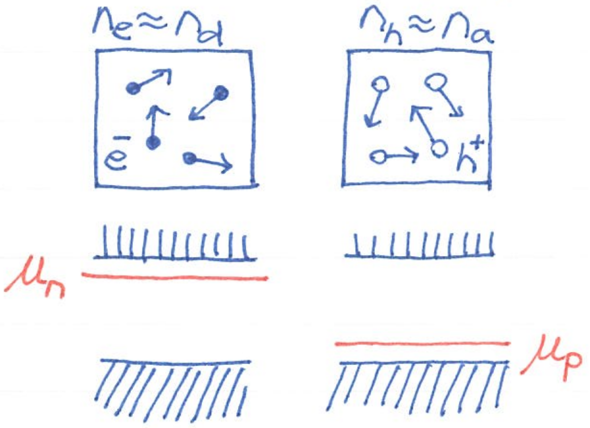
HH0042 p-n Junction

When a semiconductor is NOT uniformly doped, interesting transport properties show up and are of crucial importance for device applications. Here we just demonstrate the simple p-n junction by putting p-type and n-type semiconductors together with sharp interface.

① **Qualitative understanding:** Consider two pieces of extrinsic semiconductors: p-type and n-type. As discussed before, the chemical potentials μ_n, μ_p may not be the same. For simplicity, we assume complete ionization so that

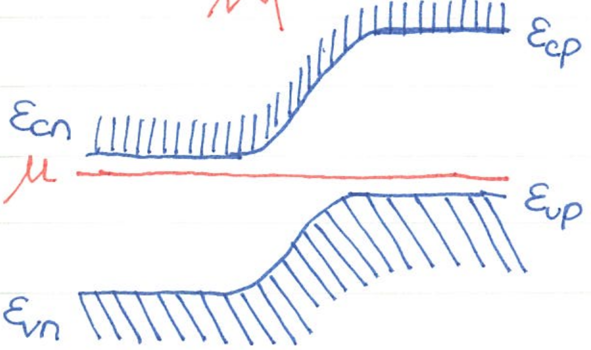


$$\boxed{n_e \approx n_d^+ \approx n_d \ (n\text{-type})}$$ electrons are majority.

$$\boxed{n_h \approx n_a^- \approx n_a \ (p\text{-type})}$$ holes are majority.

After carrier migration, there is just ONE chemical potential. Thus, one can guess the bands are shifted....

$$\boxed{\begin{array}{l} \mathcal{E}_c(x) = \mathcal{E}_c(-\infty) - e\varphi(x) \\ \text{with } \varphi(-\infty) = 0 \text{ and } \mathcal{E}_c(-\infty) = \mathcal{E}_{cn} \end{array}}$$

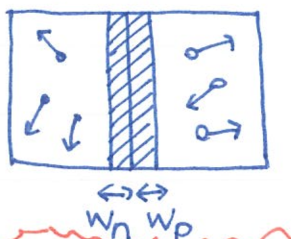


From the figure on the left, it's clear that $\mathcal{E}_c(\infty) = \mathcal{E}_{cp}$

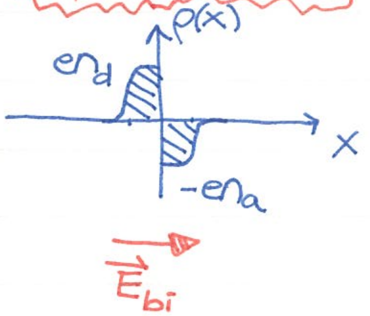
For the valence band edge, $\boxed{\mathcal{E}_v(x) = \mathcal{E}_v(-\infty) - e\varphi(x)}$ where $\mathcal{E}_v(-\infty) = \mathcal{E}_{vn}$

Similarly, it's easy to see that $\mathcal{E}_v(\infty) = \mathcal{E}_{vp}$. The band gap is assumed to be robust $\mathcal{E}_g \equiv \mathcal{E}_c(x) - \mathcal{E}_v(x)$ and a build-in voltage $eV_{bi} = \mathcal{E}_{cp} - \mathcal{E}_{cn} = \mathcal{E}_{vp} - \mathcal{E}_{vn}$ is present due to charge migration ☺

Simple picture



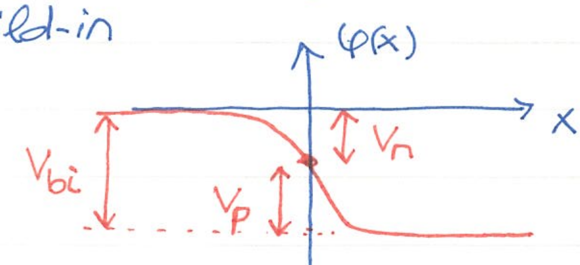
depletion zone



Suppose the interface is at the origin $x=0$. The holes in the p-type migrate over the interface and annihilate with electrons. Similarly, the electrons in the n-type migrate and annihilate with holes on the other side. $\to$ formation of depletion zone! Inside the depletion zone, there is almost no mobile electrons and holes

$$\to \boxed{\rho(x) \approx \begin{cases} en_d & \text{on the n-type side} \\ -en_a & \text{on the p-type side} \end{cases}}$$

The charges give rise to a build-in electric field $\vec{E}_{bi}$, pointing from n-type to p-type and stopping further charge migration from both sides. The build-in electric field generates a potential profile $\varphi(x)$ as shown on the right. $\varphi(-\infty) \equiv 0$ as the zero for the electrostatic potential.



The profile agrees with our previous guess ☺

Ⓘ Finding build-in voltage: We assume the extrinsic S.C. is in the regime satisfying

(1) $n_i \ll n_d, n_a \ll n_c, n_v$ ← "classical"

(2) $n_e \approx n_d$, $n_h \approx n_a$ ← fully ionized!

So, starting from the n-type side, $n_e = n_c e^{-(\varepsilon_{cn}-\mu)/\tau} \longrightarrow \varepsilon_{cn} = \mu + \tau \log\left(\frac{n_c}{n_e}\right)$

Making use of $n_e \simeq n_d \longrightarrow \boxed{\varepsilon_{cn} = \mu + \tau \log(n_c/n_d)}$

Now turn to the p-type side.

$n_h = n_v e^{-(\mu-\varepsilon_{vp})/\tau} \longrightarrow \boxed{\varepsilon_{vp} = \mu - \log\left(\frac{n_v}{n_h}\right) = \mu - \log\left(\frac{n_v}{n_a}\right)}$

Thus, $\varepsilon_{cn} - \varepsilon_{vp} = \tau \log\left(\frac{n_c n_v}{n_d n_a}\right)$ we are almost there ☺

From the band-bending diagram, $\varepsilon_{vp} = \varepsilon_{vn} + eV_{bi}$

$\varepsilon_{cn} - \varepsilon_{vp} = \varepsilon_{cn} - \varepsilon_{vn} - eV_{bi} = \varepsilon_g - eV_{bi}$, ε_g is the band gap.

Finally, it is straightforward to solve for the build-in voltage.

$$\boxed{eV_{bi} = \varepsilon_g - \tau \log\left(\frac{n_c n_v}{n_d n_a}\right) < \varepsilon_g}$$

Note that eV_{bi} is of the same order of ε_g, but slightly smaller.

② The tougher part – finding $\varphi(x)$: To find the profile of the electrostatic potential, one needs to solve the Poisson eq. self-consistently

$$\boxed{\frac{d^2\varphi}{dx^2} = -\frac{1}{\epsilon}\rho}$$

$\rho = \rho[\varphi]$ depends on $\varphi(x)$ ..... tough ☹///

Again, one needs to carry out the calculations on two sides. On the n-type side, $\rho(x) = e[n_d - n_e(x)]$. w/ $n_e(x) = n_c e^{-[\varepsilon_c(x)-\mu]/\tau}$

$$\rightarrow n_e(x) = \underbrace{n_c e^{-(\varepsilon_{cn}-\mu)/\tau}}_{= n_e(-\infty) \approx n_d} e^{e\varphi(x)/\tau} = n_d e^{e\varphi(x)/\tau}$$

depending on $\varphi(x)$!

Now we can write down the Poisson eq. on the n-type side,

$$\boxed{\frac{d^2\varphi}{dx^2} = -\frac{en_d}{\epsilon}\left[1 - e^{e\varphi(x)/\tau}\right] < 0}$$

Must be solved self consistently!

On the p-type side, $n_h(x) = n_v e^{-[\mu - \varepsilon_v(x)]/\tau}$ w/ $\varepsilon_v(x) = \varepsilon_v(-\infty) - e\varphi(x)$

Because $\varepsilon_v(-\infty) = \varepsilon_{vn} = \varepsilon_{vp} - eV_{bi}$, the hole concentration becomes

$$n_h(x) = n_v e^{-(\mu - \varepsilon_{vp})/\tau} e^{-e[\varphi(x) + V_{bi}]} \approx n_a e^{-e[\varphi(x)+V_{bi}]}$$

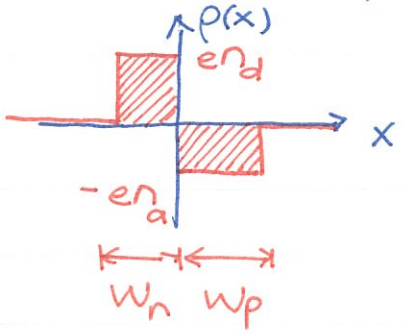
Following similar steps, the Poisson eq. now takes the form,

$$\boxed{\frac{d^2\varphi}{dx^2} = \frac{en_a}{\epsilon}\left[1 - e^{-e[\varphi(x)+V_{bi}]}\right] > 0}$$

$\frac{d^2\varphi}{dx^2}$ has opposite signs

Without computers, it is very messy to solve the above equations self consistently. Thus, we are going to simplify them .... ☺

Assume the depletion zone has the step-like charge distribution.



The approximation actually makes sense. On the n-type size, $\rho(x) = en_d\left[1 - e^{e\varphi(x)/\tau}\right]$

$$\rightarrow \boxed{\rho(x) \cong \begin{cases} en_d, & -W_n < x < 0 \\ 0, & x < -W_n \end{cases}}$$

B.C.: $\varphi(-\infty) = 0$

The Poisson equation in the regime $-W_n < x < 0$ is

$$\frac{d^2\varphi_<}{dx^2} = -\frac{en_d}{\epsilon}$$

SO SIMPLE !!

$$\boxed{\varphi_<(x) = \begin{cases} -\frac{1}{2}\frac{en_d}{\epsilon}(x+W_n)^2 & -W_n < x < 0 \\ 0 & x < -W_n \end{cases}}$$

On the p-type side, $\rho(x) = -en_a\left[1 - e^{-e\varphi(x) - eV_{bi}}\right]$

$$\rightarrow \rho(x) \cong \begin{cases} -en_a, & 0 < x < W_p \\ 0, & x > W_p \end{cases}$$

The Poisson equation also takes a very simple form in the depletion zone $0 < x < W_p$:

$$\frac{d^2\varphi_>}{dx^2} = \frac{en_a}{\epsilon}$$

SIMPLE

$$\boxed{\varphi_>(x) = \begin{cases} \frac{1}{2}\frac{en_a}{\epsilon}(x-W_p)^2 - V_{bi}, & 0 < x < W_p \\ -V_{bi}, & x > W_p \end{cases}}$$

B.C.: $\varphi(\infty) = -V_{bi}$

But, we are NOT done yet.....

The depletion width W_n, W_p can be solved by matching B.C. at the interface $x=0$.

(1) $\varphi_<(0) = \varphi_>(0)$ $\qquad -\frac{1}{2}\frac{en_d}{\epsilon}W_n^2 = \frac{1}{2}\frac{en_a}{\epsilon}W_p^2 - V_{bi}$

$$\rightarrow \boxed{\frac{1}{2}\frac{en_d}{\epsilon}W_n^2 + \frac{1}{2}\frac{en_a}{\epsilon}W_p^2 = V_{bi}}$$

$\leftarrow$ 1st equation for (W_n, W_p).

(2) $\frac{d\varphi_<}{dx}(0) = \frac{d\varphi_>}{dx}(0)$ because there is no surface charge. So the electric field at the origin is well defined.

$$\boxed{E(0) = -\frac{d\varphi}{dx}(0) = \frac{en_d}{\epsilon}W_n = \frac{en_a}{\epsilon}W_p}$$

$\leftarrow$ 2nd equation for (W_n, W_p).

To compare with the results in Kittel, we can solve $E(0)$ first.

$$\frac{1}{2}\frac{\epsilon}{en_d}\Big(\underbrace{\frac{en_d}{\epsilon}W_n}_{=E(0)}\Big)^2+\frac{1}{2}\frac{\epsilon}{en_a}\Big(\underbrace{\frac{en_a}{\epsilon}W_p}_{=E(0)}\Big)^2=V_{bi}$$

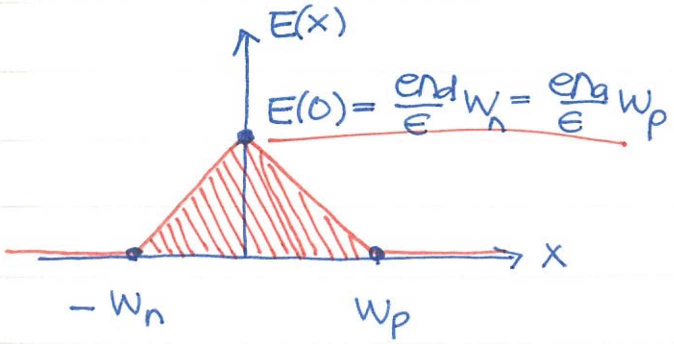
One can find the electric field at the interface $x=0$:

$$\frac{\epsilon}{2e}\left(\frac{1}{n_d}+\frac{1}{n_a}\right)E^2(0)=V_{bi}\ \longrightarrow\ \boxed{E(0)=\sqrt{\frac{2e}{\epsilon}\frac{n_a n_d}{n_a+n_d}V_{bi}}}$$

The above result is basically the same as in Kittel's textbook. Now you know the physical picture behind the approximation made in the textbook. Finding the depletion length W_n, W_p is now quite trivial.

$$\boxed{W_n=\frac{\epsilon}{en_d}E(0)\ ,\quad W_p=\frac{\epsilon}{en_a}E(0)}$$

I shall not bore you to write down the expressions explicitly. It's helpful to plot $E(x)$ within this approximation,



The area under the $E(x)$ curve is the build-in voltage

$$\frac{1}{2}\frac{en_d}{\epsilon}W_n^2+\frac{1}{2}\frac{en_a}{\epsilon}W_p^2=V_{bi}$$

The linear dependence of $E(x)$ explains why the depletion length $W_n, W_p \propto \sqrt{V_{bi}}$ as derived previously.

2012.0501

清大東院