Chapter 7

External Forced Convection
In this course, discussion is limited to low speed, forced convection, with no phase change within the fluid.

7.1 The Empirical Method

\[ Nu_x = f(x^*, Re_x, Pr) \]  
\[ \overline{Nu_x} = f(Re_x, Pr) \]

The empirical correlation of the form

\[ \overline{Nu}_L = CRe_L^m Pr^n \]

may be determined experimentally, as shown in Figs. 7.1-7.2.

For fixed \( Pr \)
\[ \log \overline{Nu}_L = \log C + m \log Re_L \]

For fixed \( Re_L \)
\[ \log \overline{Nu}_L = \log C + n \log Pr \]

**Figure 7.2** Dimensionless representation of convection heat transfer measurements.
Note: To account for the effect of non-uniform temperature on fluid properties, there are two methods:
(1) To evaluate fluid properties at the *film temperature* $T_f = \frac{T_s + T_\infty}{2}$

$$\overline{N_u}_L = CRe_L^m Pr^n$$ (7.1)

$$\overline{S_h}_L = CRe_L^m Sc^n$$ (7.3)

(2) To evaluate all properties at and to correct with a parameter of the form $(Pr_\infty/Pr_s)^r$ or $(\mu_\infty/\mu_s)^r$. 
7.2 The Flat Plate in Parallel Flow
7.2.1 Laminar Flow Over an Isothermal Plate: A Similarity Solution

Read the textbook pp. 405-410.

With stream function $\psi(x, y)$ and the transformation of $\eta, f(\eta)$ defined in (7.9), (7.10), the continuity equation (7.4) together with the steady-state momentum equation (7.5) can be reduced to a single ordinary differential equation (7.17), subjected to BCs (7.18). The numerical solution is shown in Table 7.1. The boundary layer thickness $\delta$ and the local friction coefficient $C_{f,x}$ can be determined as (7.19) and (7.20).

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$f$</th>
<th>$\frac{df}{d\eta} = \frac{u}{u_\infty}$</th>
<th>$\frac{d^2f}{d\eta^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.332</td>
</tr>
<tr>
<td>0.4</td>
<td>0.027</td>
<td>0.133</td>
<td>0.331</td>
</tr>
<tr>
<td>0.8</td>
<td>0.106</td>
<td>0.265</td>
<td>0.327</td>
</tr>
<tr>
<td>1.2</td>
<td>0.238</td>
<td>0.394</td>
<td>0.317</td>
</tr>
<tr>
<td>1.6</td>
<td>0.420</td>
<td>0.517</td>
<td>0.297</td>
</tr>
<tr>
<td>2.0</td>
<td>0.650</td>
<td>0.630</td>
<td>0.267</td>
</tr>
<tr>
<td>2.4</td>
<td>0.922</td>
<td>0.729</td>
<td>0.228</td>
</tr>
<tr>
<td>2.8</td>
<td>1.231</td>
<td>0.812</td>
<td>0.184</td>
</tr>
<tr>
<td>3.2</td>
<td>1.569</td>
<td>0.876</td>
<td>0.139</td>
</tr>
<tr>
<td>3.6</td>
<td>1.930</td>
<td>0.923</td>
<td>0.098</td>
</tr>
<tr>
<td>4.0</td>
<td>2.306</td>
<td>0.956</td>
<td>0.064</td>
</tr>
<tr>
<td>4.4</td>
<td>2.692</td>
<td>0.976</td>
<td>0.039</td>
</tr>
<tr>
<td>4.8</td>
<td>3.085</td>
<td>0.988</td>
<td>0.022</td>
</tr>
<tr>
<td>5.2</td>
<td>3.482</td>
<td>0.994</td>
<td>0.011</td>
</tr>
<tr>
<td>5.6</td>
<td>3.880</td>
<td>0.997</td>
<td>0.005</td>
</tr>
<tr>
<td>6.0</td>
<td>4.280</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>6.4</td>
<td>4.679</td>
<td>1.000</td>
<td>0.001</td>
</tr>
<tr>
<td>6.8</td>
<td>5.079</td>
<td>1.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Similarly, the energy equation can be reduced to (7.21), subjected to BCs (7.22). Numerical integration leads to

\[
\frac{dT}{d\eta}\bigg|_{\eta=0} = 0.332Pr^{1/3}
\]

and

\[
Nu_x \equiv \frac{h_x x}{k} = 0.332Re_x^{1/2} Pr^{1/3} \quad Pr \geq 0.6
\] (7.23)

From the solution of (7.21), it also follows that \( \frac{\delta}{\delta_t} \approx Pr^{1/3} \) (7.24)

The average heat transfer coefficient is

\[
\bar{h}_x = \frac{1}{x} \int_0^x h_x \, dx = \ldots = 2h_x
\]

Hence,

\[
\bar{Nu}_x \equiv \frac{\bar{h}_x x}{k} = 0.664Re_x^{1/2} Pr^{1/3} \quad Pr \geq 0.6
\] (7.30)

Similarly,

\[
\bar{Sh}_x \equiv \frac{\bar{h}_{m,x} x}{D_{AB}} = 0.664Re_x^{1/2} Sc^{1/3} \quad Sc \geq 0.6
\] (7.31)

- For small \( Pr \), namely liquid metals, \( \delta_t \gg \delta \), we may assume \( u= u_\infty \) throughout the thermal boundary layer and obtain (7.32).
- For all \( Pr \) numbers: (7.33)
7.2.2 Turbulent Flow over an Isothermal Plate

From experiment, it is known
\[ C_{f,x} = 0.0592Re_x^{-1/5} \quad Re_x \leq 10^7 \] (7.34)

Moreover,
\[ \delta = 0.37xRe_x^{-1/5} \] (7.35)

For turbulent flow,
\[ \delta \approx \delta_t \approx \delta_c \]

Using (7.35) with the modified Reynolds analogy,
\[ Nu_x = St \ Re_x \ Pr = 0.0296Re_x^{4/5} \ Pr^{1/3} \quad 0.6 < Pr < 60 \] (7.36)
\[ Sh_x = St_m \ Re_x \ Sc = 0.0296Re_x^{4/5} \ Sc^{1/3} \quad 0.6 < Sc < 3000 \] (7.37)

Enhanced mixing causes the turbulent boundary layer to grow more rapidly than the laminar boundary layer (\( \delta \) varies as \( x^{4/5} \) in contrast to \( x^{1/2} \) for laminar flow) and to have larger friction and convection coefficients.
7.2.3 Mixed boundary Layer Conditions

When transition occurs sufficiently upstream of the trailing edge, \((x_c/L) \leq 0.95\) (Fig. 7.3), both the laminar and the boundary layers should be considered.

General expressions:

\[
\overline{Nu}_L = (0.037R_{eL}^{4/5} - A)Pr^{1/3} \quad \begin{cases} 
0.6 \leq Pr \leq 60 \\
Re_{x,c} \leq Re_L \leq 10^8 
\end{cases} \tag{7.38}
\]

\[
A = 0.037Re_{x,c}^{4/5} - 0.664Re_{x,c}^{1/2} \tag{7.39}
\]

\[
\overline{C}_{f,L} = 0.074R_{eL}^{-1/5} - \frac{2A}{Re_L} \quad \begin{cases} 
Re_{x,c} \leq Re_L \leq 10^8 
\end{cases} \tag{7.40}
\]

\[
\overline{Sh}_L = (0.037R_{eL}^{4/5} - A)Sc^{1/3} \quad \begin{cases} 
0.6 \leq Sc \leq 60 \\
Re_{x,c} \leq Re_L \leq 10^8 
\end{cases} \tag{7.41}
\]
7.2.4 Unheated Starting Length

For flat plate in parallel flow with *unheated starting length* (Fig. 7.4):

Laminar flow: \[ Nu_x = \frac{Nu_x \big|_{\xi=0}}{\left[1 - \left(\frac{\xi}{x}\right)^{3/4}\right]^{1/3}} \] (7.42)

turbulent flow: \[ Nu_x = \frac{Nu_x \big|_{\xi=0}}{\left[1 - \left(\frac{\xi}{x}\right)^{9/10}\right]^{1/9}} \] (7.43)
7.2.5 Flat Plates with Constant Heat Flux Conditions

For flat plate with a uniform surface heat flux:

laminar flow: \( \frac{h_x x}{k} = 0.453 \Re_x^{1/2} \Pr^{1/3}, \quad \Pr \geq 0.6 \) \hspace{1cm} (7.45)

\( \Leftrightarrow \) in comparison with (7.23)

\[
\frac{q_x''}{h_x} = 0.680 \Re_L^{1/2} \Pr^{1/3}
\]

turbulent flow: \( \Nu_x = St \Re_x \Pr = 0.0308 \Re_x^{4/5} \Pr^{1/3}, \quad 0.6 < \Pr < 60 \) \hspace{1cm} (7.46)

\( \Leftrightarrow \) in comparison with (7.36)

In this case, \( T_s(x) = T_\infty + \frac{q_x''}{h_x} \) is varying. The local surface temperature is

\[
\frac{(T_s - T_\infty)}{L} = \frac{1}{L} \int_0^L (T_s - T_\infty) dx = \frac{q_x''}{L} \int_0^L \frac{x}{k \Nu_x} dx
\]

\[
= \frac{q_x'' L}{k \Nu_L}
\]

where \( \Nu_L = 0.680 \Re_L^{1/2} \Pr^{1/3} \) \hspace{1cm} (7.49)

**Note:** Any of the \( \Nu_L \) results obtained for a uniform surface temperature may be used with Eq. 7.48 to evaluate \( (T_s - T_\infty) \).
7.2.6 Limitations on Use of Convection Coefficients

Errors as large as 25% may be incurred by using the expressions due to varying free stream turbulence and surface roughness.

7.3 Methodology for a Convection Calculation

Follow the six steps listed in the textbook.
1. Become immediately cognizant of the flow geometry.
2. Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature.
3. In mass transfer problems the pertinent fluid properties are those of species B.
4. Calculate the Reynolds number.
5. Decide whether a local or surface average coefficient is required.
6. Select the appropriate correlation.

EXs 7.1-7.3
7.4 The Cylinder in Cross Flow

Boundary layer *separation* may occur due to the adverse pressure gradient \((dp/dx > 0)\). Boundary layer transition, from laminar b. l. to turbulent b. l., depends on \(Re_D (\equiv \rho VD/\mu)\).

Drag coeff. \(C_D \rightarrow\) Fig. 7.8
Figure 7.6  Velocity profile associated with separation on a circular cylinder in cross flow.

Figure 7.7  The effect of turbulence on separation.
**Figure 7.8** Drag coefficients for a smooth circular cylinder in cross flow and for a sphere [2]. Boundary layer separation angles are for a cylinder. Adapted with permission.
7.4.2 Convection Heat and Mass Transfer

The local $Nu_\theta \rightarrow$ Fig. 7.9

The average (of more engineering interest):

\[
\overline{Nu_D} \equiv \frac{\bar{h}D}{k} = C\left(\frac{Re_D}{Pr}\right)^{1/3} \quad \text{--empirical}
\]  

(7.52)

Constants for circular cylinders: Table 7.2

<table>
<thead>
<tr>
<th>(Re_D)</th>
<th>(C)</th>
<th>(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4–4</td>
<td>0.989</td>
<td>0.330</td>
</tr>
<tr>
<td>4–40</td>
<td>0.911</td>
<td>0.385</td>
</tr>
<tr>
<td>40–4000</td>
<td>0.683</td>
<td>0.466</td>
</tr>
<tr>
<td>4000–40,000</td>
<td>0.193</td>
<td>0.618</td>
</tr>
<tr>
<td>40,000–400,000</td>
<td>0.027</td>
<td>0.805</td>
</tr>
</tbody>
</table>
\[
\frac{Nu_D}{k} \equiv \frac{\overline{h}D}{k} = CRe_D^{m}Pr^{1/3}
\]  \hspace{1cm} (7.52)

Constants for noncircular cylinders: Table 7.3

<table>
<thead>
<tr>
<th>Geometry</th>
<th>( Re_D )</th>
<th>( C )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>( 5 \times 10^{3} - 10^{5} )</td>
<td>0.246</td>
<td>0.588</td>
</tr>
<tr>
<td>( V \rightarrow \quad \square )</td>
<td>( \overline{D} )</td>
<td>( \overline{D} )</td>
<td>( \overline{D} )</td>
</tr>
<tr>
<td>Hexagon</td>
<td>( 5 \times 10^{3} - 1.95 \times 10^{4} )</td>
<td>0.160</td>
<td>0.638</td>
</tr>
<tr>
<td>( V \rightarrow \quad \hexagon )</td>
<td>( \overline{D} )</td>
<td>( \overline{D} )</td>
<td>( \overline{D} )</td>
</tr>
<tr>
<td>( V \rightarrow \quad \hexagon )</td>
<td>( 1.95 \times 10^{4} - 10^{5} )</td>
<td>0.0385</td>
<td>0.782</td>
</tr>
<tr>
<td>Vertical plate</td>
<td>( 5 \times 10^{3} - 10^{5} )</td>
<td>0.153</td>
<td>0.638</td>
</tr>
<tr>
<td>( V \rightarrow \quad \square )</td>
<td>( \overline{D} )</td>
<td>( \overline{D} )</td>
<td>( \overline{D} )</td>
</tr>
<tr>
<td>( V \rightarrow \quad \square )</td>
<td>( 4 \times 10^{3} - 1.5 \times 10^{4} )</td>
<td>0.228</td>
<td>0.731</td>
</tr>
</tbody>
</table>

Other correlations are shown in (7.53), (7.54).

EX 7.4
7.5 The Sphere

Whitaker:

\[ Nu_D = 2 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.4} \left( \frac{\mu}{\mu_s} \right)^{1/4} \]  (7.56)

\[
\begin{bmatrix}
0.71 \leq Pr \leq 380 \\
3.5 \leq Re_D \leq 7.6 \times 10^4 \\
1.0 \leq (\mu / \mu_s) \leq 3.2
\end{bmatrix}
\]

- For liquid drops the Ranz and Marshall correlation (7.57). More accurate modifications for it are also available.

\[ Nu_D = 2 + 0.6Re_D^{1/2} Pr^{1/3} \]  (7.57)

EX 7.6
7.6 Flow across Banks of Tubes (brief introduction)

**Figure 7.10** Schematic of a tube bank in cross flow.

**Figure 7.12** Flow conditions for (a) aligned and (b) staggered tubes.
Aligned or staggered: Fig. 7.11 and Fig. 7.12

A number of correlations for $\overline{Nu}_D$ are given in (7.58)-(7.65), with constants listed in Tables 7.5-7.8.

The $h$ for a tube in the first row is approximately equal to that for a single tube in cross flow, whereas larger heat transfer coefficients are associated with tubes of the inner rows. Mostly, the convection coefficient stabilizes for a tube beyond the fourth or fifth row.

In general, heat transfer enhancement is favored by the more tortuous flow of a staggered arrangement, particularly for small $Re_D$ (<100).

Since the fluid temperature may change over the tube bank, the heat transfer rate may be significantly over-predicted by using $\Delta T = T_s - T_\infty$. The appropriate form of $\Delta T$ is a log-mean temperature difference $\Delta T_{lm}$, as shown in (7.66). And

$$q' = N(\bar{h} \pi D \Delta T_{lm}) \quad (7.68)$$

The pressure drop (7.69)
7.7 Impinging Jets (brief introduction)

The impinging jet is a simple and effective way of cooling. The compressed boundary layer near the central stagnation point causes effective cooling of the surface. It is widely adopted, in combination of cooling fins, in CPU cooling for desktop PCs and other types of computers.

**Figure 7.15** Surface impingement of a single round or slot gas jet.

**Figure 7.16** Surface impingement of an array of slot jets.
Heat Transfer Enhancement

- Boundary layer regrowth
- Flow transition or turbulence

Air Flow
Heat Transfer Enhancement

- Laminar boundary layer
- Wake region
- Flow
- Plain fin - continuous fin
- Interrupted surface
Heat Transfer Enhancement

a. Rectangular
b. Triangular
c. Wavy
d. Offset Strip Fin
e. Perforated
f. Louvered