Chapter 8

Internal Forced Convection

8.1 Hydrodynamic Considerations

8.1.1 Flow Conditions

may be determined experimentally, as shown in Figs. 7.1-7.2.

$$Re_D \equiv \frac{\rho u_m D}{\mu} \qquad (8.1)$$

where u_m is the *mean fluid velocity* over the tube cross section.

The critical Re_D corresponding to the onset of turbulence is

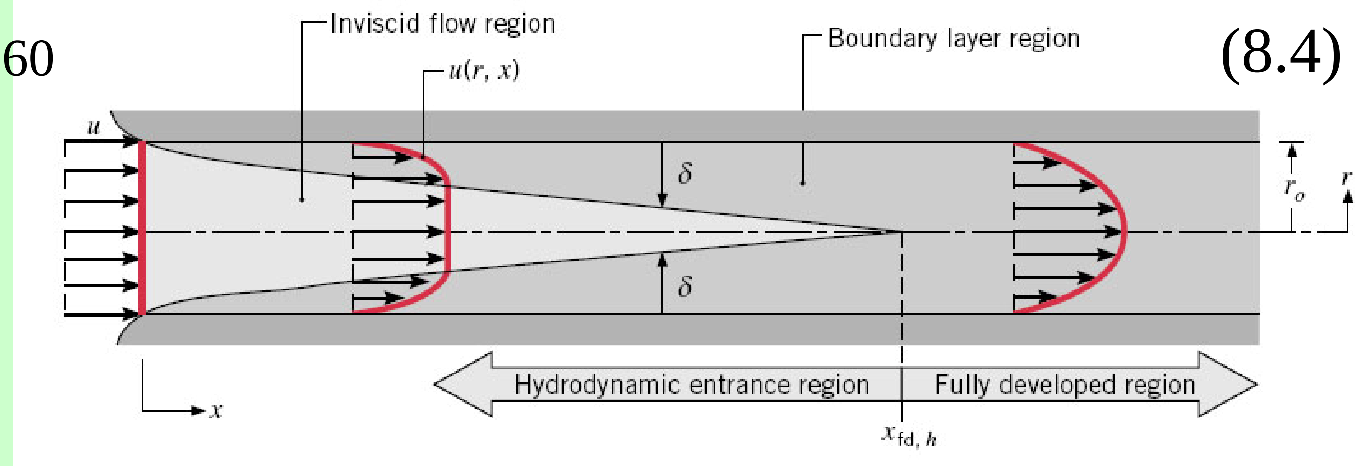
$$Re_{D,c} \approx 2300 \qquad (8.2)$$

The **entrance length** for laminar flow is

$$\left(\frac{x_{fd,h}}{D}\right)_{\text{lam}} \approx 0.05\,\text{Re}_D \qquad (8.3)$$

The entrance length for turbulent flow is approximately indep. of *Re*

$$10 \le \left(\frac{x_{fd,h}}{D}\right)_{\text{turb}} \le 60 \qquad (8.4)$$



8.1.2 The mean Velocity

$$u_m = \frac{\int_{A_c} \rho u(r,x) dA_c}{\rho A_c} = \frac{2}{r_0^2}\int_0^{r_0} u(r,x) r dr \qquad (8.8)$$

8.1.3 Velocity Profile in the Fully Developed Region

The velocity profile of a fully developed laminar flow is parabolic as shown in (8.13)-(8.15).

$$u(r) = -\frac{1}{4\mu}\left(\frac{dp}{dx}\right) r_0^2 \left[1 - \left(\frac{r}{r_0}\right)^2\right] \qquad (8.13)$$

8.1.4 Pressure Gradient and Friction Factor in Fully Developed Flow

Friction factor for laminar flow:

$$f = \frac{64}{Re_D} \qquad (8.19)$$

Friction factor for turbulent flow: (8.20) or (8.21) or Fig. 8.3.

Pressure drop:

$$\Delta p = -\int_{p_1}^{p_2} dp = f\frac{\rho u_m^2}{2D}\int_{x_1}^{x_2} dx = f\frac{\rho u_m^2}{2D}(x_2 - x_1) \qquad (8.22a)$$

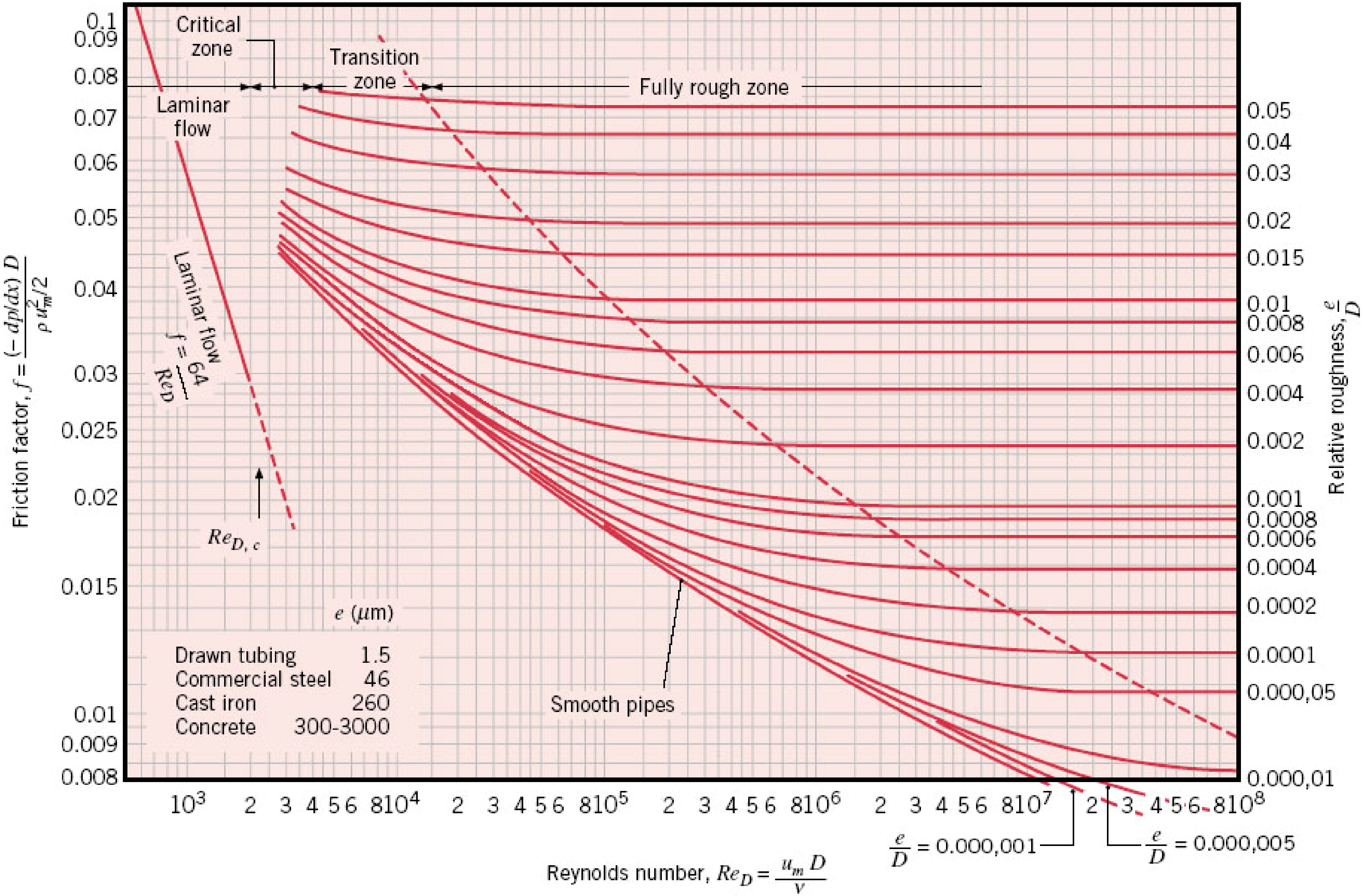


FIGURE 8.3 Friction factor for fully developed flow in a circular tube [3]. Used with permission.

8.2 Thermal Considerations

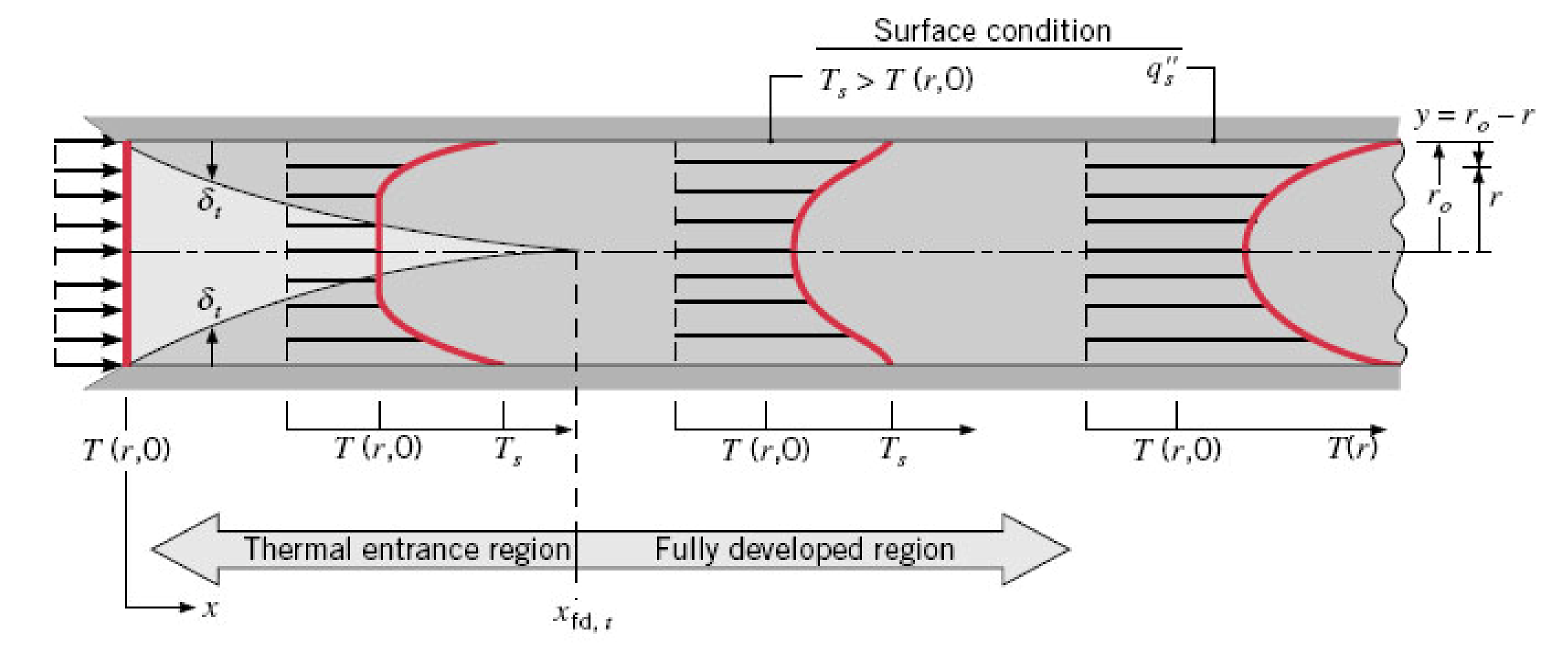


FIGURE 8.4 Thermal boundary layer development in a heated circular tube.

The thermal entrance length for laminar flow is

$$\left(\frac{x_{fd,t}}{D}\right)_{lam} \approx 0.05 Re_D Pr \qquad (8.23)$$

The thermal entrance length for turbulent flow is

$$\left(\frac{x_{fd,t}}{D}\right)_{turb} \approx 10 \qquad (8.24)$$

8.2.1 The Mean Temperature

$$T_m = \frac{\int_{A_c} \rho u c_p T dA_c}{\dot{m} c_p} \qquad (8.25)$$

For incompressible flow in a circular tube with constant c_p,

$$T_m = \frac{2}{u_m r_0^2} \int_0^{r_0} uTrdr \qquad (8.26)$$

8.2.2 Newton's Law of Cooling

$$q_s'' = h(T_s - T_m) \qquad (8.27)$$

where h is the *local* convection heat transfer coefficient.

- This relation applies for different wall conditions, e.g., $T_s = C$ or $q'' = C$.
- Since T_m usually varies along the tube, the variation of T_m must be accounted for.

8.2.3 Fully Developed Conditions

The *thermally fully developed* condition is when the *relative* shape of the profile no longer changes and is stated as

$$\frac{\partial}{\partial x}\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]_{\mathrm{fd},t}=0 \qquad (8.28)$$

for cases with either a *uniform surface heat flux* or *a* *uniform surface temperature*. Since the term in the bracket of (8.29) is independent of *x*, its *r*-derivative is also independent of *x*. So,

$$\frac{\partial}{\partial r}\left(\frac{T_s-T}{T_s-T_m}\right)\bigg|_{r=r_0}=\frac{-\partial T/\partial r|_{r=r_0}}{T_s-T_m}=-\frac{h}{k}\neq f(x) \qquad (8.29)$$

Hence, in the *thermally fully developed flow* of a fluid with constant properties, the *local convection coefficient is a constant*. (Fig. 8.5)

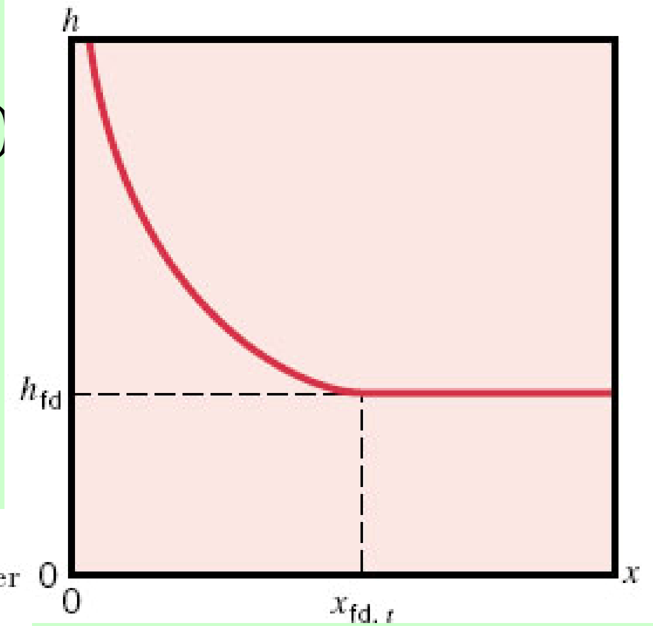


FIGURE 8.5
Axial variation of the convection heat transfer coefficient for flow in a tube.

Eq. (8.28) implies

$$\left.\frac{\partial T}{\partial x}\right|_{\mathrm{fd},t} = \left.\frac{\partial T_m}{\partial x}\right|_{\mathrm{fd},t} \quad \text{or} \quad \left.\frac{\partial (T - T_m)}{\partial x}\right|_{\mathrm{fd},t} = 0, \quad q_s^{''} = \text{constant} \qquad (8.32)$$

$$\left.\frac{\partial T}{\partial x}\right|_{\mathrm{fd},t} = \frac{(T_s - T)}{(T_s - T_m)} \left.\frac{\partial T_m}{\partial x}\right|_{\mathrm{fd},t}, \quad T_s = \text{constant} \qquad (8.33)$$

$T_m(x)$ is a very important variable for internal flows and may be obtained by applying an overall energy balance to the flow.

EX 8.1

8.3 The Energy Balance

8.3.1 General Considerations

Energy conservation for a differential control volume of Fig. 8.6 leads to

$$dq_{\mathrm{conv}} = \dot{m} c_p dT_m \qquad (8.36)$$

or $$\frac{dT_m}{dx} = \frac{q_s^{''} P}{\dot{m} c_p} = \frac{P}{\dot{m} c_p} h (T_s - T_m) \qquad (8.37)$$

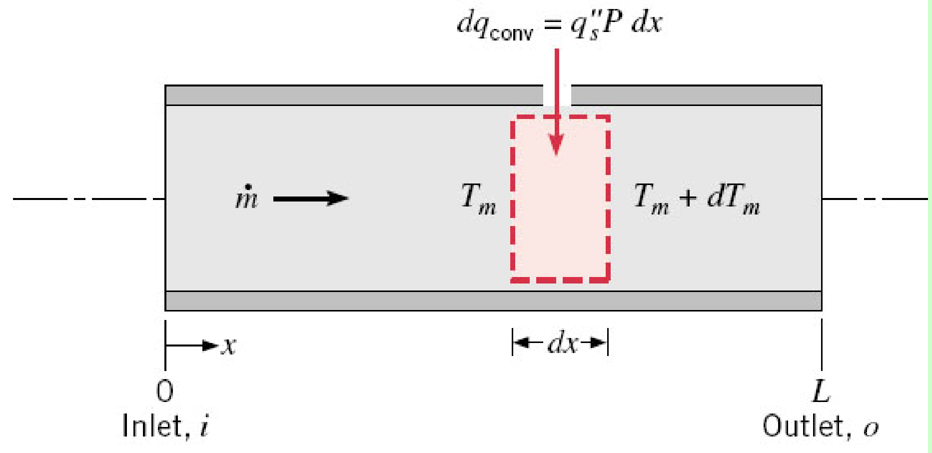


FIGURE 8.6 Control volume for internal flow in a tube.

8.3.2 Constant Surface Heat Flux

$$T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m} c_p} x, \quad q_s'' = \text{constant} \qquad (8.40)$$

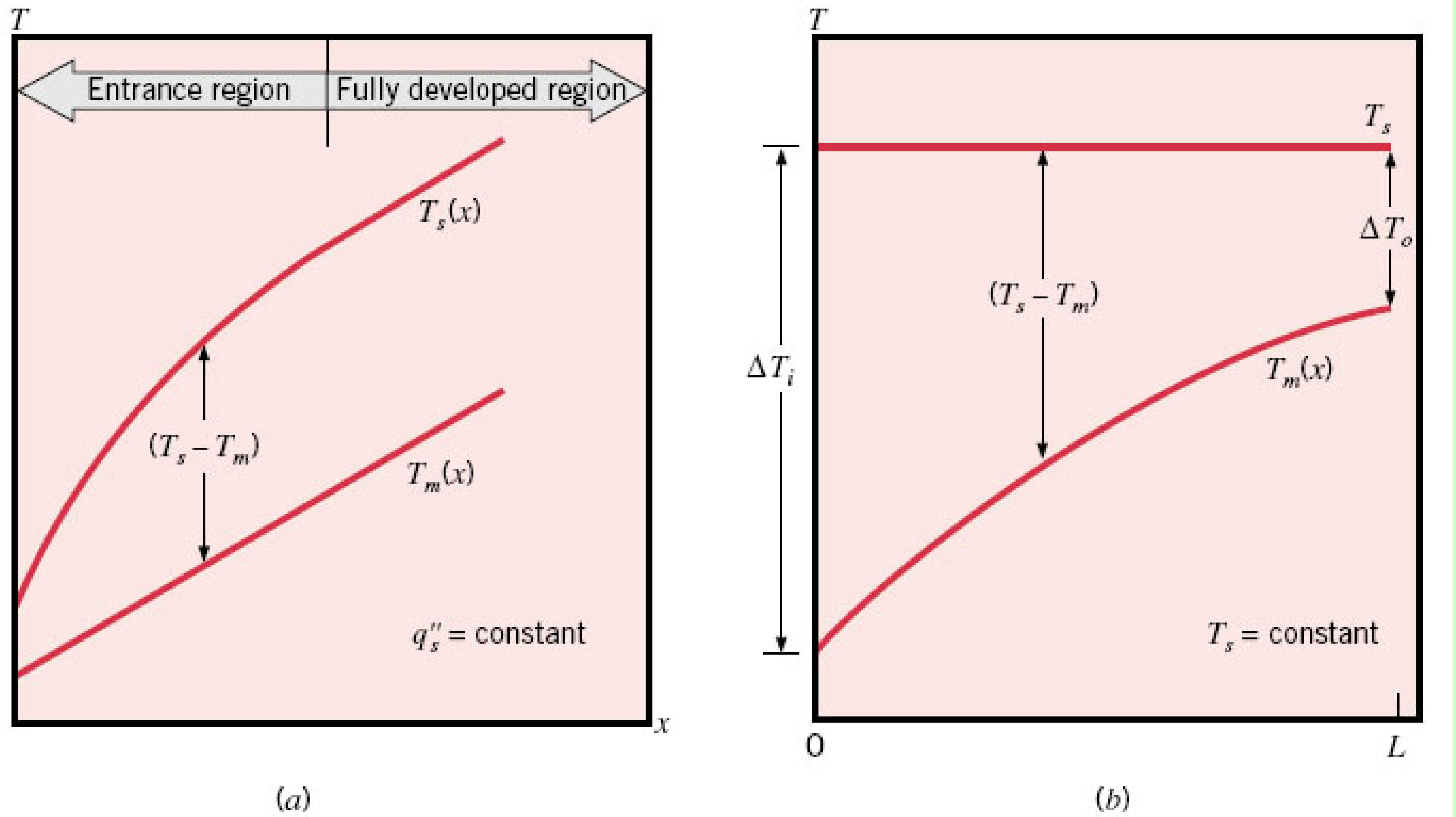


FIGURE 8.7 Axial temperature variations for heat transfer in a tube. (*a*) Constant surface heat flux. (*b*) Constant surface temperature.

EX. 8.2

8.3.3 Constant Surface Temperature

Define $\Delta T = T_s - T_m$, Eq. 8.37 becomes

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h\Delta T$$

Integration leads to

$$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\left(\int_0^L \frac{1}{L} h dx\right) = -\frac{PL}{\dot{m}c_p}\bar{h}_L \qquad (8.41a)$$

Then, $$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}\right), \quad T_s = \text{constant} \qquad (8.42)$$

From energy balance,

$$q_{\text{conv}} = \dot{m}c_p(T_{m,o} - T_{m,i}) = \dot{m}c_p[(T_s - T_{m,i}) - (T_s - T_{m,o})] = \dot{m}c_p(\Delta T_i - \Delta T_o)$$

Substituting for $\dot{m}c_p$ from (8.41a), we obtain

$$q_{\text{conv}} = \bar{h}A_s\Delta T_{\text{lm}}, \quad \Delta T_{\text{lm}} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}, \quad T_s = \text{constant} \qquad (8.44)$$

- This means for constant T_s, the total q_{conv} is proportional to $\bar{h}$ and the **log mean temperature difference ΔT_{lm}**.

- In many applications, the *temperature of an external fluid*, rather than the tube surface temperature, *is fixed* (Fig. 8.8). The previous formulae can still be applied (with modification adopting the concept of thermal resistance), as indicated by Eqs. (8.45) and (8.46).

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\bar{U}A_s}{\dot{m}c_p}\right) \quad (8.45a)$$

$$q = \bar{U}A_s \Delta T_{\text{lm}} \quad (8.45b)$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{1}{\dot{m}c_p R_{\text{tot}}}\right) \quad (8.46a)$$

$$q = \frac{\Delta T_{\text{lm}}}{R_{\text{tot}}} \quad (8.46b)$$

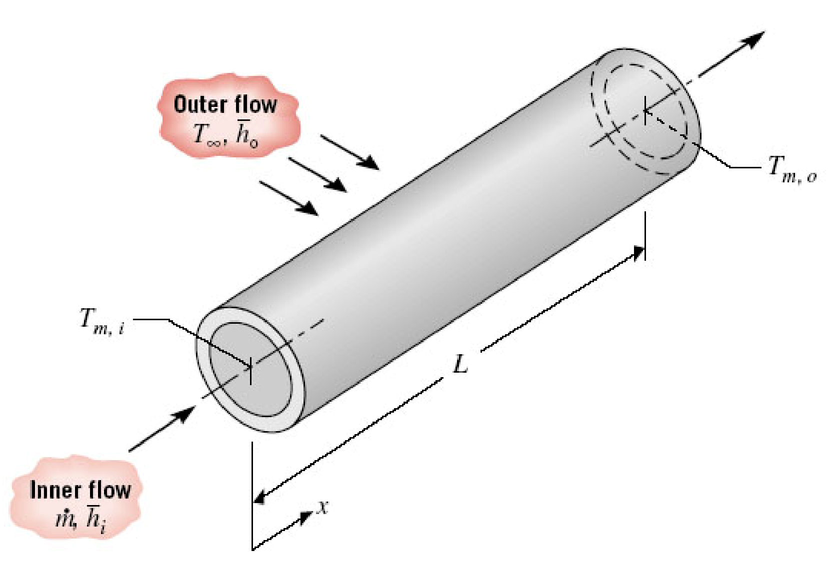


FIGURE 8.8 Heat transfer between fluid flowing over a tube and fluid passing through the tube.

EX 8.3

8.4 Laminar Flow in Circular Tubes: Thermal Analysis

8.4.1 The Fully Developed Region

For the case of uniform surface heat flux, the exact correlation is

$$Nu_D \equiv \frac{hD}{k} = 4.36, \quad q_s'' = \text{constant} \tag{8.53}$$

For the case of constant surface temp., with $(\partial^2 T/\partial^2 x) << (\partial^2 T/\partial^2 r)$

$$Nu_D = 3.66, \quad T_s = \text{constant} \tag{8.55}$$

EXs. 8.4, 8.5

8.4.2 The Entry Region

Thermal entry length problem → (8.56)

Combined entry length problem → (8.57)

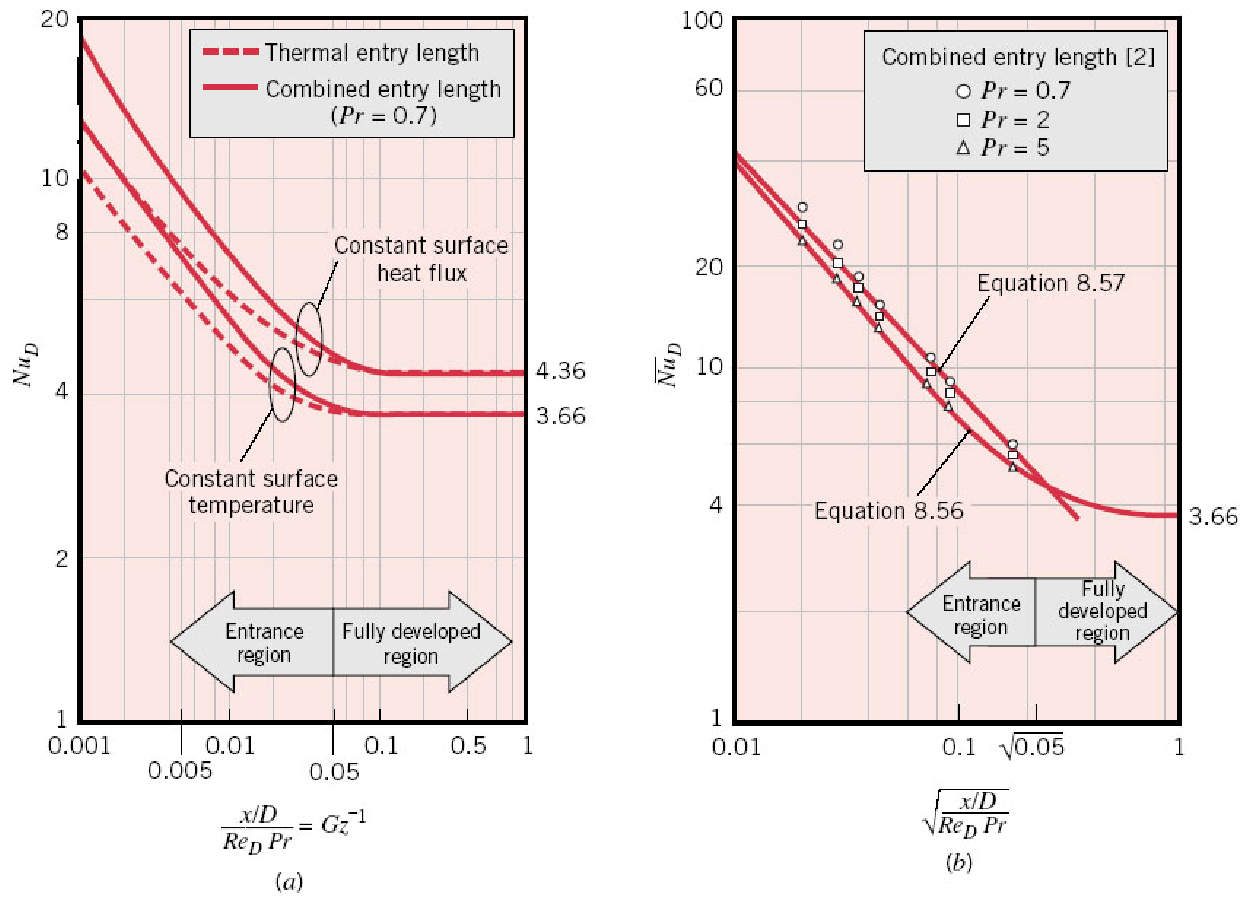
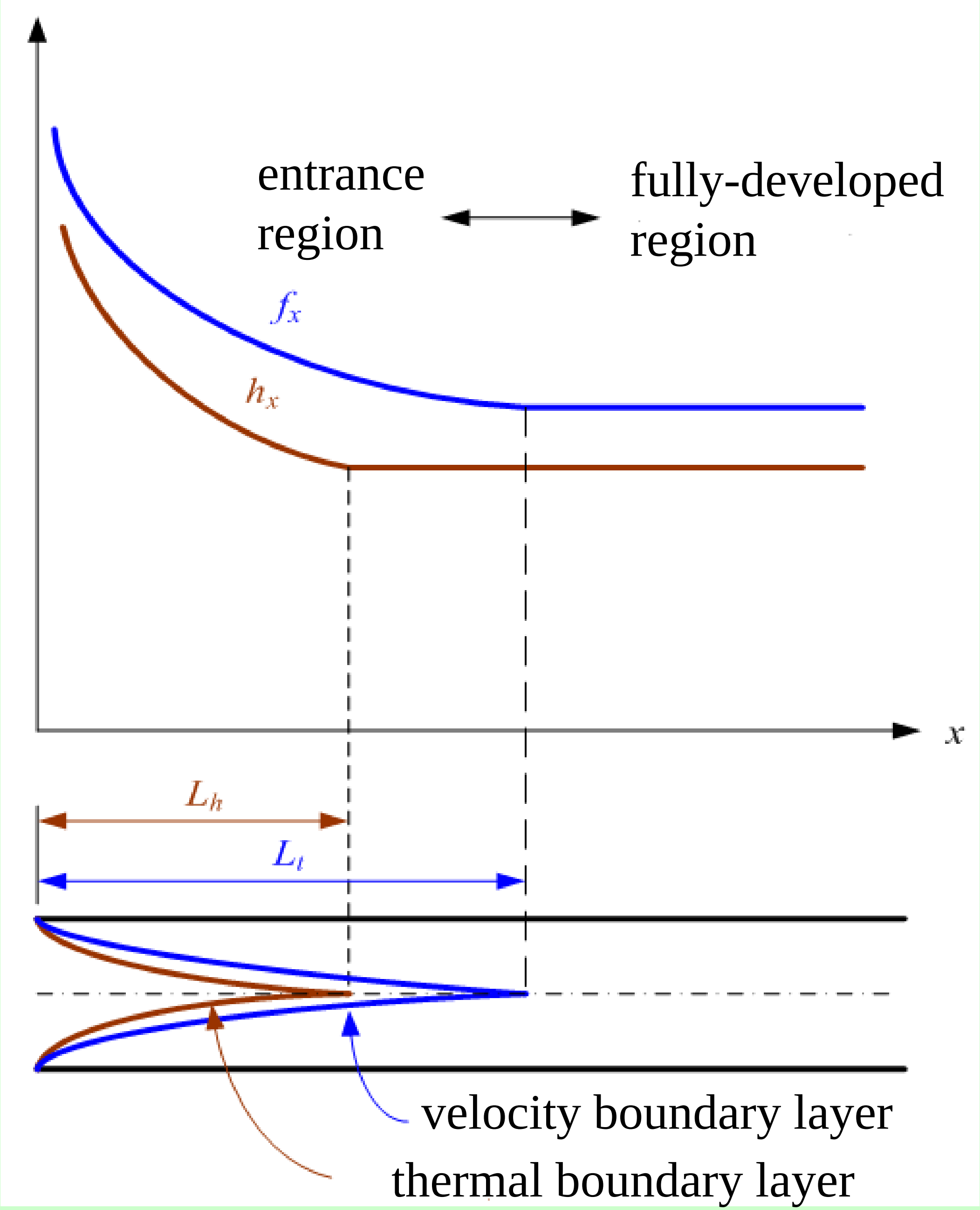


FIGURE 8.10 Results obtained from entry length solutions for laminar flow in a circular tube: (a) local Nusselt numbers and (b) average Nusselt numbers [2].

Fully Developed Pipe Flow

For $Pr < 1$



8.5 Convection Correlations: Turbulent Flow in Circular Tubes

Using the modified Reynolds analogy, there are (8.58)-(8.60).

(Note the exponent of Re_D is 4/5).

For higher accuracies, use (8.61)-(8.62).

EX 8.6

8.6 Convection Correlations: Noncircular Tubes and the Concentric Tube Annulus

Hydraulic diameter: $D_h \equiv \frac{4A_c}{P}$ (8.66)

It is this diameter that should be used in calculating parameters such as Re_D and Nu_D.

- For turbulent flow, it is reasonable to use the correlations of Section 8.5 for $Pr > 0.7$. However, the coefficient is presumed to be an average over the perimeter.
- For laminar flow, the use of circular tube correlations is less accurate, particularly with cross sections characterized by sharp corners. For laminar fully developed conditions: use Table 8.1

Fully Developed Laminar Flow in Noncircular Tubes

Hydraulic diameter:

$$D_h \equiv \frac{4A_c}{P}$$

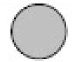
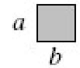
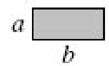
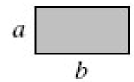
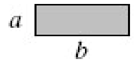
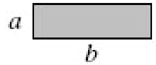
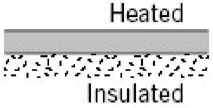
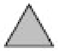
$$Nu = \frac{hD_h}{k} = \text{constant}$$

$$f\,\mathrm{Re}_{D_h} = \text{constant}$$

$\therefore$ as $D_h \downarrow$

$\rightarrow$ *h and* $\Delta P \uparrow$

TABLE 8.1 Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q_s'')	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	8.0	6.49	5.60	82
	∞	8.23	7.54	96
	∞	5.39	4.86	96
	—	3.11	2.49	53

Used with permission from W. M. Kays and M. E. Crawford, *Convection Heat and Mass Transfer*, 3rd ed. McGraw-Hill, New York, 1993.

8.7 Heat Transfer Enhancement

Heat transfer enhancement may be achieved by increasing the convection coefficient (inducing turbulence or secondary flow) and/or by increasing the convection surface area. See Figs. 8.12 and 8.13

To induce secondary flow or turbulence

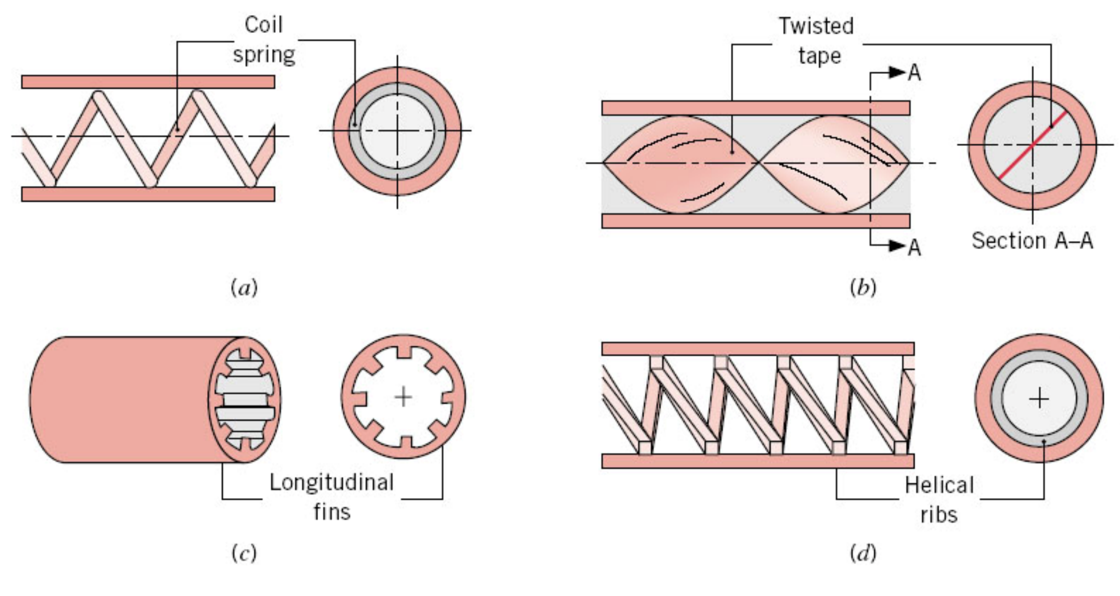


FIGURE 8.12 Internal flow heat transfer enhancement schemes: (a) longitudinal section and end view of coil-spring wire insert, (b) longitudinal section and cross-sectional view of twisted tape insert, (c) cut-away section and end view of longitudinal fins, and (d) longitudinal section and end view of helical ribs.

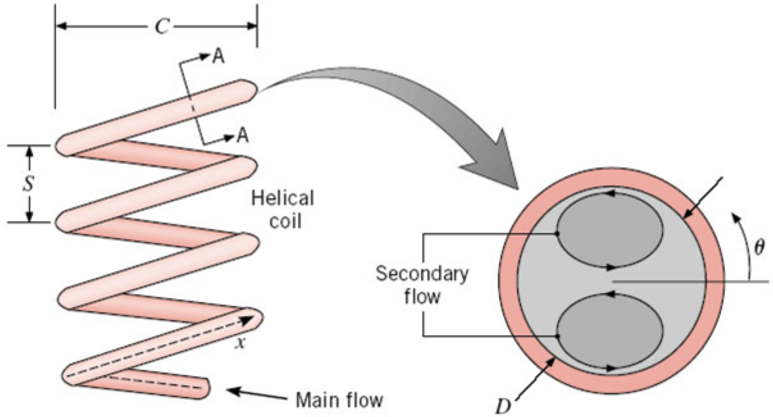


FIGURE 8.13 Schematic of helically coiled tube and secondary flow in enlarged cross-sectional view.

8.8 Microscale Internal Flow

Many new technologies involve microscale internal flow with $D_h \leqq 100\mu m$.

- For gases, the results of Chapters 6 through 8 are not expected to apply when $D_h / \lambda_{mfp} \leqq 10$.
- For liquids, some features of previous results, e.g., friction factor (8.19), pressure drop (8.22a), and transition criterion (8.2) remain applicable.

8.9 Convection Mass Transfer

Omitted.