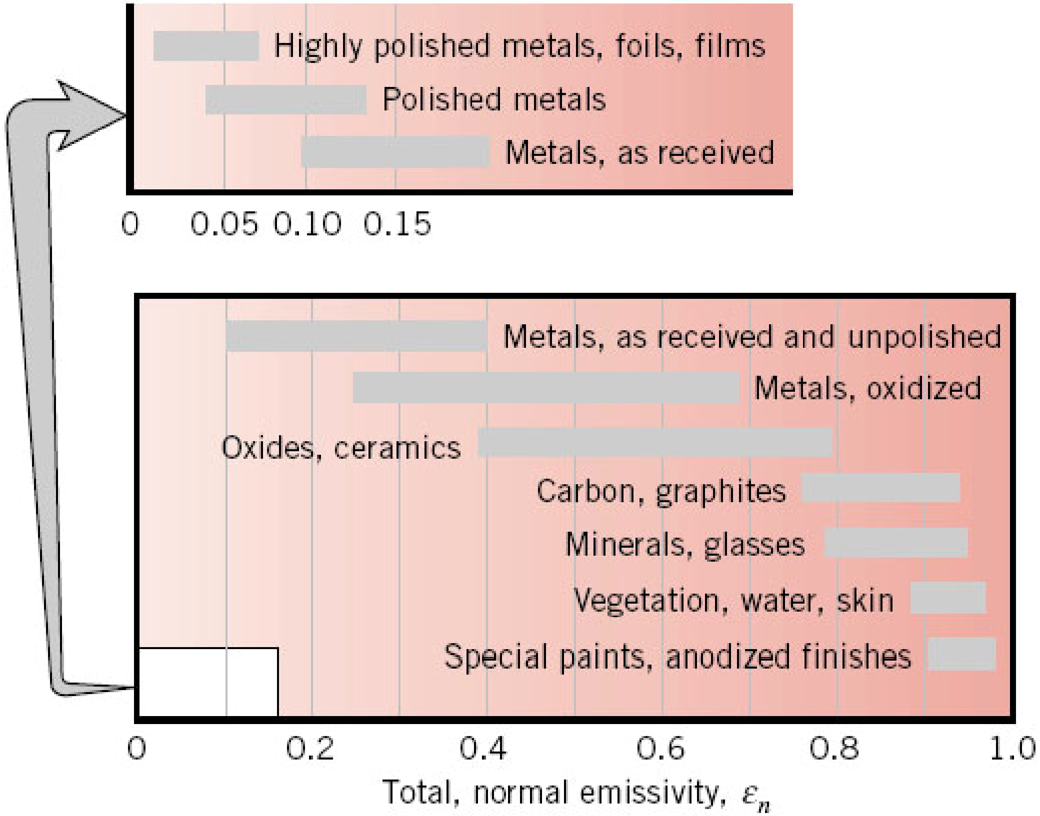


FIGURE 12.19 Representative values of the total, normal emissivity ε_n.

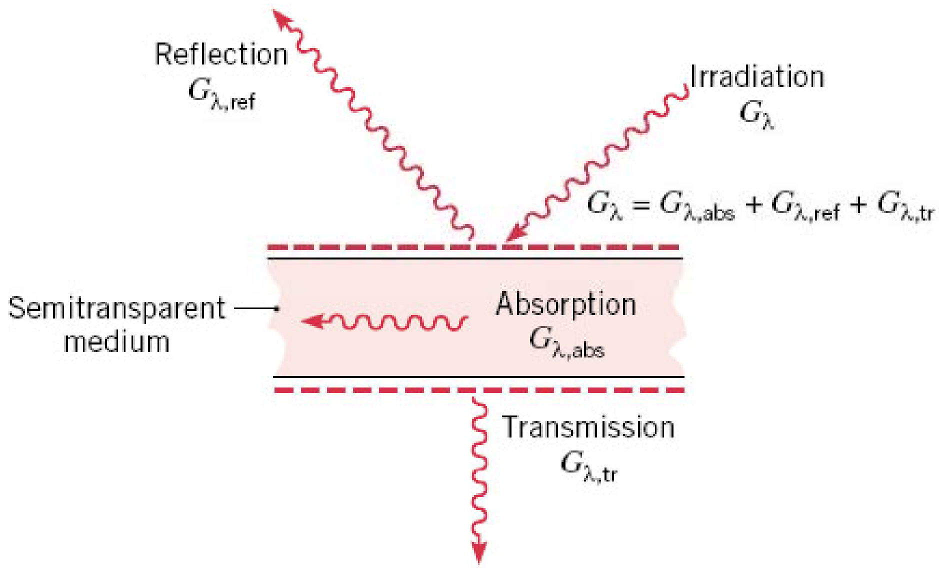


FIGURE 12.20 Absorption, reflection, and transmission processes associated with a semitransparent medium.

EX 12.5

12.5 Absorption, Reflection, and Transmission by Real Surface

(Fig.12.20)

$$G_\lambda = G_{\lambda,\mathrm{ref}} + G_{\lambda,\mathrm{abs}} + G_{\lambda,\mathrm{tr}} \tag{12.38}$$

- *Spectral, directional* absorptivity, $\alpha_{\lambda,\theta}$

$$\alpha_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,\mathrm{abs}}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)} \tag{12.39}$$

- *Spectral, hemispherical* absorptivity, α_λ

$$\alpha_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{abs}}(\lambda)}{G_\lambda(\lambda)} \tag{12.40}$$

- *Total, hemispherical* absorptivity, α

$$\alpha \equiv \frac{G_{\mathrm{abs}}}{G} = \frac{\int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda} \tag{12.43, 44}$$

  - *Total* absorptivity to solar radiation, α_S

$$\alpha_S \approx \frac{\int_0^\infty \alpha_\lambda E_{\lambda,b}(\lambda, 5800K)\, d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda, 5800K)\, d\lambda} \tag{12.45}$$

- Definitions for *reflectivities* ρ : (12.46-50)
  - **Diffuse** and **specular** reflection:

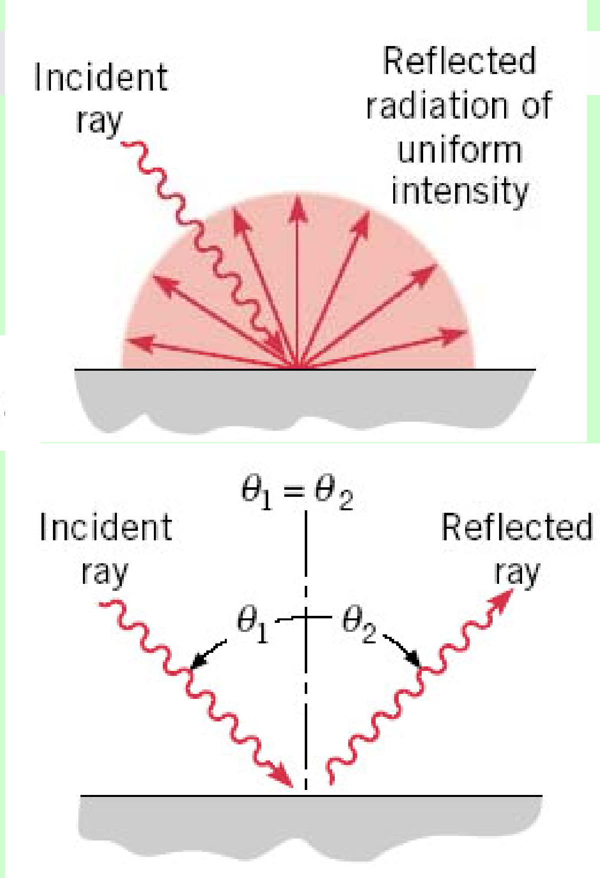


FIGURE 12.21 Diffuse and specular reflection.

- *Spectral, hemispherical* transmissivity:

$$\tau_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{tr}}(\lambda)}{G_\lambda(\lambda)} \qquad (12.51)$$

- *Total* transmissivity: $\tau \equiv \dfrac{G_{\mathrm{tr}}}{G}$ (12.52)

Figs. 12.22: spectral normal absorptivities and reflectivities

Figs. 12.23: spectral transmissivities

Eq. (12.38) $\rightarrow$ $\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1$ (12.54)

Also $\rho + \alpha + \tau = 1$ (12.55)

For *opaque* medium: $\rho_\lambda + \alpha_\lambda = 1;\ \ \rho + \alpha = 1$ (12.56, 57)

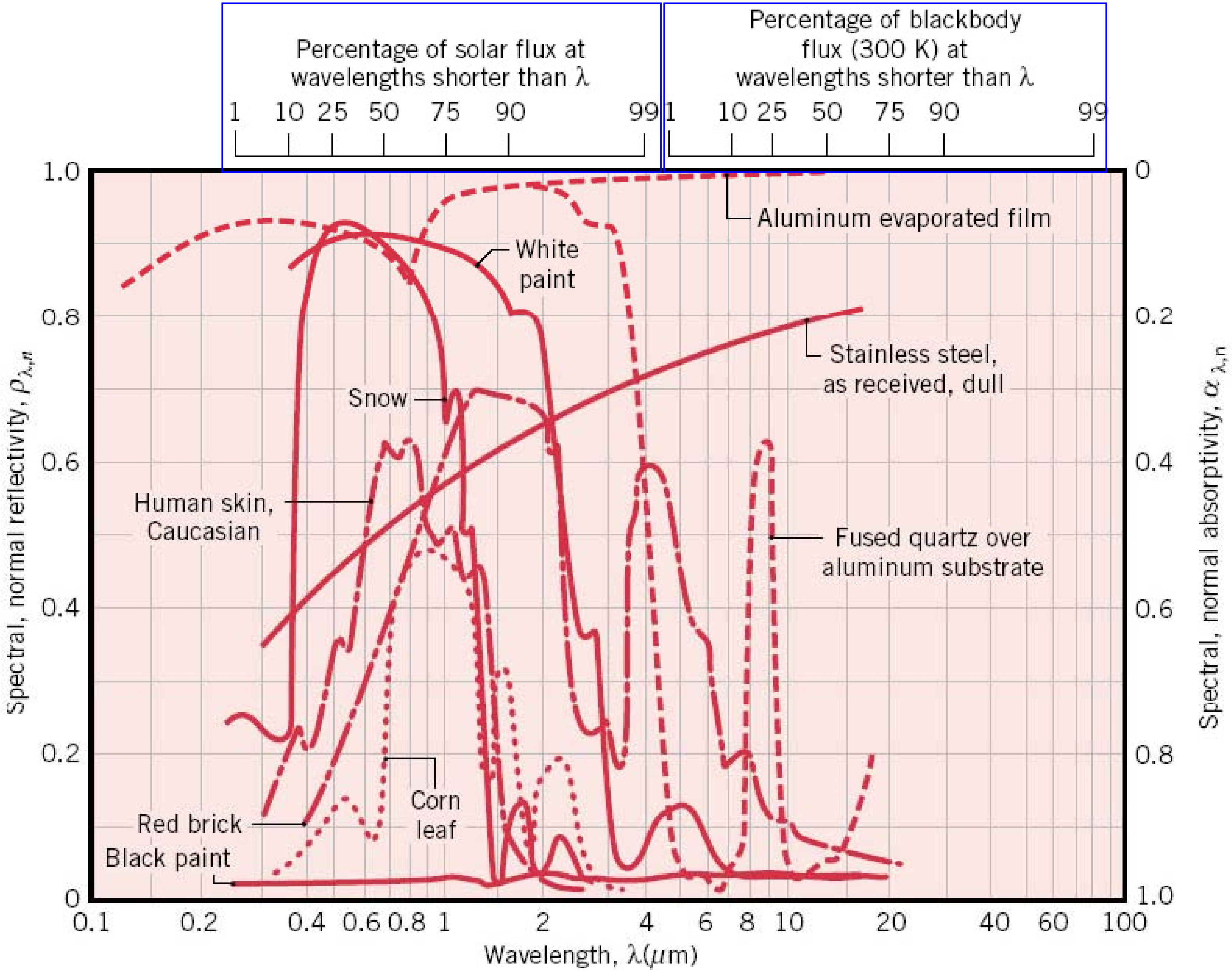


FIGURE 12.22 Spectral dependence of the spectral, normal absorptivity $\alpha_{\lambda,n}$ and reflectivity $\rho_{\lambda,n}$ of selected opaque materials.

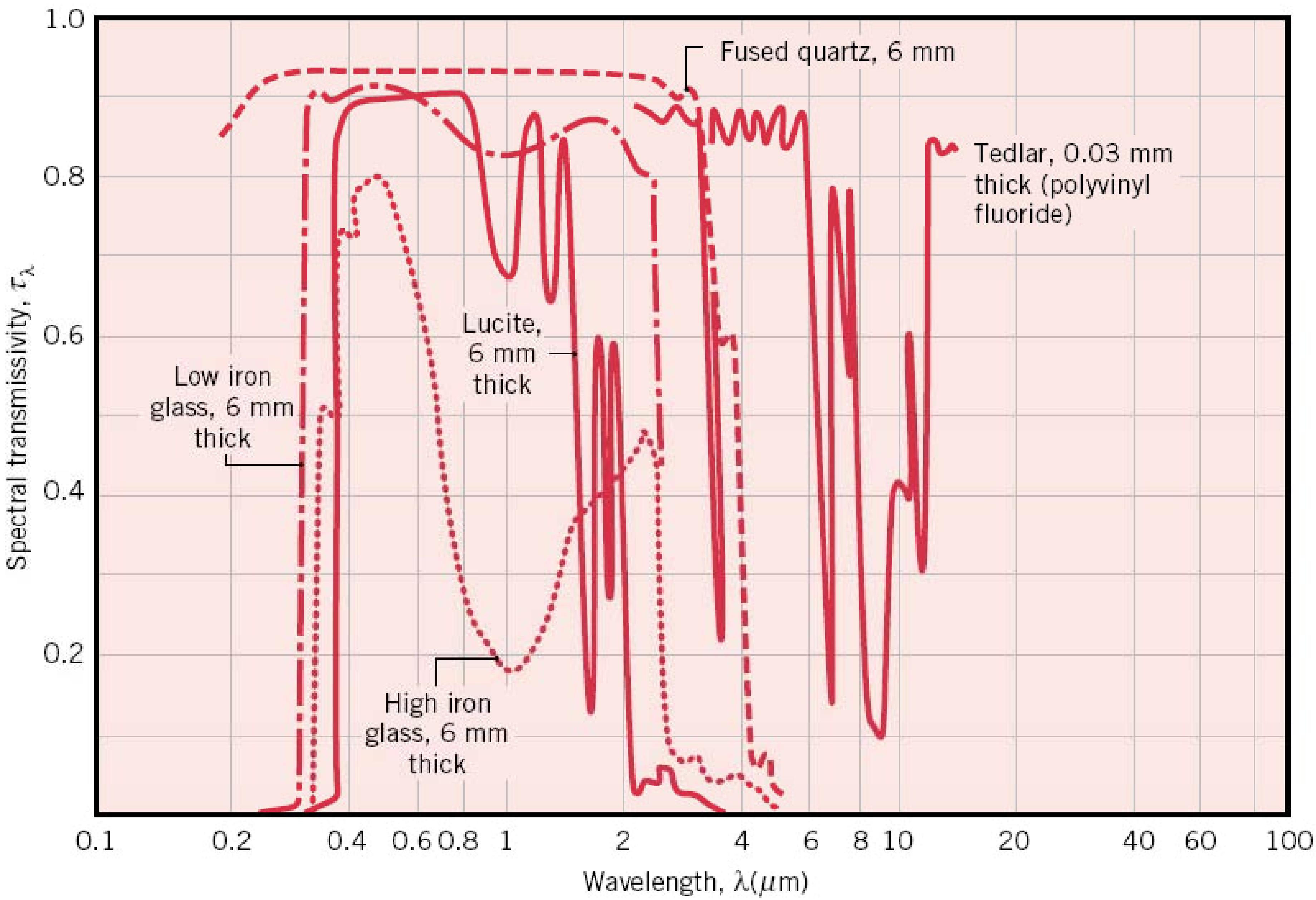


FIGURE 12.23 Spectral dependence of the spectral transmissivities τ_λ of selected semitransparent materials.

12.6 Kirchhoff's Law

Considering the thermal equilibrium between a body in an enclosure under steady-state conditions (Fig. 12.24), the following relations, called **Kirchhoff's law**, can be obtained:

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T)=\alpha_{\lambda,\theta}(\lambda,\theta,\phi,T)\ \text{, with no restriction} \qquad (12.62)$$

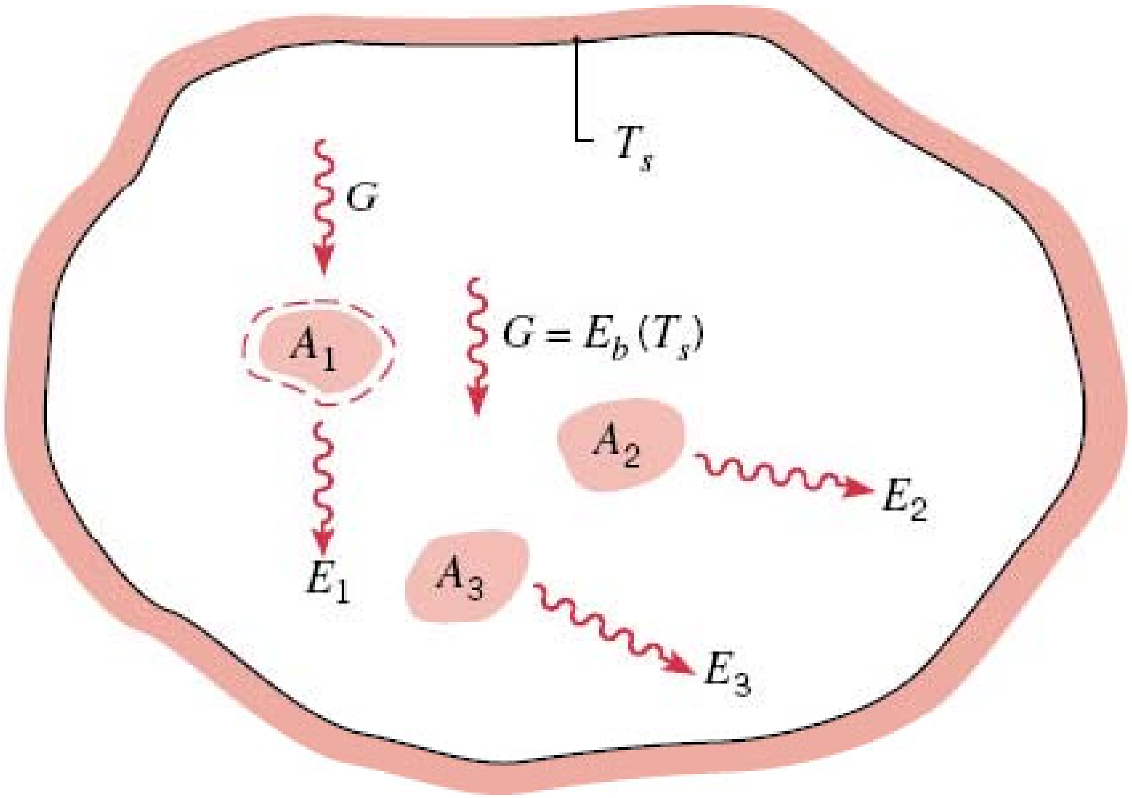


FIGURE 12.24
Radiative exchange in an isothermal enclosure.

12.6 Kirchhoff's Law (cont'd)

Under special restrictions, Kirchhoff's law can be rewritten in various simplified forms:

$$\varepsilon_\lambda(\lambda, T) = \alpha_\lambda(\lambda, T) \qquad (12.61)$$

- if the irradiation is diffuse; or
- if the surface is diffuse.

$$\varepsilon(T) = \alpha(T) \qquad (12.60)$$

- if the irradiation is emission from a black-body at the same temperature as the surface; or
- if the surface is gray and diffuse; or ..

12.7 The Gray Surface

From the definitions,

$$\varepsilon_\lambda = \frac{\int_0^{2\pi}\int_0^{\pi/2} \varepsilon_{\lambda,\theta} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi} \overset{?}{=} \frac{\int_0^{2\pi}\int_0^{\pi/2} \alpha_{\lambda,\theta} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi} = \alpha_\lambda \quad (12.63)$$

$$\rightarrow \varepsilon_\lambda(\lambda, T) = \alpha_\lambda(\lambda, T)$$

if

1. the irradiation is diffuse (reasonable approximation for many engineering calculations), or
2. the surface is diffuse ($\varepsilon_{\lambda,\theta}$ and $\alpha_{\lambda,\theta}$ independent of θ and ϕ, reasonable particularly for nonconducting materials, Fig. 12.16)

For (12.60) to be valid, additional conditions must be satisfied:

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda,T)d\lambda}{E_b(T)} \overset{?}{=} \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda)d\lambda}{G} = \alpha \qquad (12.64)$$

Since $\varepsilon_\lambda = \alpha_\lambda$, then (12.64) $\rightarrow \varepsilon = \alpha$, if

1. the irradiation corresponds to emission from a blackbody at the same temperature as the surface, or
2. the surface is gray.

Because the total absorptivity of a surface depends on the spectral distribution of the irradiation, it cannot be stated unequivocally that $\varepsilon = \alpha$. (Fig. 12.25)

Practically speaking, a gray surface may be defined as one for which α_λ and ε_λ *are independent of* λ *over the spectral regions* of the irradiation and the surface emission. (Fig. 12.26)

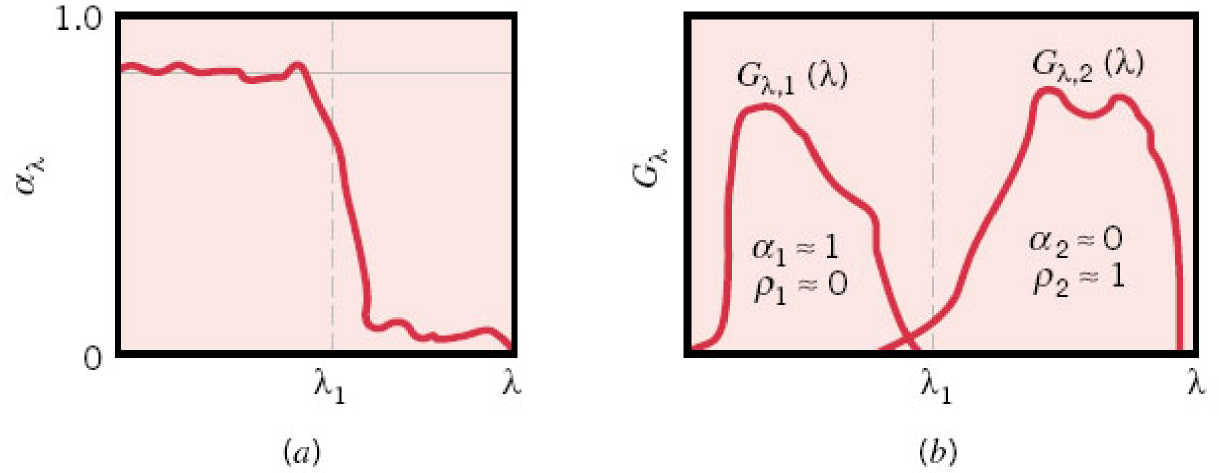


FIGURE 12.25 Spectral distribution of (*a*) the spectral absorptivity of a surface and (*b*) the spectral irradiation at the surface.

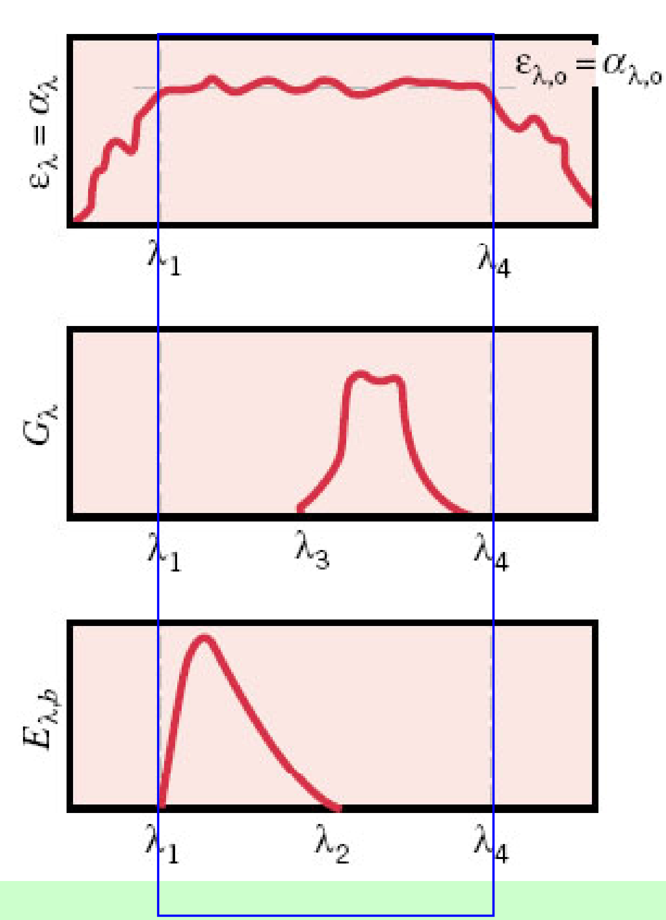


FIGURE 12.26
A set of conditions for which gray surface behavior may be assumed.

In summary, for a ***diffuse*** (direction-independent) surface,

$$\varepsilon_\lambda(\lambda, T) = \alpha_\lambda(\lambda, T) \qquad (12.61)$$

For a ***diffuse*** (direction-indep.) *and* ***gray*** (wavelength-indep.) surface,

$$\varepsilon(T) = \alpha(T) \qquad (12.60)$$

EXs 12.9, 12.10

12.8 Environmental Radiation

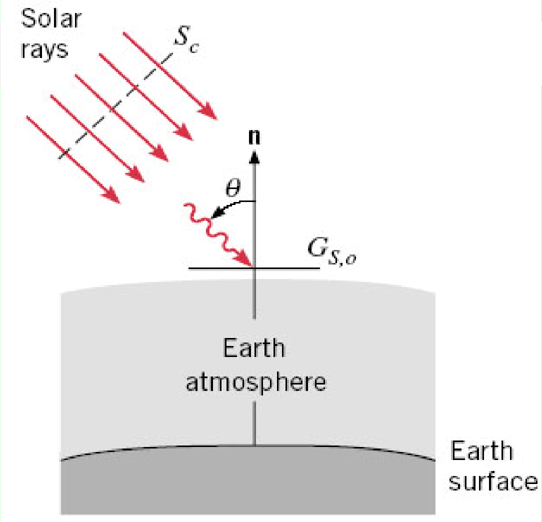


FIGURE 12.27 Directional nature of solar radiation outside the earth's atmosphere.

Solar radiation: Fig. 12.27 and Eq.12.65:

$$G_{S,o} = S_c \cdot f \cdot \cos\theta \qquad (12.65)$$

where

$G_{S,o}$: extraterrestrial solar irradiation

S_c: solar constant, $S_c = 1353\ \mathrm{W/m^2}$

f: correction factor for orbit eccentricity

$(0.97 \leq f \leq 1.03)$

short wavelength $(0.2 \leqq \lambda \leqq 3\mu\mathrm{m})$:

- **absorption**: mainly by O_3, H_2O, O_2, CO_2 (Fig. 12.28)

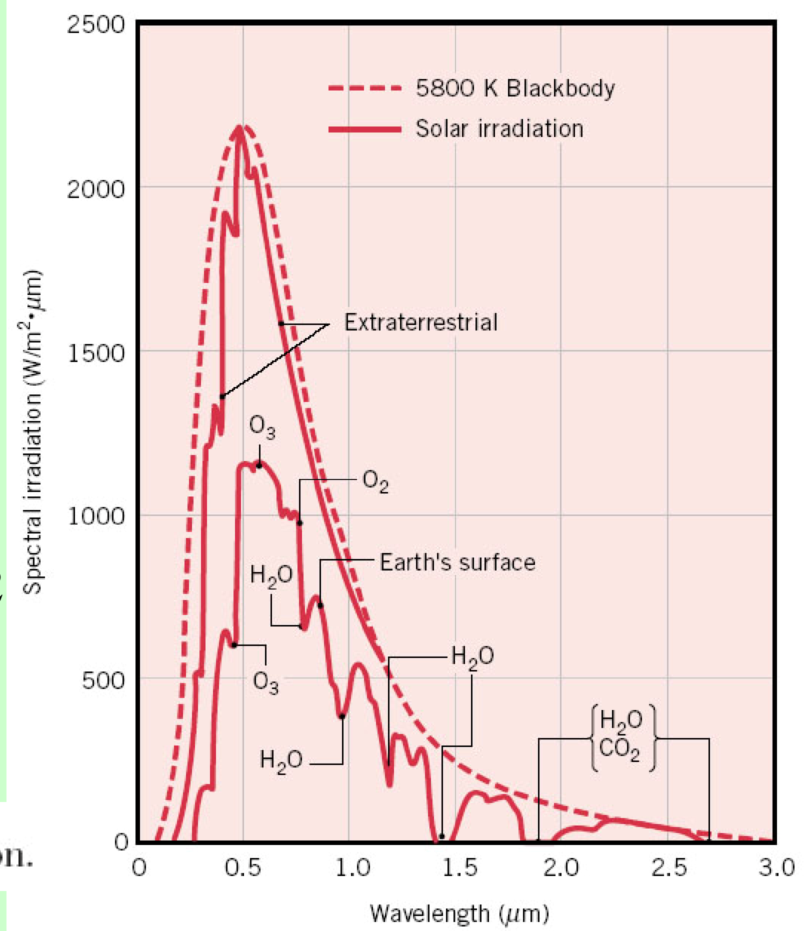


FIGURE 12.28 Spectral distribution of solar radiation.

short wavelength ($0.2 \leqq \lambda \leqq 3\mu m$):

- **scattering**: Rayleigh and Mie scattering (Fig. 12.29)

  --Rayleigh scattering: by particles of size $D << \lambda$, $\propto 1/\lambda^4$

  (Rayleigh scattering by the molecules of the atmosphere accounts for the sky being blue, and for the sun becoming red at sunset.)

Direct and scattered radiation: Figs. 12.29; 12.30

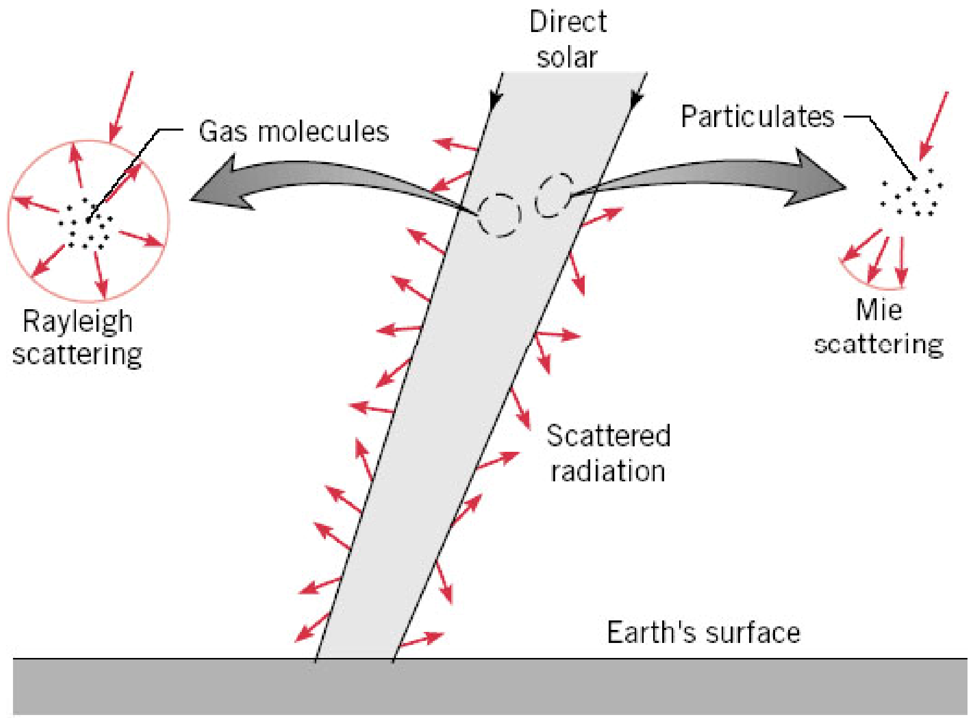


FIGURE 12.29 Scattering of solar radiation in the earth's atmosphere.

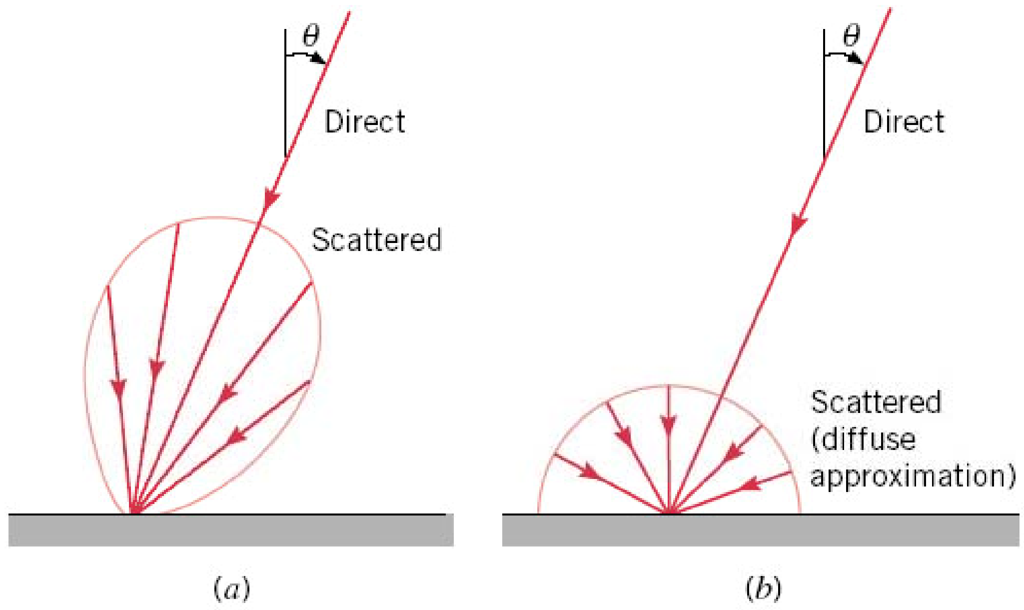


FIGURE 12.30 Directional distribution of solar radiation at the earth's surface. (a) Actual distribution. (b) Diffuse approximation.

Emission from the earth's surface: long wavelength, 4-40μm

$$E = \varepsilon \sigma T^4 \tag{12.66}$$

where $\varepsilon \sim 0.97$ and T is from 250 to 320 K.

Atmospheric emission: largely from CO_2 and H_2O, long wavelength from 5-8 μm and above 13 μm.

$$G_{\text{atm}} = \sigma T_{\text{sky}}^4 \tag{12.67}$$

where T_{sky} is the effective sky temperature, ranging from 230-285 K.

Since solar radiation is concentrated in the short wavelengths, it follows that many surfaces may not be approximated as gray in their response to solar irradiation. (Table 12.2)

TABLE 12.2 Solar Absorptivity α_S and Emissivity ε of Surfaces Having Spectral Absorptivity Given in Figure 12.22

Surface	α_S	ε (300 K)	α_S/ε
Evaporated aluminum film	0.09	0.03	3.0
Fused quartz on aluminum film	0.19	0.81	0.24
White paint on metallic substrate	0.21	0.96	0.22
Black paint on metallic substrate	0.97	0.97	1.0
Stainless steel, as received, dull	0.50	0.21	2.4
Red brick	0.63	0.93	0.68
Human skin (Caucasian)	0.62	0.97	0.64
Snow	0.28	0.97	0.29
Corn leaf	0.76	0.97	0.78

EX 12.11