Chapter 8

Internal Forced Convection
8.1 Hydrodynamic Considerations

8.1.1 Flow Conditions

may be determined experimentally, as shown in Figs. 7.1-7.2.

\[ Re_D \equiv \frac{\rho u_m D}{\mu} \]

(8.1)

where \( u_m \) is the mean fluid velocity over the tube cross section. The critical \( Re_D \) corresponding to the onset of turbulence is

\[ Re_{D,c} \approx 2300 \]

(8.2)

The entrance length for laminar flow is

\[ \left( \frac{x_{fid,h}}{D} \right)_{\text{lam}} \approx 0.05 \text{Re}_D \]

(8.3)

The entrance length for turbulent flow is approximately indep. of \( Re \)

\[ 10 \leq \left( \frac{x_{fid,h}}{D} \right)_{\text{turb}} \leq 60 \]

(8.4)
8.1.2 The mean Velocity

\[ u_m = \frac{\int_{A_c} \rho u(r, x) dA_c}{\rho A_c} = \frac{2}{r_0^2} \int_0^{r_0} u(r, x) r dr \]  

(8.8)

8.1.3 Velocity Profile in the Fully Developed Region

The velocity profile of a fully developed laminar flow is parabolic as shown in (8.13)-(8.15).

\[ u(r) = -\frac{1}{4 \mu} \left( \frac{dp}{dx} \right) r_0^2 \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right] \]  

(8.13)

8.1.4 Pressure Gradient and Friction Factor in Fully Developed Flow

Friction factor for laminar flow:

\[ f = \frac{64}{Re_D} \]  

(8.19)

Friction factor for turbulent flow: (8.20) or (8.21) or Fig. 8.3.

Pressure drop:

\[ \Delta p = -\int_{p_1}^{p_2} dp = f \frac{\rho u_m^2}{2D} \int_{x_1}^{x_2} dx = f \frac{\rho u_m^2}{2D} (x_2 - x_1) \]  

(8.22a)
Figure 8.3 Friction factor for fully developed flow in a circular tube [3]. Used with permission.
8.2 Thermal Considerations

The thermal entrance length for laminar flow is

\[
\left( \frac{x_{fd,t}}{D} \right)_{\text{lam}} \approx 0.05 \text{Re}_D \text{Pr}
\]  

(8.23)

The thermal entrance length for turbulent flow is

\[
\left( \frac{x_{fd,t}}{D} \right)_{\text{turb}} \approx 10
\]  

(8.24)
8.2.1 The Mean Temperature

\[ T_m = \frac{\int_{A_c} \rho u c_p T dA_c}{\dot{m} c_p} \]  

(8.25)

For incompressible flow in a circular tube with constant \( c_p \),

\[ T_m = \frac{2}{u_m r_0^2} \int_{r_0} u T r dr \]  

(8.26)

8.2.2 Newton's Law of Cooling

\[ q'' = h(T_s - \overline{T_m}) \]  

(8.27)

where \( h \) is the local convection heat transfer coefficient.

- This relation applies for different wall conditions, e.g., \( T_s = C \) or \( q'' = C \).
- Since \( T_m \) usually varies along the tube, the variation of \( T_m \) must be accounted for.
8.2.3 Fully Developed Conditions

The *thermally fully developed* condition is when the *relative shape* of the profile no longer changes and is stated as

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right]_{fd,t} = 0 \tag{8.28}$$

for cases with either a *uniform surface heat flux* or a *uniform surface temperature*. Since the term in the bracket of (8.29) is independent of $x$, its $r$-derivative is also independent of $x$. So,

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)_{r=r_0} = -\frac{\partial T/\partial r}{T_s - T_m} = -\frac{h}{k} \neq f(x) \tag{8.29}$$

Hence, in the *thermally fully developed flow* of a fluid with constant properties, the *local convection coefficient is a constant.* (Fig. 8.5)
Eq. (8.28) implies
\[
\frac{\partial T}{\partial x} \bigg|_{fd,t} = \frac{\partial T_m}{\partial x} \bigg|_{fd,t} \quad \text{or} \quad \frac{\partial (T - T_m)}{\partial x} \bigg|_{fd,t} = 0, \quad q'_s = \text{constant} \quad (8.32)
\]

\[
\frac{\partial T}{\partial x} \bigg|_{fd,t} = \frac{(T_s - T)}{(T_s - T_m)} \frac{\partial T_m}{\partial x} \bigg|_{fd,t}, \quad T_s = \text{constant} \quad (8.33)
\]

$T_m(x)$ is a very important variable for internal flows and may be obtained by applying an overall energy balance to the flow.

EX 8.1

8.3 The Energy Balance

8.3.1 General Considerations

Energy conservation for a differential control volume of Fig. 8.6 leads to

\[
dq_{conv} = \dot{m} c_p \, dT_m \quad (8.36)
\]

or

\[
\frac{dT_m}{dx} = \frac{q'_s P}{\dot{m} c_p} = \frac{P}{\dot{m} c_p} h(T_s - T_m) \quad (8.37)
\]

Figure 8.6 Control volume for internal flow in a tube.
8.3.2 Constant Surface Heat Flux

\[ T_m(x) = T_{m,i} + \frac{q_s'' P}{m c_p} x, \quad q_s'' = \text{constant} \]  \hspace{1cm} (8.40)

**Figure 8.7** Axial temperature variations for heat transfer in a tube. (a) Constant surface heat flux. (b) Constant surface temperature.

**EX. 8.2**
8.3.3 Constant Surface Temperature

Define $\Delta T = T_s - T_m$, Eq. 8.37 becomes

$$\frac{dT_m}{dx} = - \frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h \Delta T$$

Integration leads to

$$\ln \frac{\Delta T_o}{\Delta T_i} = - \frac{PL}{\dot{m}c_p} \left( \int_0^L \frac{1}{h} dx \right) = - \frac{PL}{\dot{m}c_p} \bar{h}_L$$

(8.41a)

Then,

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp \left( - \frac{Px}{\dot{m}c_p} \bar{h} \right), \quad T_s = \text{constant}$$

(8.42)

From energy balance,

$$q_{\text{conv}} = \dot{m}c_p (T_{m,o} - T_{m,i}) = \dot{m}c_p [(T_s - T_{m,i}) - (T_s - T_{m,o})] = \dot{m}c_p (\Delta T_i - \Delta T_o)$$

Substituting for $\dot{m}c_p$ from (8.41a), we obtain

$$q_{\text{conv}} = \bar{h} A_s \Delta T_{\text{lm}}, \quad \Delta T_{\text{lm}} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}, \quad T_s = \text{constant}$$

(8.44)

- This means for constant $T_s$, the total $q_{\text{conv}}$ is proportional to $\bar{h}$ and the log mean temperature difference $\Delta T_{\text{lm}}$. 
In many applications, the temperature of an external fluid, rather than the tube surface temperature, is fixed (Fig. 8.8). The previous formulae can still be applied (with modification adopting the concept of thermal resistance), as indicated by Eqs. (8.45) and (8.46).

\[
\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp \left( - \frac{\overline{UA}_s}{mc_p} \right) \quad (8.45a)
\]

\[
q = \overline{UA}_s \Delta T_{lm} \quad (8.45b)
\]

\[
\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp \left( - \frac{1}{mc_p R_{tot}} \right) \quad (8.46a)
\]

\[
q = \frac{\Delta T_{lm}}{R_{tot}} \quad (8.46b)
\]

**EX 8.3**
8.4 Laminar Flow in Circular Tubes: Thermal Analysis

8.4.1 The Fully Developed Region

For the case of uniform surface heat flux, the exact correlation is

\[ Nu_D \equiv \frac{hD}{k} = 4.36, \quad q_s'' = \text{constant} \]  \hspace{1cm} (8.53)

For the case of constant surface temp., with \((\partial^2 T/\partial x)^2 < (\partial^2 T/\partial r^2)\)

\[ Nu_D = 3.66, \quad T_s = \text{constant} \]  \hspace{1cm} (8.55)

**EXs. 8.4, 8.5**
8.4.2 The Entry Region

Thermal entry length problem $\rightarrow$ (8.56)

Combined entry length problem $\rightarrow$ (8.57)

Figure 3.10  Results obtained from entry length solutions for laminar flow in a circular tube: (a) local Nusselt numbers and (b) average Nusselt numbers [2].
Fully Developed Pipe Flow

For $Pr < 1$

- entrance region
- fully-developed region

$\frac{f}{f_0} = rac{4}{Re} + \frac{4}{Pr} \left( \frac{4}{Pr} \right)^{1/2}$

$\frac{h}{h_0} = 0.4 \left( \frac{4}{Pr} \right)^{1/2}$

$\frac{L_1}{d} = 0.5 (Pr)^{1/2}$

$\frac{L_2}{d} = 0.31 (Pr)^{1/2}$
8.5 Convection Correlations: Turbulent Flow in Circular Tubes

Using the modified Reynolds analogy, there are (8.58)-(8.60). (Note the exponent of $Re_D$ is 4/5).
For higher accuracies, use (8.61)-(8.62).

**EX 8.6**

8.6 Convection Correlations: Noncircular Tubes and the Concentric Tube Annulus

Hydraulic diameter: $D_h \equiv \frac{4A_c}{P}$ (8.66)

It is this diameter that should be used in calculating parameters such as $Re_D$ and $Nu_D$.

- For **turbulent flow**, it is reasonable to use the correlations of Section 8.5 for $Pr > 0.7$. However, the coefficient is presumed to be an average over the perimeter.

- For **laminar flow**, the use of circular tube correlations is less accurate, particularly with cross sections characterized by sharp corners. For laminar fully developed conditions: use Table 8.1
Fully Developed Laminar Flow in Noncircular Tubes

Hydraulic diameter:

\[ D_h \equiv \frac{4 A_c}{P} \]

\[ Nu = \frac{hD_h}{k} = \text{constant} \]

\[ f \ \text{Re}_{D_h} = \text{constant} \]

\[ \therefore \text{as } D_h \downarrow \]

\[ \rightarrow h \text{ and } \Delta P \uparrow \]

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8.7 Heat Transfer Enhancement

Heat transfer enhancement may be achieved by increasing the convection coefficient (inducing turbulence or secondary flow) and/or by increasing the convection surface area. See Figs. 8.12 and 8.13

To induce secondary flow or turbulence

**Figure 8.12** Internal flow heat transfer enhancement schemes: (a) longitudinal section and end view of coil-spring wire insert, (b) longitudinal section and cross-sectional view of twisted tape insert, (c) cut-away section and end view of longitudinal fins, and (d) longitudinal section and end view of helical ribs.

**Figure 8.13** Schematic of helically coiled tube and secondary flow in enlarged cross-sectional view.
8.8 Microscale Internal Flow

Many new technologies involve microscale internal flow with $D_h \leq 100\mu m$.

- For gases, the results of Chapters 6 through 8 are not expected to apply when $D_h / \lambda_{mfp} \leq 10$.
- For liquids, some features of previous results, e.g., friction factor (8.19), pressure drop (8.22a), and transition criterion (8.2) remain applicable.

8.9 Convection Mass Transfer

Omitted.