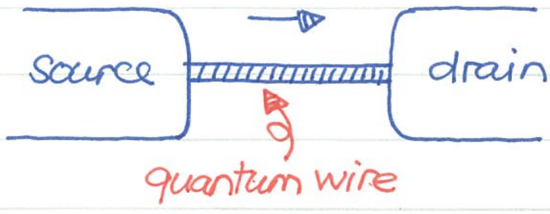
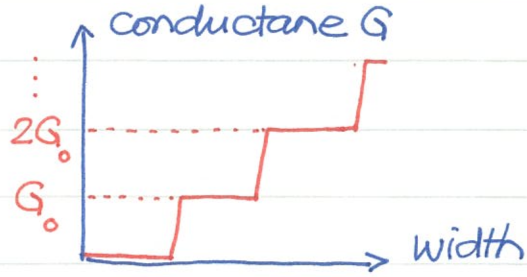
HH0052 Quantum Transport

Transport properties are rather different in quantum regime. For instance, the conductane of a clean quantum wire is quantized in units of G_0,



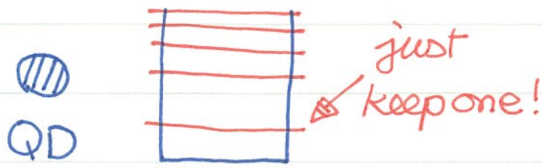
$$\boxed{G_0 = \frac{e^2}{h} = (25.8\,k\Omega)^{-1}}$$



By changing the width of the wire, the conductance increases in steps. How can we understand this surprising phenomena?

⊘ One-level quantum dot:

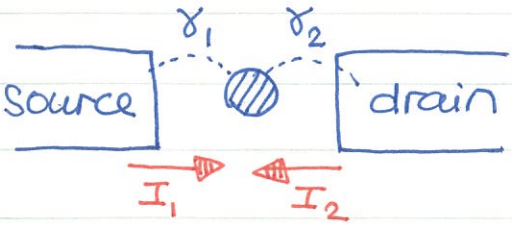
Let us start with a simple system: quantum dot. To simplify the problem, just keep one energy level $E = \varepsilon$. The source and drain are described by Fermi-Dirac dist.,

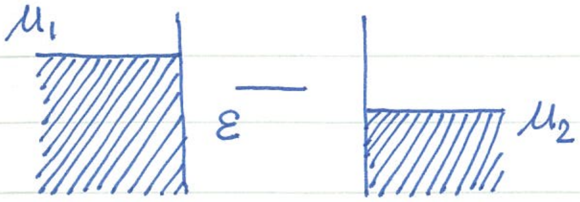


$$\boxed{\begin{aligned} f_1(E) &= \frac{1}{e^{(E-\mu_1)/\tau}+1} \\ f_2(E) &= \frac{1}{e^{(E-\mu_2)/\tau}+1} \end{aligned}}$$

① $\mu_1 > \mu_2$

② $\mu_1 - \mu_2 = q V_{sd}$





The tunneling rates between the dot and the leads are γ_1 and γ_2. We can now write down the current ∴

$$I_1 = q\gamma_1 \left[f_1(1-N) - N(1-f_1) \right] \longrightarrow \boxed{I_1 = q\gamma_1 \left[f_1(\varepsilon) - N \right]}$$

Similarly, the current from drain into the quantum dot is

$$I_2 = q\gamma_2 \left[f_2(1-N) - N(1-f_2) \right] \longrightarrow \boxed{I_2 = q\gamma_2 \left[f_2(\varepsilon) - N \right]}$$

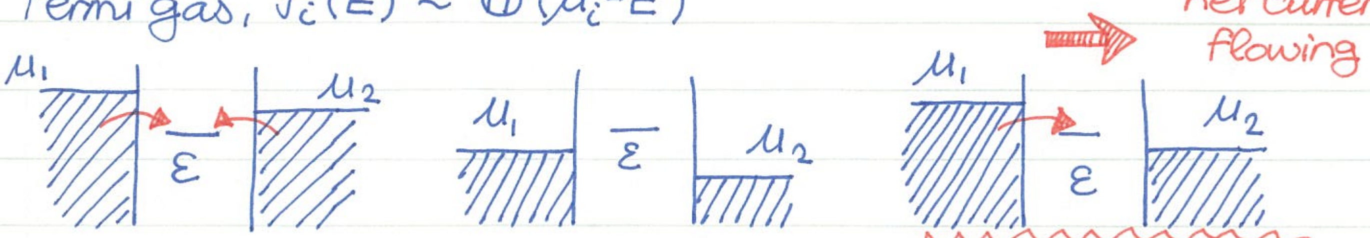
In steady state, the charge on QD is constant $\rightarrow$ $I_1 + I_2 = 0$

One can then solve for the number N on QD.

$$\boxed{N = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}} \rightarrow \boxed{I = I_1 = -I_2 = q\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left[f_1(\varepsilon) - f_2(\varepsilon)\right]}$$

Let's try to understand the tunneling current better. For degenerate Fermi gas, $f_i(E) \approx \Theta(\mu_i - E)$

net current flowing



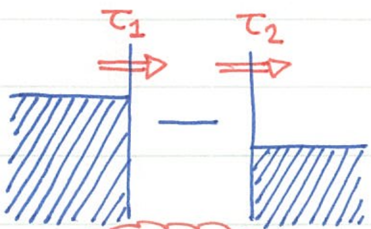
$f_1(\varepsilon)=1,\ f_2(\varepsilon)=1$ — no current!

$f_1(\varepsilon)=0,\ f_2(\varepsilon)=0$ — no current!

$f_1(\varepsilon)=1,\ f_2(\varepsilon)=0$

$$I = q\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} = q\,\frac{1}{\left(\frac{1}{\gamma_1}\right)+\left(\frac{1}{\gamma_2}\right)}$$

We can understand the current in the case $f_1(\varepsilon)=1$ & $f_2(\varepsilon)=0$ in the following picture:



$\tau = \tau_1 + \tau_2$

The transport consists two steps:

(Source $\rightarrow$ QD takes time) τ_1

+(QD $\rightarrow$ drain takes time) τ_2

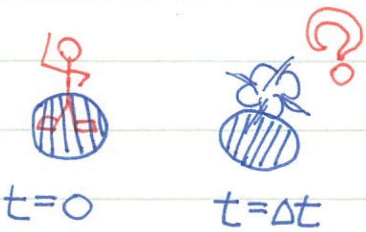
total time is $\tau = \tau_1 + \tau_2$

$$I = \frac{q}{\tau} = \frac{q}{\tau_1+\tau_2} = q\,\frac{1}{\frac{1}{\gamma_1}+\frac{1}{\gamma_2}} = q\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}$$

Very nice and simple picture ☺

There is only one problem about the above calculations — <u>the result is complete WRONG</u> ...

⊘ Uncertainty principle: The main problem is the quantum dot is not a closed system. According to uncertainty principle, $\Delta E \Delta t \sim \hbar$



$$\rightarrow \boxed{\Delta E \sim \frac{\hbar}{\Delta t} = \hbar\gamma}$$

uncertainty in energy.

We cannot say the energy is ε any more.

Instead, we should use a probability distribution $P(E)$ to describe the single level.

$$\boxed{P(E)\,dE = \text{probability to find the level in } (E, E+dE)}$$

$$\int dE\, P(E) = 1$$

After some advanced calculations, the probability density is

$$\boxed{P(E) = \frac{1}{\pi}\frac{(\hbar\gamma/2)}{(E-\varepsilon)^2+(\hbar\gamma/2)^2}} \xrightarrow{\gamma\to 0} P(E) = \delta(E-\varepsilon)$$

Some of you may recognize that the probability density $P(E)$ is just a special case of our good old friend: $D(E)$. Yes, the density of states —

$$\boxed{D(E)\,dE = \text{number of states in } (E, E+dE)} \qquad \int dE\, D(E) = \text{number of ALL states.}$$

It's clear that $P(E) = D(E)$ with (number of all states = 1).

◎ Re-calculate the tunneling current: Since the energy level is not fixed at $E=\varepsilon$, the tunneling current now becomes

$$\boxed{I = q\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\int dE\, D(E)\left[f_1(E)-f_2(E)\right]}$$

Chemical potentials $\mu_1, \mu_2 = \varepsilon \pm \frac{1}{2} q V_{sd}$ ☺

For degenerate Fermi gas, $f_i(E) \approx \Theta(\mu_i - E)$

$$\int dE\, D(E)\left[f_1(E)-f_2(E)\right] = \int_{-\infty}^{\mu_1} dE\, D(E) - \int_{-\infty}^{\mu_2} dE\, D(E) = \int_{\mu_2}^{\mu_1} dE\, D(E)$$

$$\rightarrow \boxed{I = q\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\int_{\mu_2}^{\mu_1} dE\, D(E)}$$

The conductance $G = dI/dV_{sd}$ can be computed,

$$\left.\frac{dI}{dV_{sd}}\right|_{V_{sd}=0} = q\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\cdot\left[D(\varepsilon)\cdot\tfrac{1}{2}q - D(\varepsilon)\left(-\tfrac{1}{2}q\right)\right] = q^2 D(\varepsilon)\,\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}$$

Note that the density of states takes the Lorentzian shape and its value at resonance $E=\varepsilon$ is

$$D(\varepsilon) = \frac{1}{\pi}\frac{(\cancel{\hbar\gamma/2})}{(\varepsilon-\varepsilon)^2+(\hbar\gamma/2)^{\cancel{2}}} = \frac{2}{\pi\hbar\gamma} = \frac{4}{h(\gamma_1+\gamma_2)}$$

$\gamma = \gamma_1+\gamma_2$ ☺

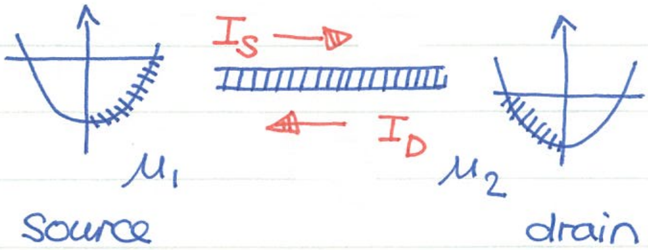
Substitute into the expression for the conductance,

$$G = q^2 D(\varepsilon) \frac{\gamma_1 \gamma_2}{\gamma_1+\gamma_2} \longrightarrow \boxed{G = \frac{q^2}{h} \frac{4\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2} \leq \frac{q^2}{h}}$$

The maximum conductance occurs at the symmetrical point $\gamma_1 = \gamma_2$,

$G_{max} = G_0 = q^2/h$! ⟵ The conductance has an upper limit (G_0) and does not go to infinity!

⊘ 1D wire : Now we are ready to compute the 1D wire case.



$$I_S = q\int_0^\infty \frac{dp}{2\pi\hbar} f_1(E)\cdot v$$

$$= \frac{q}{2\pi\hbar}\int_0^\infty dE\, f_1(E)\cdot\left(\frac{dp}{dE}\cdot v\right)$$

1 !

Similarly, $I_D = q\int_{-\infty}^{0} \frac{dp}{2\pi\hbar} f_2(E)\cdot v = -\frac{q}{2\pi\hbar}\int_0^\infty dE\, f_2(E)$

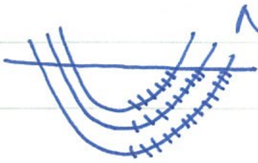
Total current $I = I_S + I_D = \frac{q}{2\pi\hbar}\int_0^\infty dE\,(f_1 - f_2) = \frac{q}{2\pi\hbar}(\mu_1-\mu_2)$

Making use of the relation $\mu_1 - \mu_2 = q V_{sd}$,

$$I = \frac{q^2}{2\pi\hbar} V_{sd} = \frac{q^2}{h} V_{sd} \longrightarrow \boxed{G = G_0 = \frac{q^2}{h}}$$

One conducting channel, one $G_0 = q^2/h$!

Generalization to multiple bands is trivial — just count bands.



N=3 (number of active bands) $I = N\cdot(q^2/h)\cdot V_{sd}$.

⟶ $G = NG_0$ explains the conductance steps.

Note that no impurity scattering is included in the above at all.

清大東院
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